Quadratic MIMO Contrast Functions for Blind Source Separation in a Convolutive Context

Saloua Rhioui[1], Marc Castella[2], and Eric Moreau[1]

[1] STD, ISITV, av. G. Pompidou, BP56,
F-83162 La Valette du Var, Cedex, France
{rhioui, moreau}@univ-tln.fr
[2] GET/INT, Département CITI, UMR-CNRS 5157,
9 rue Charles Fourier, 91011 Évry Cedex, France
marc.castella@int-evry.fr

Abstract. This paper considers the problem of blind separation of sources mixed by a MIMO convolutive system. For both i.i.d. and non i.i.d. sources, quadratic separation criteria previously designed for the extraction of a single source are extended to parallel extraction in the MIMO case. These criteria are based on the use of so-called reference signals and a condition is given under which we obtain MIMO contrast functions. Simulations demonstrate that a particular choice of a set of reference signals ensures the contrast property. The performance offered by these criteria is investigated through simulations: it is shown that the proposed contrast functions avoid accumulation errors, contrary to deflation methods.

1 Introduction

We consider the problem of blind equalization of Linear Time Invariant (LTI) Multi-Input / Multi-Output (MIMO) systems. Such a problem is of interest e.g. in multi-user wireless communications where observed signals have to be equalized both in space and time in order to eliminate both intersymbol and cochannel interferences. These interferences are due to possible delays introduced by multi-path propagation and to possible multi-users. Examples are found in Space Division Multiple Access (SDMA) or Code Division Multiple Access (CDMA) communication systems.

Our approach is based on the use of a contrast function [6, 4]. In particular, this has the advantage to yield a sufficient condition for separation. In the context of MIMO systems and parallel extraction of all sources, classical contrast functions generally first require a pre-whitening stage on the observation signals in order to constrain the searched system to be para-unitary [4, 5, 8]. On the other hand, recent solutions have been shown to be very efficient when so-called reference signals are considered, either for equalization of a SISO or SIMO systems [3] or for extraction of one source [1] from a MIMO system. Our main goal in this paper is to propose a generalization of the latter results to the case of parallel extraction in convolutive MIMO systems. In particular notice that our results

J. Rosca et al. (Eds.): ICA 2006, LNCS 3889, pp. 230–237, 2006.

generalize the one in [2] and more importantly that we do not require any pre-whitening stage. The usefulness of such a wide family of criteria is illustrated by computer simulations.

2 Model and Problem Formulation

We consider a Q-dimensional ($Q \in \mathbb{N}, Q \geq 2$) discrete-time signal which is called vector of *observations* and denoted by $\mathbf{x}(n)$ (in the whole paper, n stands for any integer: $n \in \mathbb{Z}$). It results from a linear time invariant (LTI) multichannel system $\{\mathbf{M}\}$ described by the input-output relation:

$$\mathbf{x}(n) = \sum_{k \in \mathbb{Z}} \mathbf{M}(k)\mathbf{s}(n-k) \triangleq \{\mathbf{M}\}\mathbf{s}(n), \tag{1}$$

where $\mathbf{M}(n)$ is the sequence of (Q, N) impulse response matrices and $\mathbf{s}(n)$ is an N-dimensional ($N \in \mathbb{N}^*$) unknown and unobserved column vector, which is referred to as the vector of *sources*.

The multichannel blind deconvolution problem consists in estimating a multivariate LTI system $\{\mathbf{W}\}$ operating on the observations, such that the vector

$$\mathbf{y}(n) = \sum_{k \in \mathbb{Z}} \mathbf{W}(k)\mathbf{x}(n-k) \triangleq \{\mathbf{W}\}\mathbf{x}(n) \tag{2}$$

restores the N input sources. The problem is referred to as the blind source separation (BSS) problem, where blind means that no information is available on the mixing system and that the sources are unobservable. It is useful to define the (N, N) global LTI filter $\{\mathbf{G}\}$ by the following impulse response:

$$\mathbf{G}(n) = \sum_{k \in \mathbb{Z}} \mathbf{W}(k)\mathbf{M}(n-k). \tag{3}$$

We have then:

$$\mathbf{y}(n) = \sum_{k \in \mathbb{Z}} \mathbf{G}(n-k)\mathbf{s}(k) \triangleq \{\mathbf{G}\}\mathbf{s}(n). \tag{4}$$

In order to be able to solve the BSS problem, we have to introduce some assumptions on the source signals. The following one is known to play a key role:

A.1. The source vector components $s_i(n), i \in \{1, \dots, N\}$ are *mutually independent*, stationary and zero-mean processes with unit variance. Their respective covariance functions are denoted by $\gamma_i(k), k \in \mathbb{Z}$ and are positive definite functions (i.e the corresponding spectrum density is positive).

