17 Entropy and Gaia: Is There a Link Between MEP and Self-Regulation in the Climate System?

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Summary. The Gaia hypothesis posits that the Earth's climate is self-regulating, while the maximum entropy production (MEP) principle suggests that the climate system self-organizes in a state of maximum entropy production due to turbulent dissipative processes. We explore the relationship between the two by applying MEP to a toy model based on Daisyworld in which the temperature-albedo feedback is dependent on the heat transport rates within the system. We initially assume that the dynamical response of the climate system to differential radiative heating is to create heat fluxes such that a steady state satisfying a maximum entropy-production (MEP) condition is obtained. The resulting system, which does not depend on free parameters, turns out to be thermostatic and to favour the existence of two, but not several, daisy species simultaneously. Furthermore, it maximizes the range of luminosity over which daisies exist, that is, the lifespan of Daisyworld. However, if the daisy coverage is assumed to adjust more slowly than the heat fluxes, the range of habitation is narrowed. Imposing a sinusoidal forcing allows more than two species to coexist, but only occasionally and not to a significant extent.

17.1 Introduction

The Gaia hypothesis (Lovelock 1972, Lovelock and Margulis 1974) emerged from the realisation that Earth's atmospheric composition is in an extreme state of thermodynamic disequilibrium (low entropy), and that this is in turn a product of the presence of myriad non-equilibrium life forms maintaining a highly ordered (low entropy) state (Lovelock 1975). In its later incarnation, the Gaia theory proposed that atmospheric composition and climate are self-regulated by the whole surface Earth system of life plus its abiotic environment ("Gaia"). Self-regulation can be defined in terms of resistance to external forcing or resilience to perturbation (Lenton 2002). Here we focus on self-regulation of the climate in response to gradually changing external forcing (increasing solar luminosity).

Paltridge (1975, 1978, 1981) proposed that, in analogy to simpler dynamical systems that are subject to a given forcing, the response of the climate

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system to heterogeneous solar input is to maximize its internal entropy production (and thus maximize the entropy export) due to turbulent dissipation associated with heat transport. This idea is referred to as the maximum entropy production (MEP) principle, and is reviewed elsewhere (O'Brien and Stephens 1995, Ozawa et al. 2003, also Kleidon and Lorenz, this volume). Its applicability to planetary atmospheres and the the Earth is reviewed and discussed in Kleidon and Lorenz (this volume) and in Lorenz (this volume), while Dewar (this volume) provides a theoretical framework.

To investigate the possible effects of large-scale biotic feedbacks, we focus on a simple albedo mechanism, which operates indirectly on the entropy production of the climate through the amount of solar energy absorbed locally. We need thus to distinguish between total planetary entropy export and internal generation of entropy within the system. The Earth's total planetary entropy export is due entirely to radiation, and includes the thermalization of high-energy solar radiation into equilibrium black-body infrared radiation. However, as long as non-linear radiative processes, such as photochemical processes, can be neglected, internal entropy production is nearly independent of the nature of the heating source, but only on its distribution, and is due mainly to turbulent dissipation by the atmospheric and oceanic circulation. Theory and observations suggest that the statistically preferred state can be found by maximizing internal, material entropy production only.

Absorption of radiation is essentially a linear process (Ozawa et al. 2003). Feedback mechanisms can tie albedo and absorption together, via an albedo dependence on temperature. Such feedback may be abiotic or biotic. Well-known abiotic feedbacks include snow/ice-albedo positive feedback, and cloud cover-temperature negative or positive feedbacks. Biotic feedbacks include taiga/tundra-albedo positive feedback, vegetation-water cycle positive feedback, vegetation-cloud cover negative feedback, and the feedback between marine dimethyl sulphide (DMS) production and cloud albedo, the sign of which is uncertain (see also Kleidon and Fraedrich, this volume).

Here we consider whether there is a link between maximization of entropy production due to heat transports (i.e., mechanical and thermal dissipation) within the climate system and self-regulation of the climate system. As a first attempt to address this issue we refer to the simplest 'Gaian' paradigm available: Daisyworld.

17.2 Daisyworld

The Daisyworld model (Watson and Lovelock 1983), describes a multi-component system, forced by a prescribed, homogeneous solar radiation field, with the capacity to vary its albedo and thus the amount of heat taken up. Within a range of solar forcing, the system has built-in feedback mechanisms that keep the surface temperature close to a pre-defined value. This is some-what analogous to chemical buffer solutions. The concentrations, or amounts, of the different components (e.g., of the carbonate ion in a sodium bicarbonate solution, or the black daisies in Daisyworld) depend on the quantity on which the feedback operates (the pH in the buffer, the temperature in Daisyworld). Pushing the analogy further, in Daisyworld, the "reaction rate" is prescribed, taking the form of a heat exchange rate between the different components of the system. It is this heat exchange rate that makes the feedback operative, and defines the range of external forcing within which it does operate. The analogy, however, ends here. The different daisy species of Daisyworld interact indirectly through the mediation of heat transport, which must be provided by a dynamical response of the climate system. Furthermore, while the near-equilibrium properties of chemical solutions with generic parameters are well known, those of the climate system are not. If the system "chose" to transport no heat, or to guarantee a uniform local temperature everywhere, Daisyworld would not work. Indeed, if poleward heat transport were not taking place in the real Earth climate, daisies and other species would have a pretty hard time almost everywhere.

In order to determine the forcing, and thus the heat exchange rates and local temperatures obtained, most existing MEP studies (Paltridge 1975, Paltridge 1978, Lorenz et al. 2001) prescribe not the external solar radiation field, but the actual heat locally taken up by the climate system, i.e., the local albedo. Here we apply the MEP principle to the Daisyworld system, such that none of the albedo values or heat transport rates have to be prescribed. The surface temperature follows just from the intrinsic properties of the feedback and from the magnitude of the solar luminosity. Moreover, the natural formulation of the equations is such that it is not necessary to assume a given number of species. In principle, any number of species is allowed, and the actual number corresponding to the steady-state solution for a given external radiative field is determined as the one compatible with the MEP principle.