Since the sources are assumed to be unobservable, some inherent indeterminations in their restitution remain: in the general case, their order cannot be restored and each of them is only recovered up to a permutation and a scalar filtering ambiguity. Consequently, the sources are said to be separated when the global transfer matrix $\mathbf{G}(z) \triangleq \sum_k \mathbf{G}(k)z^{-k}$ reads:

$$\mathbf{G}(z) = \mathbf{D}(z)\mathbf{P} \tag{5}$$

where $\mathbf{P}$ is permutation matrix and $\mathbf{D}(z) = \mathrm{Diag}(d_1(z), \ldots, d_N(z))$ is a matrix with scalar filters on its main diagonal. Whenever the sources are assumed to be temporally i.i.d. (independent and identically distributed), the scalar filtering ambiguity reduces to a scaling factor and time delay. In this case, the sources are said to be separated when:

$$\mathbf{G}(z) = \mathbf{D}(z)\mathbf{\Lambda}\mathbf{P} \tag{6}$$

where $\mathbf{\Lambda}$ is a constant diagonal matrix and $\mathbf{D}(z) = \mathrm{Diag}(z^{-l_1}, \ldots, z^{-l_N})$ with $(l_1, \ldots, l_N) \in \mathbb{Z}^N$. Naturally, we assume that the mixing filter is invertible (which implies $Q \geq N$) in the sense that it is possible to obtain (6) or (5).

3 MIMO Separation Criteria

The concept of contrast function has been introduced in BSS so as to reduce the problem to an optimization one: by definition, a contrast function is a criterion which maximization leads to a separating solution. When a pre-whitening procedure has been applied, and under certain conditions, one of the simplest contrast [4, 5] in the context of a MIMO parallel extraction of all sources is given by

$$\mathcal{C}_R\{\mathbf{y}(n)\} \triangleq \sum_{i=1}^{N} |\mathrm{Cum}\{\underbrace{y_i(n), y_i(n), y_i(n), \ldots, y_i(n)}_{R \text{ times, } R \geq 3}\}| \tag{7}$$

where Cum denotes the cumulant. The main contribution of the paper consists in using criteria based on R-th order cross-cumulants, where $R-2$ variables are fixed. This choice yields a quadratic dependence with respect to the optimized parameter, which greatly simplifies the optimization task. We define the following R-th order cumulant, where $R \geq 3$:

$$\kappa_{R,z_i}\{y_i(n)\} = \mathrm{Cum}\{y_i(n), y_i(n), \underbrace{z_i(n), \ldots, z_i(n)}_{R-2 \text{ times}}\} \tag{8}$$

where $z_i(n)$ are given signals to be precisely defined later. In previous works [1], they have been referred to as *reference* signals determined from prior information, but we will see that they may be chosen as observations whitened. We now define the following criterion:

$$\mathcal{C}_{R,\mathbf{z}}\{\mathbf{y}(n)\} \triangleq \sum_{i=1}^{N} |\kappa_{R,z_i}\{y_i(n)\}| \,. \tag{9}$$

This criterion is a MIMO extension of the results in [1]: it will allow the parallel extraction of all sources, contrary to [1] which allows the extraction of the sources one after the other. (9) also generalizes a result in [2].

3.1 Case of i.i.d. Sources

Main result. Since the sources have unit variance, one can restrict the multiplicative factors in (6) to $|\mathbf{\Lambda}| = \mathbf{I}$ by imposing the constraint $\mathrm{E}\{|y_i(n)|^2\} = 1$ for all $i \in \{1, \ldots, N\}$. For i.i.d. sources, this condition also reads:

$$\forall i \in \{1, \ldots, N\}\ , \quad \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |G_{ij}(k)|^2 = 1 \tag{10}$$

We need to define the following supremum:

$$\kappa_{R,i}^{\max} \triangleq \max_{j=1}^{N} \sup_{k \in \mathbb{Z}} |\kappa_{R,z_i}\{s_j(n-k)\}| \tag{11}$$

The proof of Proposition 1 requires the following assumption:

A.2. $\forall i \in \{1, \ldots, N\}$, there exits (j_i, l_i) such that:

$$\kappa_{R,i}^{\max} = |\kappa_{R,z_i}\{s_{j_i}(n - l_i)\}| < +\infty \tag{12}$$

We can then state:

Proposition 1. *In the case of i.i.d. source signals and under the constraint* (10)*, the criterion* $\mathcal{C}_{R,\mathbf{z}}$ *is a contrast function if and only if each set*

$$\mathcal{I}_i \triangleq \{(j,k) \in \{1, \ldots, N\} \times \mathbb{Z} \mid |\kappa_{R,z_i}\{s_j(n-k)\}| = \kappa_{R,i}^{\max}\}, \tag{13}$$

where $i \in \{1, \ldots, N\}$*, contains a single element* (σ_i, k_i)*, where* σ *denotes a permutation in* $\{1, \ldots, N\}$*.*

Proof: We can write: $\kappa_{R,z_i}\{y_i(n)\} = \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} G_{ij}(k)^2 \kappa_{R,z_i}\{s_j(n-k)\}$ and, using (11) and (10), it follows

$$\mathcal{C}_{R,\mathbf{z}}\{\mathbf{y}(n)\} \leq \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |G_{ij}(k)|^2 |\kappa_{R,z_i}\{s_j(n-k)\}| \tag{14}$$

$$\leq \sum_{i=1}^{N} \kappa_{R,i}^{\max} \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |G_{ij}(k)|^2 = \sum_{i=1}^{N} \kappa_{R,i}^{\max} \tag{15}$$

$$\leq \sum_{i=1}^{N} |\kappa_{R,z_i}\{s_{\sigma_i}(n - l_i)\}| = \mathcal{C}_{R,\mathbf{z}}\{\mathbf{s}(n - \mathbf{l})\}. \tag{16}$$

where $\mathbf{s}(n - \mathbf{l}) = (s_{\sigma_1}(n - l_1), \ldots, s_{\sigma_N}(n - l_N))^T$ is a vector of N source signals delayed. If the above upper-bound is reached (which is possible according to assumption A.2), then

$$\sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k \in \mathbb{Z}} |\mathbf{G}_{ij}(k)|^2 \big(\kappa_{R,i}^{\max} - |\kappa_{R,z_i}\{s_j(n-k)\}|\big) = 0. \tag{17}$$

All terms in the above sum being positive, if $\mathcal{I}_i$ contains a single element and σ is a permutation, one deduces, that $\{\mathbf{G}\}$ satisfies the equalization condition (6). Conversely, one can see that if $\mathcal{I}_i$ contains several elements, there exist non separating filters which maximize $\mathcal{C}_{R,\mathbf{z}}$. ■

Comments and alternative results. One should notice that no prewhitening has been required to prove the validity of the contrast $\mathcal{C}_{R,\mathbf{z}}$. This means that the global filter $\{\mathbf{G}\}$ need not be para-unitary for the result to hold but that (10) is sufficient. Incidently, one can notice that if (10) is replaced by: $\forall j \in \{1,\dots,N\} \quad \sum_{i=1}^{N}\sum_{k\in\mathbb{Z}}|G_{ij}(k)|^2 = 1$, a result similar to the one given by Proposition 1 can be proved by changing the roles of i and j. Finally, one should notice also that, although no assumption has been explicitly made on the cumulants of the sources, no source should have vanishing R-th order cumulants in order to satisfy the conditions of Proposition 1.