Much of the work described herein was undertaken independently of, and at a similar time to, an existing study (Pujol 2002). Our formulation of the problem in Sect. (17.3) is, however, more general, while the results for a two-daisy system (17.4) are equivalent to those of Pujol (2002) given certain caveats. In the subsequent sections we consider what happens with multiple components/daisies (17.5), with saturating growth responses (17.6), with two boxes that represent equatorial- and polar-regions and exchange heat (17.7), and when the daisies are assumed to adjust more slowly than the climate system (17.8). We close with a discussion in Sect. 17.9.

17.3 Model Formulation

The system is determined by three sets of equations. The first set follows Watson and Lovelock (1983), and describes the biological equilibrium between growth rate and death rate as determined by the competition for resources and by the physiological properties of each daisy species. A key point is that the growth rate of each daisy species is assumed to be proportional to the area of free soil, α_g. The growth equation for species i is:

$$\frac{d\ln\alpha_i}{dt} = \alpha_g\beta_i\left(T_i\right) - \gamma_i\,, \quad (i = 1, 2, \ldots, n) \tag{17.1}$$

where $\alpha_g = 1 - \sum_i \alpha_i$, α_i is the fractional area occupied by daisy species i, $\beta_i(T)$ is a function of temperature characterizing the growth rate of daisies of species i, γ_i is the death rate of that species, n is the number of available daisy species.

The functions β_i are assumed to peak at a certain temperature $T_{0,i}$ and reduce to zero (or to negligible values) outside an interval of width $2T_{s,i}$ centred on the peak. We assume:

$$\beta_i\left(T\right) = \beta_{0,i}\left[1 - \left(\frac{T - T_{0,i}}{T_{s,i}}\right)^2\right]\,, \quad (i = 1, 2, \ldots, n)\,. \tag{17.2}$$

The detailed form of β_i is relatively unimportant, especially when (in thermodynamic units) $T_{s,i} \ll T_{0,i}$, since the daisies are present in significant amounts only near their optimum temperature $T_{0,i}$, where an approximate form (17.2) always holds. In a steady-state, (17.1) can be satisfied only if:

$$\left|T_i - T_{0,i}\right| \leq T_{s,i}\sqrt{1 - \gamma_i/\beta_{0,i}}\,. \tag{17.3}$$

In the following, we restrict our attention to steady states, so that all time derivatives will be assumed to vanish. In order to make use of the MEP constraint, we must assume that the equilibration of the daisy system is much faster than any characteristic dynamical time scale of the climate system. Additionally, we assume (quite reasonably) that the radiative forcing, T_r, changes slowly compared to the evolution of the daisies.

Equation (17.1) can be written in the more convenient form:

$$b_i\left(\theta_i\right) = \phi \qquad (i = 1, 2, \ldots, n) \tag{17.4}$$

where $b_i = \beta_i/\gamma_i$ is the normalized growth function for species i, and $\theta_i = T_i/T_{0,i}$. The quantity $\phi = 1/(1 - \sum_i \alpha_i)$ is the inverse of the bare-soil fraction, and it determines the local temperatures of the existing daisy species, since

$$T_i = T_{0,i}b_i^{-1}\left(\phi\right) \qquad (i = 1, 2, \ldots, n)\,. \tag{17.5}$$

The second set of equations for the MEP-Daisyworld model expresses the steady-state energy balance of the system. Assuming grey-body spectra:

$$\alpha_i \left(T_i^4 - N_i T_r^4\right) = Q_i \qquad (i = 1, 2, \ldots, n)\,, \tag{17.6}$$

$$T_g^4 - N_g T_r^4 = -\phi \sum_i Q_i \tag{17.7}$$

Here, the subscript g refers to the bare soil; $N_i = 1 - A_i$, where A_i is the albedo of the daisies of species i, and similarly $N_g = 1 - A_g$; T_r is the effective (or equivalent black-body) temperature of the solar radiation field.

The quantities Q_i are heat fluxes per unit area (divided by the Stefan-Boltzmann constant), which it is assumed the climate system provides to each daisy species. In the original model (Watson and Lovelock 1983) they took the explicit form $Q_i = \alpha_i q(A - A_i)$, where A is the average albedo as resulting from the fractional area covers α_i of the daisy species, and q is a free parameter, which must lie in the range $0 \leq q \leq T_r^4$ in order to satisfy Kelvin's 2nd law of thermodynamics. The range of T_r over which the albedo feedback operates (if at all), and the number of daisy species allowed by the system at a certain T_r, all depend on the adopted value of q.

In the present formulation, the heat fluxes Q_i are treated as independent quantities. Then, by substituting α_i from (17.6) and T_i from (17.7), the definition of ϕ can be written

$$\frac{1}{\phi} = 1 - \sum_i \frac{Q_i}{\left[T_{0,i} b_i^{-1}(\phi)\right]^4 - N_i T_r^4} \tag{17.8}$$

The solution of this equation, $\phi = \phi(\underline{Q})$, falling within the interval $1 \leq \phi \leq \max_i \{\beta_{0,i}/\gamma_i\}$ characterizes the response of the Daisyworld system to a given set of values of the heat fluxes Q_i. The local temperatures T_i and the fractional area coverages α_i are given by (17.5–17.6). The heat fluxes Q_i are determined as functions of the forcing radiation brightness temperature T_r by a final set of equations which follow from the MEP requirement, i.e., that the entropy production within the system attains a maximum. The entropy production is given by:

$$\dot{S} = \sum_i Q_i \left(\frac{1}{T_i} - \frac{1}{T_g}\right)\,, \tag{17.9}$$

where T_g is given by (17.7), and the MEP conditions can be written:

$$\partial_{Q_i} \dot{S} = 0 \qquad (i = 1, 2, \ldots, n)\,. \tag{17.10}$$

The form of (17.6) gives a parameterization of the heat transports, with respect to which $\dot{S}$ as given in (17.9) is to be maximized, that is consistent with the application of the MEP principle. The entropy production is a function of quantities defining the heat transfer rates between different components of the system under a fixed external forcing, i.e., given an external energy input into the system. The novelty of the present system is that it can control the external energy flux by changing the fractional area coverage of the daisies and thus the average albedo. The heat-flux rates, though, do not depend on the fractional areas.

The possible values of Q_i are constrained by two conditions. First, the range of values for the temperature T_i, obtained from (17.6) when α_i varies between 0 and $1 - \min_i(\gamma_i/\beta_{0,i})$, should overlap with the interval defined by (17.3). Second, from Kelvin's second law of thermodynamics, the temperature of species i cannot be higher than the equilibrium temperature for the species with the largest available albedo when no heat is exchanged, and cannot be lower than the corresponding case for the lowest available albedo.