3.2 Case of Non i.i.d. Sources

Non i.i.d. sources can only be recovered up to a scalar filtering. It is hence natural in this case to work on the scalar filters components which compose the MIMO system. Thanks to the definite positiveness assumed in A.1 we define the following j-norm:

$$\|h\|_j \triangleq \left(\sum_{(k_1,k_2)\in\mathbb{Z}^2} h(k_1)h^*(k_2)\gamma_j(k_2-k_1)\right)^{\frac{1}{2}} \tag{18}$$

Similarly to (10) we will impose a variance constraint on the output, which corresponds to a unit norm constraint on the row filters:

$$\forall i \in \{1,\dots,N\} \quad \sum_{j=1}^{N}\|G_{ij}\|_j^2 = 1 \tag{19}$$

Corresponding to (11) we define $\tilde{\kappa}_{R,i}^{\max} \triangleq \max_{j=1}^{N} \sup_{\|\{\widetilde{G}_{ij}\}\|_j=1} |\kappa_{R,z_i}\{\{\widetilde{G}_{ij}\}s_j(n)\}|$ and corresponding to (12) we assume:

A.3. For all $i \in \{1,\dots,N\}$ there exists $j_i \in \{1,\dots,N\}$ and a filter $h_i^\sharp$ such that:

$$\tilde{\kappa}_{R,i}^{\max} = |\kappa_{R,z_i}\{\{h_i^\sharp\}s_{j_i}(n)\}| < +\infty \tag{20}$$

We can then state:

Proposition 2. *In the case of non i.i.d. sources and under the constraint* (19)*, the criterion $\mathcal{C}_{R,\mathbf{z}}$ is a contrast if and only if each set*

$$\mathcal{I}_i \triangleq \{j \in \{1,\dots,N\} \mid \sup_{\|h\|_j=1} |\kappa_{R,z_i}\{\{h\}s_j(n)\}| = \tilde{\kappa}_{R,i}^{\max}\}, \tag{21}$$

where $i \in \{1,\dots,N\}$, contains a single element σ_i, where σ denotes a permutation in $\{1,\dots,N\}$.

Proof: We have:$y_i(n) = \sum_{j=1}^{N}\{G_{ij}\}s_j(n) = \sum_{j=1}^{N}\|G_{ij}\|_j\{\widetilde{G}_{ij}\}s_j(n)$ where $\{\widetilde{G}_{ij}\}$ is defined by $\{\widetilde{G}_{ij}\} = \{\widetilde{G}_{ij}\}/\|\widetilde{G}_{ij}\|_j$ if $\|\widetilde{G}_{ij}\|_j \neq 0$ and $\{\widetilde{G}_{ij}\} = 0$ otherwise. Now we easily obtain:

$$\mathcal{C}_{R,\mathbf{z}}\{\mathbf{y}(n)\} \le \sum_{i=1}^{N}\sum_{j=1}^{N} \|G_{ij}\|_j^2 |\kappa_{R,z_i}\{\{\widetilde{G}_{ij}\}s_j(n)\}| \le \sum_{i=1}^{N}\sum_{j=1}^{N} \|G_{ij}\|_j^2 \tilde{\kappa}_{R,i}^{\max} \quad (22)$$

and by arguments similar to the proof in the i.i.d. case, one obtains that the above upper-bound is reached if and only if the global filter is separating in the sense of Equation (5). ■

4 Simulations

4.1 Separation Procedure

Our MIMO contrast being quadratic, the optimization can be performed with a similar method to the one presented in [1] for MISO contrasts. The reference signals $z_i(n), i \in \{1,\ldots,n\}$ must be chosen according to A.1. Interestingly, the simulations have clearly demonstrated that it is practically a valid choice to choose them as the output of a prewhitening operation on the observations. This makes our method efficient and competitive compared to other methods.

By the optimization of $\mathcal{C}_{R,\mathbf{z}}$, one can estimate the sources. As has been done in [1], these estimation of the sources can in turn serve as reference signals: when this procedure is repeated iteratively, the number of iteration is denoted by N_I.

4.2 Results

Computer simulations are now presented to compare a deflation procedure to our proposed MIMO contrast $\mathcal{C}_{R,\mathbf{z}}$. We have used fourth order cumulants ($R = 4$). The separation criteria are the mean square estimation error (MSE) on each source for the PAM-4 i.i.d. source signals and $\tau_i \triangleq 1 - \frac{\max_j \|G_{ij}\|_j^2}{\sum_{j=1}^{N}\|G_{ij}\|_j^2}$ ($i \in \{1,\ldots,N\}$) for the CPM (Continuous Phase Modulation) non i.i.d source signals (modulation indices: 0.4, 0.7, 0.3 and 0.6). Note that $0 \le \tau_i < 1$ and $\tau_i = 0$ if and only if $y_i(n)$ corresponds perfectly to one source. The mixing filter coefficients have systematically been randomly chosen according to a normal distribution.