When one of the Q_i is equal to zero (say, for $i = i_0$), (17.6) has two solutions. One is $\alpha_{i_0} = 0$, and it is also found by solving (17.8); the other is $T_{i_0} = N_{i_0}^{1/4} T_r$. For this second case, ϕ is obtained from (17.5) for $i = i_0$, the remaining temperatures from the same equation with $i \neq i_0$, the values of α_i for $i \neq i_0$ from (17.6), and finally $\alpha_{i_0} = \phi^{-1} - \sum_{i \neq i_0} \alpha_i$. Similar considerations apply if more than one of the Q_i's is zero. A solution with $Q_i = 0$ and $\alpha_i \neq 0$ does not represent a MEP solution. For such solutions, a set of infinitesimal variations δQ_i, $i = 1, 2 \ldots$, can always be found such that the entropy production increases. Therefore, such solutions are not considered further in the MEP context.

In view of the above, it is convenient to simplify the computation by using (17.6) to change variables to the fractional areas α_i. Given a vector of values $\underline{\alpha}$ for the fractional areas, one can calculate ϕ, and thus the temperatures $\underline{T}$ from (17.5), the fluxes $\underline{Q}$ from (17.6), and finally formally maximize $\dot{S}$ with respect to $\underline{\alpha}$. Since $Q_i \approx 0$ as $T_i \rightarrow N_i^{1/4} T_r$, the transformation is smooth. Thus, the $\dot{S}$ surface is mapped faithfully from one variable space into the other, single maxima corresponding to single maxima. This is not guaranteed if a different parameterization is chosen for the heat fluxes, for example, following Watson and Lovelock (1983), $Q_i = \alpha_i q(A - A_i)$, and allowing q to be a function of daisy species (i.e., $q = q_i$), leads to multiple local entropy maxima. In the present formulation, instead, the maximum seems to be unique in continuum space (see below). This property may be relevant when the global maximum changes location discontinuously, since the implied "phase transition" of the climate system will be of a different order.

A complication arises from the fact that the solution for a given $\underline{Q}$ or a given $\underline{\alpha}$ is not unique when the growth-rate functions β have multiple-valued inverse, as those defined in (17.2). In particular, whenever the growth function has a maximum, (17.5) has, for each i, two solutions, one lying above and the other below the peak-growth temperature $T_{0,i}$ (with the exception of the case $T_i = T_{0,i}$). As a result, in an N-daisy "world" there are, for each set of areas $\underline{\alpha}$, 2^N different solutions, i.e., 2^N sets of temperatures $\underline{T}$ and of corresponding heat fluxes $\underline{Q}$. We may express this multiple-valued-ness by assuming that the solution depends on an additional, discrete variable, say k, with $k \in 1, 2, \ldots, 2^N$. Suppose the system is found in one particular state $\{\underline{\alpha}, \underline{T}, \underline{Q}\}$. Statistical fluctuation in the system will ensure that the local neighbourhood in the continuum heat flux (or area-) space is "explored", and thus an extremal principle formulated in these variables is justifiable. The up to 2^N solutions to (17.5), however, are not continuously connected to one another. On the one hand, if the level of fluctuations within the system is such as to bring it from one value of k to the other, transients will be significant, and the equilibrium states will not be sufficient to characterize the solution. If, on the other hand, fluctuations are small compared to the phase-space gap between solutions at different k, there is no guarantee that the global maximum of entropy production is always achieved. Therefore, conformity to the MEP principle can be interpreted in the sense that $\dot{S}$ is a maximum in heat-flux space, while it may not be a maximum with respect to the discrete variable k.

17.4 Two-Component System

The simplest system displaying significant homeostatic properties is that allowing a maximum of two daisy types simultaneously. The intervals within which each daisy type can be expected to cover a non-zero area are once again found from (17.3) and (17.6). If the subscript 'w' ('b') indicates the daisy species with the higher (lower) albedo, the brightness temperature of the radiation field must lie in the range

$$\frac{\max\left(T_{0,i} - T_{s,i}\sqrt{1 - \gamma_i/\beta_{0,i}}\right)}{N_b^{1/4}} \leq T_r \leq \frac{\min\left(T_{0,i} + T_{s,i}\sqrt{1 - \gamma_i/\beta_{0,i}}\right)}{N_w^{1/4}} \quad (17.11)$$

when both daisy species are present. The max and min functions arise because the additional condition $T_b \geq T_w$ holds. For at least one daisy species to be present, T_r must be larger than the smaller of the expressions $\left(T_{0,i} - T_{s,i}\sqrt{1 - \gamma_i/\beta_{0,i}}\right) \Big/ N_i^{1/4}$, and similarly for the upper bound.

An example of a MEP Daisyworld solution is shown in Fig. 17.1. The entropy production, fractional areas α_i, daisy temperatures T_i, and heat fluxes Q_i are plotted for each combination of root choice in (17.5). The MEP solutions provide heat fluxes such that the range of T_r within which daisies can survive is maximal. In a q-prescription as in Watson and Lovelock (1983), this range is significantly reduced with respect to the MEP case. An example

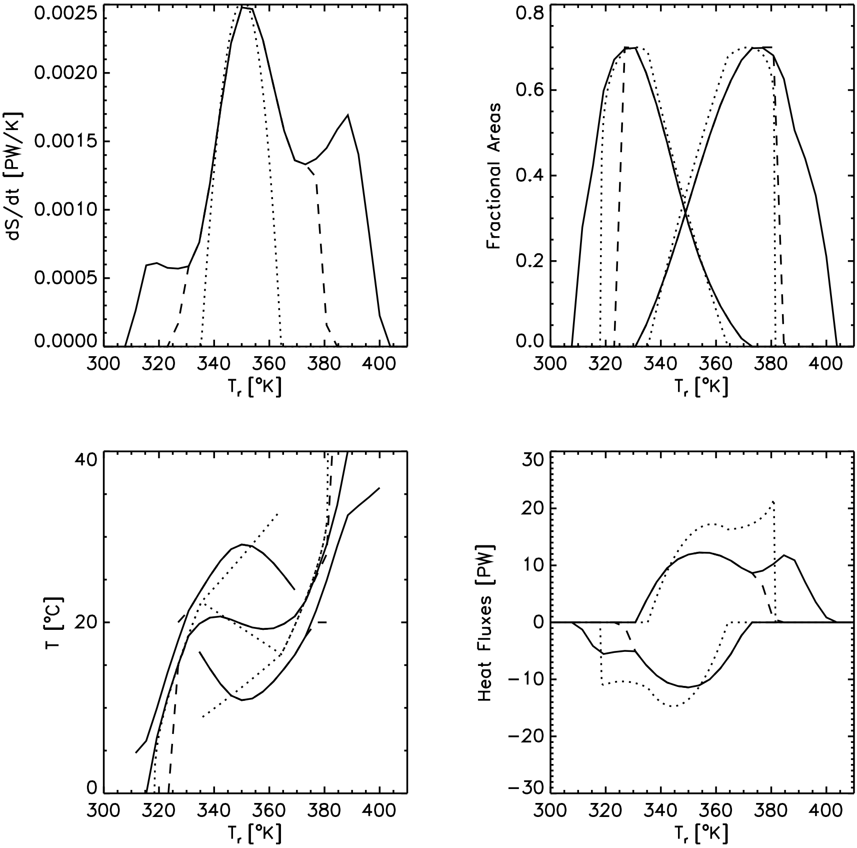

Fig. 17.1. Two-species "Fast daisy" MEP system (*solid and broken lines*) compared to the original Daisyworld solution (*dotted lines*). The *broken line* shows the MEP solution with $k = 0$; the *solid line* is the overall MEP solution among the continuous (constant-k) solutions. Panels show entropy production (*upper left*), fractional area coverage for the two species (*upper right*), the temperatures for each species and average temperature (*lower left*), and the heat fluxes towards each species (*lower right*). The parameters of the model are $N_1 = 0.35$, $N_2 = 0.65$, $T_{0,1} = T_{0,2} = 20°\text{C}$, and $T_{\text{s},1} = T_{\text{s},2} = 20°\text{C}$

of such a solution is shown together with the MEP solutions in Fig. 17.1. The MEP closure implies that both the range over which daisies exist and that over which both species exist are increased. Note, however, that the MEP solutions imply less comfortable temperatures for the daisies, and hence at some times a smaller total daisy area coverage. Within the MEP constraint, the climate system does not support the heat fluxes required for the daisies to spread further. Thus, in the present model framework, homeostasis arises not as a consequence of self-regulation of the daisies alone, but as a preferred state of the overall climate system when it comprises the daisies as a sub-system.

17.5 Multi-component System

The present model is clearly formulated for an arbitrary, but finite, number of species. Among those, it selects the species that are supported by the dynamical system "responsible" for providing the heat fluxes. In some sense, therefore, the model is parameter-free. The functional form of the growth-functions β_i does not need to be the same for each species either. Obviously, it is practically impossible to provide a "complete" input to the numerical maximization routine (even within the restrictive model assumptions). In fact, the basic character of the solutions can be investigated using only a small number of species. In a three species system we find that there are no continuous solutions in which the three species coexist. The overall MEP solution in k-space also only supports two species: those with maximum albedo contrast. This even reaches the paradoxical situation when the species whose optimum temperature $T_{0,i}$ is at its effective radiation temperature $N_i^{1/4}T_r$ is not supported. This species would tend to occupy a large area without exchanging heat with the surroundings, a situation evidently far from MEP. Even if the species with intermediate albedo is a "super species" with a very wide range of operating temperatures, when the other two species can manage to survive, it tends to be replaced. Thus the three species system tends to behave as a "piecewise two-species" system. Once again, whether any such condition is achieved depends on whether there is an amount of fluctuations sufficient to bring the system from one MEP branch to the other, or not.

17.6 Saturated Growth

The ambiguity in the solution is avoided if one chooses monotonic growth functions β_i, which saturate to γ_i either for small or for large values of the temperature. Such functions may describe non-biological components like sea-ice, or rising sea level. For the purpose of illustrating the case, we assume a form

$$\beta_i = \max\left(\beta_{0,1} \tanh\left(2[(T - T_{0,i})/T_{s,i} + 1]\right), 0\right) \tag{17.12}$$

where $T_{s,i}$ may be positive (saturation at high T) or negative (saturation at low T), and $T_{0,i} - T_{s,i}$ determines where the function goes to zero, i.e., the constants are chosen such that $T_{s,i}$ has a similar width-determining role as in (17.2).

Figure 17.2 illustrates a 2-species case for the same parameters as in Fig. 17.1, except that T_s has the opposite sign and a smaller magnitude for the light species. The similarity is obvious, and stems simply from the fact that the $\beta_i(T)$ curves are very similar in both cases when one restricts the attention to the sloping parts at $T > T_0$ for the dark and $T < T_0$ for the light species. The physical meaning of this is that regulation originates from the