Experiment 1. In Figure 1 (resp. Figure 2), we have plotted the cumulative distribution of the empirical values of the MSE (resp. values of τ_i) over 1000 Monte-Carlo runs. We have considered $N = 3$ source signals mixed on $Q = 4$ sensors with a filter of length $L = 3$, using $K = 10000$ samples and $N_I = 3$ iterations. We clearly notice that in all Monte-Carlo runs, the proposed method succeeded particularly well to separate the three different sources. This illustrates that choosing the output of a whitening filter for the references is a successful method. In addition, the proposed approach has an equal performance for the extraction of the three sources. As classically observed in deflation separation methods, the performance is worse for the last extracted source signals than for the first ones.

Experiment 2. We have considered $N = 3$ source signals mixed on $Q = 4$ sensors with a filter of length $L = 3$. The number of iterations for each source

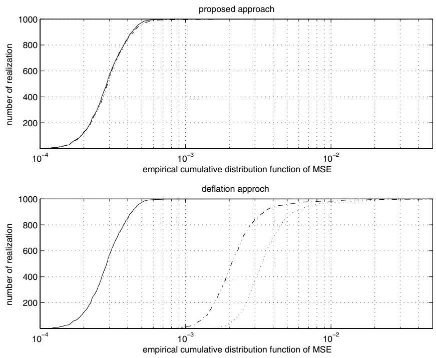

Fig. 1. Empirical cumulative distribution function of the MSE

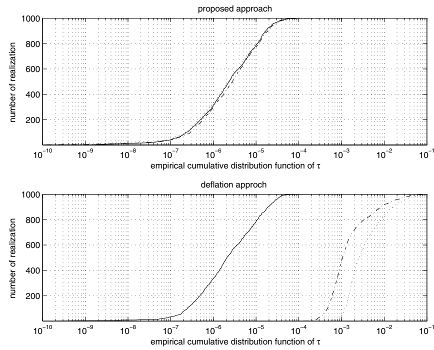

Fig. 2. Empirical cumulative distribution function of τ_i

Table 1. MSE for PAM-4 i.i.d sources and τ_i for CPM non i.i.d sources versus number of samples

	K	5000	10000	15000	20000	25000
PAM-4 sources	s_1	$6.20\ 10^{-4}$	$2.92\ 10^{-4}$	$1.92\ 10^{-4}$	$1.42\ 10^{-4}$	$1.17\ 10^{-4}$
proposed MIMO contrast	s_2	$6.08\ 10^{-4}$	$3.01\ 10^{-4}$	$1.97\ 10^{-4}$	$1.45\ 10^{-4}$	$1.17\ 10^{-4}$
	s_3	$6.07\ 10^{-4}$	$3.01\ 10^{-4}$	$1.94\ 10^{-4}$	$1.43\ 10^{-4}$	$1.17\ 10^{-4}$
PAM-4 sources	s_1	$6.22\ 10^{-4}$	$2.94\ 10^{-4}$	$1.97\ 10^{-4}$	$1.46\ 10^{-4}$	1.1710^{-4}
deflation appoach +	s_2	$6.87\ 10^{-3}$	$2.73\ 10^{-3}$	$2.21\ 10^{-3}$	$1.20\ 10^{-3}$	$9.68\ 10^{-4}$
quadratic MISO contrast	s_3	$1.17\ 10^{-2}$	$4.35\ 10^{-3}$	$4.35\ 10^{-3}$	$2.80\ 10^{-3}$	$1.71\ 10^{-3}$
CPM sources	τ_1	$5.76\ 10^{-6}$	$2.24\ 10^{-6}$	$1.21\ 10^{-6}$	$1.04\ 10^{-6}$	7.5110^{-7}
proposed MIMO contrast	τ_2	$4.82\ 10^{-6}$	$1.97\ 10^{-6}$	$1.44\ 10^{-6}$	$8.47\ 10^{-7}$	$7.9\ 10^{-7}$
	τ_3	$5.69\ 10^{-6}$	$2.29\ 10^{-6}$	$1.07\ 10^{-6}$	$8.72\ 10^{-7}$	$8.27\ 10^{-7}$
CPM sources	τ_1	$5.37\ 10^{-6}$	$2.67\ 10^{-6}$	$1.44\ 10^{-6}$	$1.08\ 10^{-6}$	5.2310^{-7}
deflation appoach +	τ_2	$5.03\ 10^{-3}$	$3.09\ 10^{-3}$	$4.06\ 10^{-3}$	$2.27\ 10^{-3}$	$2.16\ 10^{-3}$
quadratic MISO contrast	τ_3	$1.03\ 10^{-2}$	$8.51\ 10^{-3}$	$7.30\ 10^{-3}$	$6.75\ 10^{-3}$	$4.22\ 10^{-3}$

extraction was $N_I = 5$. In Table 1 we report both the average MSE of each source and τ_i for $i = 1, 2, 3$ on 100 Monte-Carlo runs for the three estimated sources versus the number of samples K. As intuitively expected, using the proposed MIMO contrast, a constant performance has been obtained for the three sources, contrary to the deflation procedure for which the performance is degraded for the extraction of s_2 and s_3.

Experiment 3. We now compare a deflation approach combined with the kurtosis based contrast $|\mathrm{Cum}\{y(n), y(n), y(n), y(n)\}|^2$ with our quadratic contrast. The kurtosis contrast has been optimized using a gradient ascent method. We have considered $N = 3$ source signals mixed on $Q = 4$ sensors with a filter of length $L = 3$. The number of samples is $K = 10000$. We plotted in Figure 3 the cumulative distribution of the empirical values of τ_i for the three

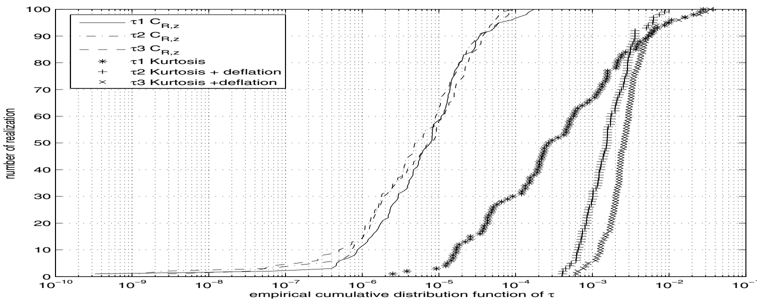

Fig. 3. Comparison of MIMO results by using the kurtosis based contrast and the proposed contrast $\mathcal{C}_{R,\mathbf{z}}$

CPM estimated sources on 100 Monte-Carlo runs for both contrasts. One can see that the quadratic approach outperforms the results obtained by the kurtosis contrast. Besides, the optimization of our contrast is much quicker than gradient optimization of the kurtosis.

References

1. M. Castella, S. Rhioui, E. Moreau and J.-C. Pesquet: "Source separation by quadratic contrast functions: a blind approach based on any higher-order statistics", Proc. of ICASSP 2005, pp. 569-572, Philadelphia, USA.
2. A. Adib, E. Moreau and D. Aboutajdine, "Source separation contrasts using a reference signal", *IEEE Signal Processing Letters*, Vol. 11, No. 3, pp 312-315, 2004.
3. B. Jelonnek, D. Boss and K.-D. Kammeyer, "Generalized Eigenvector Algorithm for Blind Equalization" *Signal Processing*, 61(3), pp. 237-264, September 1997.
4. P. Comon, "Contrasts for multichannel blind deconvolution", *IEEE Signal Processing Letters*, Vol. 3, No. 7, pp. 209-211, July 1996.
5. E. Moreau and J.-C. Pesquet, "Generalized contrasts for multichannel blind deconvolution of linear systems", *IEEE Signal Processing Letters*, Vol. 4, No. 6, pp. 182-183, June 1997.
6. C. Simon, P. Loubaton and C. Jutten, "Separation of a class of convolutive mixtures: a contrast function approach", *Signal Processing* Vol. 81, pp. 883-887, 2001
7. J.K. Tugnait, "Identification and deconvolution of multi-channel linear non-gaussian processes using higher order statistics and inverse filter criteria", *IEEE Trans. Signal Processing*, Vol. 45, No. 3, pp. 658-672, March 1997
8. R. Liu and Y. Inouye, "Blind equalization of MIMO-FIR channels driven by white but higher order colored source signals", *IEEE Trans. Information Theory*, Vol. 48, No. 5, pp. 1206-1214, May 2002.