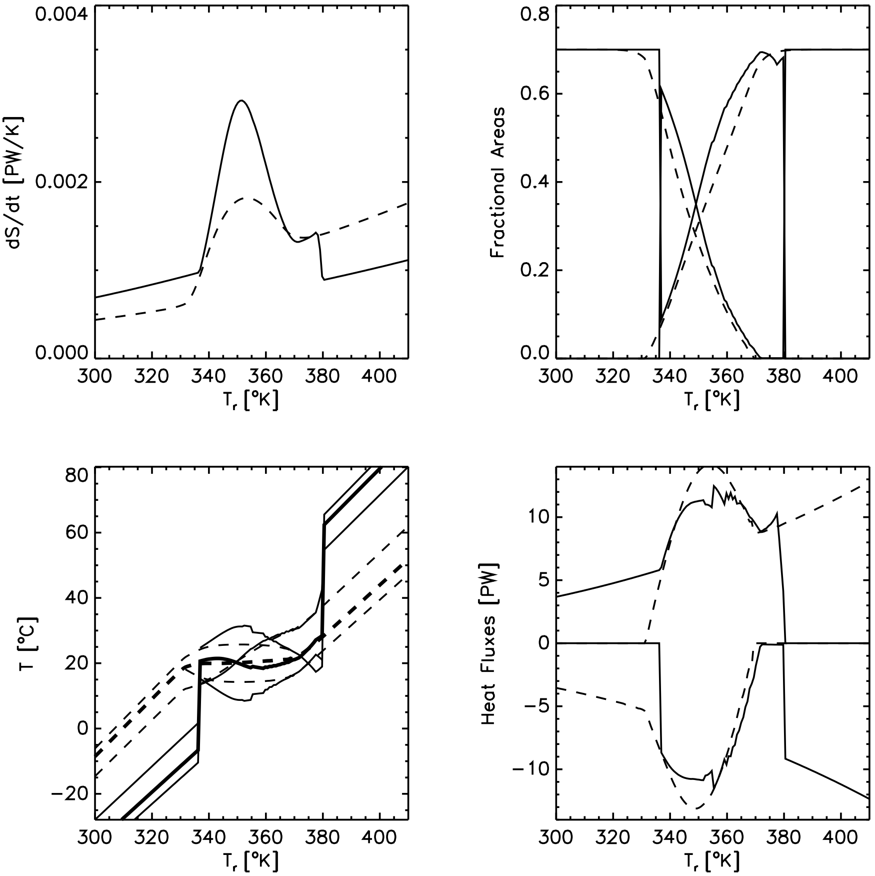

Fig. 17.2. Two-species saturated-growth "Fast daisy" MEP system. The panels are as in Fig. 17.1, but the *solid lines* now show the case in which the higher-albedo species ($N_1 = 0.35$) is "cold-loving" (i.e., its growth function saturates to 1 at low temperatures) and the lower-albedo species ($N_2 = 0.65$) is "warm-loving" (growth-rate 1 at high temperatures), while the *broken lines* show the opposite case ("cold-loving" dark species and "warm-loving" bright species). The other model parameters are $T_{0,1} = T_{0,2} = 20°\mathrm{C}$ in both cases, $T_{\mathrm{s},1} = -10°\mathrm{C}$, $T_{\mathrm{s},2} = 20°\mathrm{C}$ in the first case, and $T_{\mathrm{s},1} = 10°\mathrm{C}$, $T_{\mathrm{s},2} = -20°\mathrm{C}$ in the second case. Note that in the first case the area coverage of both species has jumps marking the transitions between the two-species "world" and a single-species "world". The *thicker lines* in the temperature panel indicate the average (planetary) temperature, while the other three lines show the local temperatures for each species and for the bare soil

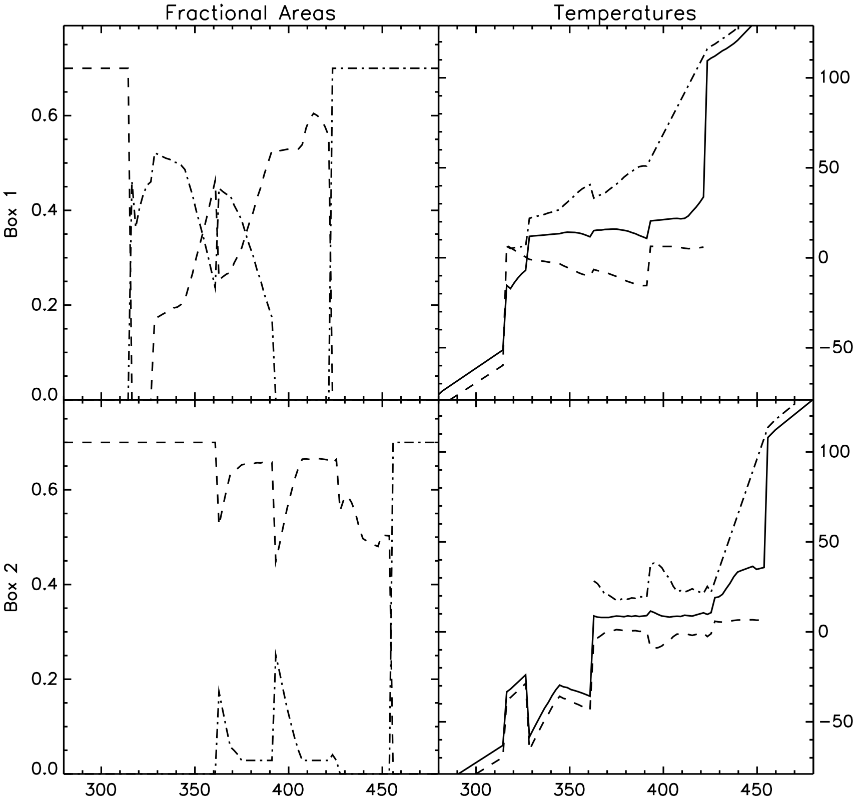

Fig. 17.3. Two-species, two-box "Fast daisy" MEP system with saturating growth-functions. Box 1 corresponds to the "tropical" box 'E', and Box 2 to the "subpolar" box 'P'. *Dashes lines* refer to the bright species ($N_1 = 0.2$), *dot-dashed lines* to the dark species ($N_2 = 0.8$). *Solid lines* in the temperature panels indicate the ("zonal") average temperatures. The other model parameters are $T_{0,1} = 0°\mathrm{C}$, $T_{\mathrm{s},1} = -10°\mathrm{C}$, $T_{0,2} = 20°\mathrm{C}$, $T_{\mathrm{s},2} = 20°\mathrm{C}$. The case presented is similar to that shown as *solid lines* in Fig. 17.2 ("cold-loving" bright species and "warm-loving" dark species)

coupling of two positive feedback mechanisms, and that MEP favours this coupling. (A similar example with three species is not worth showing, since the intermediate species seems to always be cut out when (17.12) is used.)

17.7 A Two-Box Model

An interesting modification of the homogeneously forced system is to allow two subsystems to be forced by radiation fields with different brightness temperature, couple them once again via heat exchange, and maximize the entropy production arising from this coupling. Equation (17.6) is replaced by

$$\alpha_i\left(T_i^4 - N_i T_r^4 - Q_B\right) = Q_i\,, \tag{17.13}$$

where Q_B is the heat exchanged between the two boxes. $\dot{S}$ is maximized for each box separately for a given Q_B, and then the additional quantity

$$\dot{S}_B = Q_B\left(\frac{1}{T_P} - \frac{1}{T_E}\right) \tag{17.14}$$

is maximized, where T_P ("poles") and T_E ("equator") are the average temperatures of the two boxes, e.g.,

$$T_E = \left(T_g\right)_E + \left(\sum_i \alpha_i\left(T_i - T_g\right)\right)_E \tag{17.15}$$

We adopted a radiation field of varying intensity and multiplied it by the factor $\pi/6 + \sqrt{3}/4$ for box 'E', and by $\pi/3 - \sqrt{3}/4$ for box 'P', in order to simulate the irradiation impinging on the tropical sector between 30 °N and 30 °S, and that on the remaining sub-polar regions. The same two-species system was assumed to be (potentially) present in either region. Figure 17.3 shows the resulting temperatures and area coverage for the case of saturating β_i's, (17.12). Here species 1 is even more "cold-loving" than before with $T_{0,1} = 0°$ C, $T_{\mathrm{s},1} = -10°$C. Coupled, sharp state transitions occur repeatedly in both boxes.

17.8 Slow Daisies

In the steady-state approach described so far one crucial assumption has to be made: that the daisy population equilibrates on a time-scale much shorter than that needed by the heat fluxes to adjust to changing daisy area coverage. Only under this assumption are the algebraic relationships in (17.4) valid with the equilibrium temperatures derived from entropy-production maximization.

This assumption is of course arbitrary. The atmospheric adjustments responsible for most of the heat transport in temperate land areas occur on time-scales of the order of a month to a year; deep ocean processes can take much longer, but with the exception of the North Atlantic the bulk of the ocean heat transport is related to gyre circulations and shallow wind-driven overturning cells, with time-scales of the order of a decade. It is not straightforward to think of an example of planetary-scale biota evolution that happens on timescales much shorter than this, especially when populations (i.e., area coverages in DW) are small.

Moreover, a second, possibly stronger assumption is also implied in the above treatment of the DW problem. MEP for Daisyworld involves two maximizing assumptions, not only that (i) the heat transport adjusts to MEP for given external heating rates (i.e., albedos) but also that (ii) the heating rates, or daisy fractional covers, adjust to MEP as well. While the first assumption represents the original application of MEP, it is not clear to us how justifiable the second assumption (and its combination with the first) is. For daisy areas to be regarded as a variable with respect to which EP is maximized (under the daisy-growth steady-state assumption), the climate system must have the ability not only to pick the MEP trajectory among those allowed when the external heating rates (albedos) are fixed (i.e., the original MEP principle for steady-states), but also among those different MEP states with different heating rates for which the actual entropy production is greatest.

To address these issues, a second set of calculations have been performed in which the above assumptions are relaxed in favour of another approximation, namely that the MEP-adjustment time-scale be much shorter than the daisy growth time-scale. Entropy production is given by (17.9), the temperature of each daisy species T_i from (17.6), and the temperature of daisy-free land T_g from (17.7). Entropy production is now maximized with respect to all T_i's (or, equivalently with respects to all Q_i's), with areas α_i kept fixed. This leads to the set of equations:

$$\frac{\partial \dot{S}}{\partial t} = \alpha_i T_i^3 \left[\frac{1}{T_i} \left(3 + \frac{N_i {T_r}^4}{{T_i}^4} - \frac{1}{T_g} \left(3 + \frac{N_g {T_r}^4}{{T_g}^4} \right) \right) \right] = 0 \qquad (17.16)$$

The solution method consists in taking an initial guess for

$$C = \theta_g \left(3 + N_g \theta_g^4 \right) \, , \qquad (17.17)$$

where $\theta_g \equiv T_r / T_g$, and solving the equation:

$$\theta_i \left(3 + N_i \theta_i^4 \right) = C \qquad (17.18)$$

for each i, to obtain $\theta_i = \theta(N_i, C)$. From the θ_I's, $\theta_g = \theta(C)$ is then computed. Equation (17.17) for C is solved via bisection. Because $0 < N_{i,g} < 1$ and $0 < \theta_{I,g} < 1$ in (17.18) and (17.17), the solution is guaranteed to exist, and to be unique. The Hessian of $\dot{S}$ can be shown to be negative and hence the solution represents a unique maximum for $\dot{S}$ in T (or Q) space.

Thus for each set of values of $\{\alpha_i, i = 1, ..., n\}$ we have derived, under the MEP steady-state assumption, a set of internal heat fluxes (Q_i) and hence equilibrium temperatures (T_i) for the different daisy species. Equation (17.1) can now be integrated forward in time to study the evolution of the daisies.

An example is shown in Fig. 17.4 and compared with a steady-state ("fast daisy", or FD) case, with the same parameters. For the time-dependent ("slow daisy", or SD) case shown, the time-scale over which the solar forcing (or its effective brightness temperature T_r) varies is set at 10^6 times the inverse daisy maximum growth rate. Therefore, except for very small values of α_i (not distinguishable in Fig. 17.4) the SD system is in a steady state to a very good approximation. (A "seed" value for α has to be specified in the calculation for daisies to grow. This was taken at 10^{-16}; whenever an α_i drops

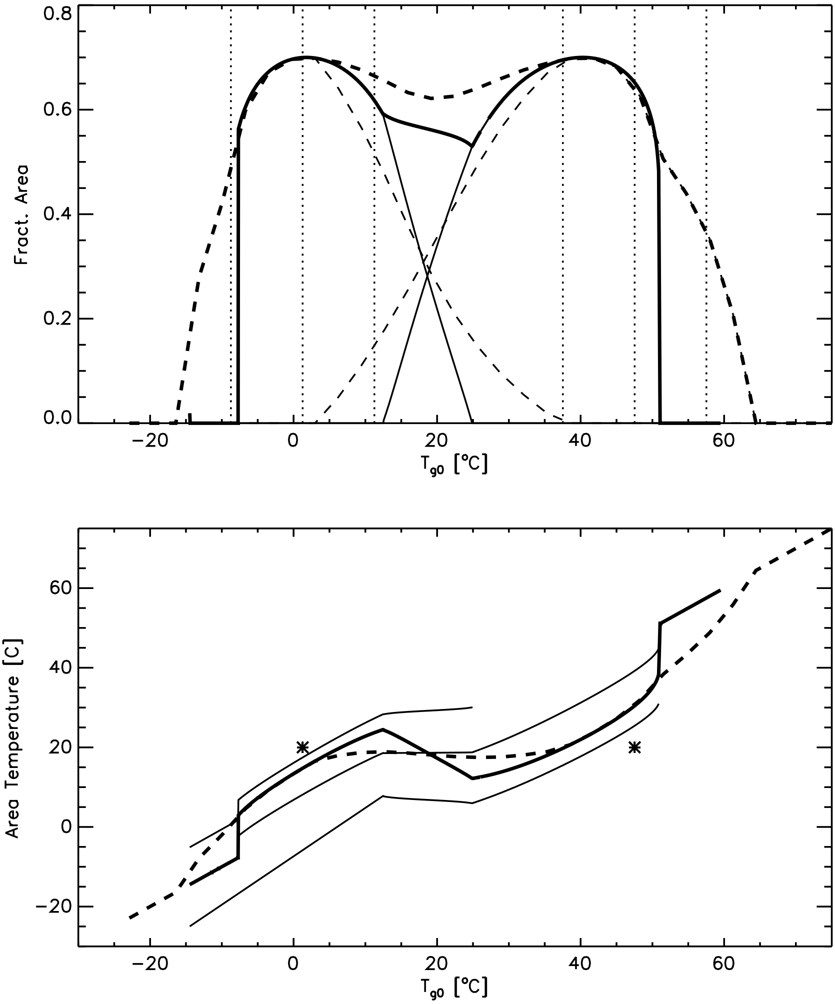

Fig. 17.4. "Fast daisy" (FD) vs. "slow daisy" (SD) MEP models. Results are shown from two calculations with $n = 2$ and $N_1 = 0.35$, $N_2 = 0.65$, optimum temperatures $T_0 = 20°\mathrm{C}$ and a tolerance range $T_\mathrm{s} = 20°\mathrm{C}$, and a death rate $\gamma = 0.3$ of the maximum growth rate. *Solid lines* are for the SD model, *broken lines* for the FD model. The *top panel* shows the fractional area coverage for each species (*thin lines*), and the total daisy area coverage (*thick lines*) in the two models. The vertical *dotted lines* indicate the bare soil temperature ranges corresponding to daisy growth temperatures if internal heat fluxes were zero. The *bottom panel* shows daisy temperatures and bare soil temperature for the SD model (*thin lines*), and planetary average temperatures (*thick lines*) for both models. The two asterisks indicate the optimum temperatures for the two daisy species in the case of no internal heat fluxes

to or below that value, it is not taken account of for EP maximization, and is reset to the seed value. Consistently, only daisy temperatures for α_i greater than seed are shown in Fig. 17.4.)

Some differences between the two cases are noteworthy. The SD case has different regulation properties than the FD case. Entropy production (not shown) is always smaller. Daisy temperatures are generally less optimal for growth and consistently the total area covered by daisies is generally smaller. This is more pronounced under more extreme external forcing conditions. In particular, black daisies do not grow until the bare soil temperature falls within their tolerance window. For white daisies, the environment actually turns hostile at higher solar forcing: the heat conveyed to them from bare-soil areas kills them off while the solar effective brightness temperature is still almost 8 degrees lower than what they could tolerate if there were no exchange heat fluxes. This property of the SD problem is opposite to that of the FD case, where the system "chooses" to keep some daisies alive by reducing heat fluxes because that is still the way to allow for some entropy production. The system with the time-evolving daisies of course does not "know" that it is killing them and just transfers the amount of heat that maximizes EP when the area covers are fixed.

Also the coexistence of daisy species is less likely in the SD case. Some heat flux is provided by the system to the white daisies at low values of T_r (their area fraction coverage is consistently above seed value), but they cannot compete with the black daisies for area occupation. That is possible only after the solar effective brightness temperature has moved out of the zero-heat-flux tolerance window for the black daisies. Thereafter the black daisy population declines and the white daisy population starts filling in the bare soil. The period of coexistence of the two species is shorter than in the FD scenario, and the soil occupation fraction during that time is particularly reduced. Note that the timescale over which these changes happen is 5 orders of magnitude larger than the time daisies would need to occupy the soil under optimal conditions: the occupation fraction is small not because the daisies have not had time to grow, but because they are subject to environmental stress. This again is an important difference from the FD scenario.

The diversity question is particularly interesting. Many-species systems do not show any significant difference from the two-species system shown. The daisy species take over one after the other, with relatively short intervals of time when two coexist, one slowly declining and the other slowly gaining ground. This is an intrinsic limitation of the model that envisages albedo as the only feedback mechanism, while not allowing for rapid fluctuations in the heat fluxes. If a fast sinusoidal forcing is superimposed on the trend (Fig. 17.5), more than two species can coexist occasionally. However, at almost all times one species dominates.

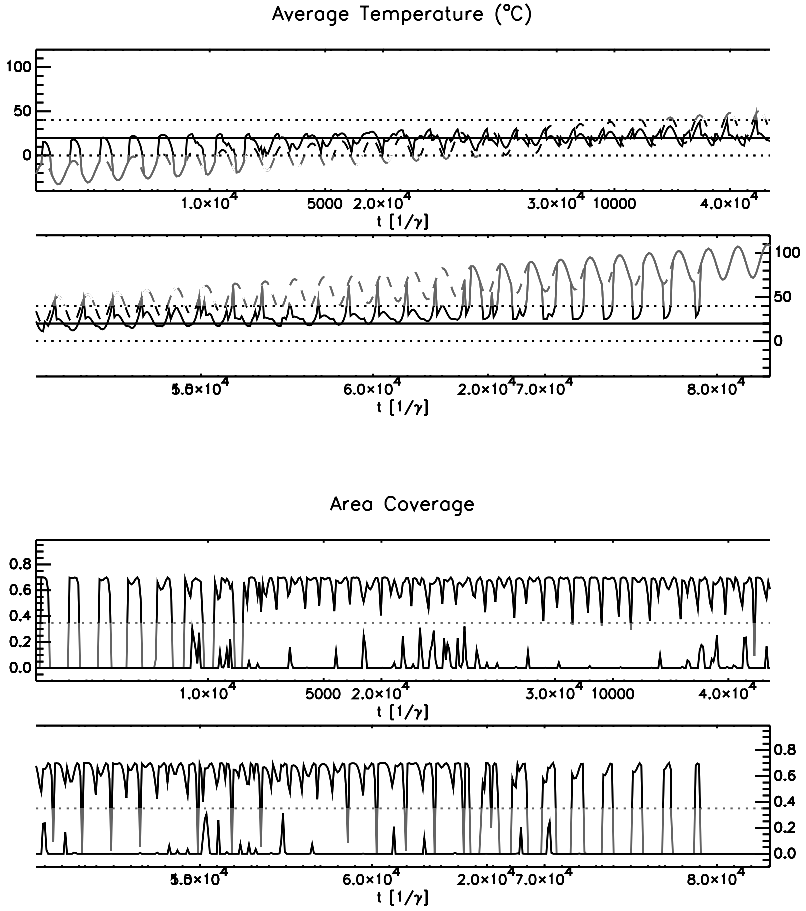

Fig. 17.5. "Slow daisy" MEP Daisyworld model with $n = 6$ and a variable forcing applied. Parameters are as in Fig. 17.1 except that daisy albedos are equal to 0.2, 0.3, 0.4, 0.6, 0.7, and 0.8. The *top panel* shows the average (planetary) temperature with (*solid line*) and without (*broken line*) daisies. Values outside the growth range for daisies are plotted in grey. The abscissa is the elapsed time in units of the inverse daisy death rate. The *bottom panel* shows the total area coverage of all daisy species (*upper line*) and the area occupied by daisy species other than the most abundant one (*bottom line*). Most of the time only one species exists, and almost all the time only one species is dominant in this model

17.9 Discussion

Strictly speaking, the MEP principle is formulated only for dynamical systems with a well-defined accessible phase-space. We have formulated a series of model systems in which there is strong temperature-albedo feedback and entropy production due to heat transports is maximized as a constraint. All of the systems have thermostatic properties and tend to favour the existence of two, but not several, daisy species simultaneously. If the biological populations are assumed to adjust faster than the heat transports, the system maximizes the range of luminosity over which daisies exist (the lifespan of the biosphere). This represents optimal self-regulation in terms of maintaining habitability in response to external forcing. However, if the populations are assumed to adjust more slowly than the heat fluxes, the range of habitation is narrowed. If a sinusoidal forcing is superimposed on the long-term trend, brief periods of coexistence of more than two species occur, but only to a very limited extent.

In model systems based on Daisyworld the total planetary entropy production, which is dominated by radiation absorption, does not show a maximization tendency. In the original model (Watson and Lovelock 1983), the appearance of black daisies lowers planetary albedo, increases solar absorption, and corresponds to a step increase in planetary entropy production. However, as solar luminosity increases and the white daisies gradually take over this amounts to the system counteracting a potential increase in planetary entropy production by maintaining a more constant net absorption flux. When the white daisies disappear this again causes a step increase in planetary entropy production. Thus, there are “phase transitions” corresponding to increases in planetary entropy production, and these are “irreversible”, in the sense that they can only be reversed with a significant decrease in solar luminosity or extinction event (i.e., there is hysteresis in the system). Imposing an MEP constraint on heat transport, as we have, removes the regions of hysteresis present in the original Daisyworld, but the system still has a tendency to counteract increasing luminosity by increasing planetary albedo.

Coupling mechanisms between the climate system and the biosphere like that considered in our Daisyworld model can imply that additional degrees of freedom are made potentially available to the climate system on the timescales of adjustment of the ocean-atmosphere system. If so, the MEP principle, as applied here, would need to be modified to account for the additional entropy associated with ‘biotic’ degrees of freedom, and thermal entropy production would no longer be the (only) relevant quantity to consider. If, on the other hand, the coupling is weak, as e.g., for biota that evolve or grow more slowly than changes in the climate, then we cannot expect to find the daisies in a steady-state. In fact, a general caveat for the application of MEP (in any form) to the climate system is that the conditions and assumptions under which the real climate system may be considered at steady state are not well established.

The Daisyworld studies suggest that while self-regulation of the climate system may be enhanced by maximization of the entropy production due to heat transports, self-regulation bears no clear relationship to total planetary entropy production. This contrasts with a recent simple model of Earth's carbon cycle and temperature. Kleidon (2004) suggests that Earth's planetary albedo is close to a minimum with respect to surface temperature, which exists because of increasing snow and ice cover in response to cooling and increasing (cumulus) cloud cover in response to warming (see also Kleidon and Fraedrich, this volume). A biotic entropy production is defined as the energy flux of respiration divided by the temperature of respiration minus the energy flux of gross primary productivity divided the temperature of radiation used in photosynthesis. This biotic entropy production, like planetary entropy production, is primarily determined by absorbed solar radiation and hence the planetary albedo. If biotic entropy production is maximized (by varying the fraction of biomass that is respired, which is assumed to represent degrees of freedom introduced by the biota), this results in a reduction in atmospheric carbon dioxide as solar luminosity increases and homeostatic regulation of global temperature. In essence, temperature is held close to the value that is assumed to minimize planetary albedo and maximize planetary entropy production (see also Kleidon and Fraedrich, this volume).

Whether a single minimum of planetary albedo exists with respect to temperature is debatable. In the real world, life affects planetary albedo and organisms typically have a peaked growth response to temperature, which could give rise to multiple minima and/or maxima of albedo with respect to temperature. Life can increase planetary entropy production by lowering surface and/or planetary albedo, as do the boreal forest and the black daisies in Daisyworld. However, there are also white daisy analogues in the real world. Phytoplankton that emit dimethyl sulphide (DMS) gas inadvertently increase cloud albedo. Vegetation can enhance low cloud cover through evapotranspiration and the emission of volatile organic carbon compounds that oxidize to form cloud condensation nuclei. Thus it is not clear whether the net effect of life is to increase or decrease planetary albedo. Kleidon (2004) estimates that land vegetation increases planetary entropy production, but a similar analysis is lacking for the marine biota. Kleidon (personal communication) argues, however, that the many ways that the biota affects clouds represent many degrees of freedom, and that is precisely why MEP should apply.

If we accept that given sufficient degrees of freedom, a climate system will tend to adopt a state that maximizes entropy production due to heat transport, the present study and other work (Gerard et al. 1990, Pujol 2002) suggests that this in turn tends to increase the range of solar forcing over which a planet remains habitable. In the Daisyworld studies this result is contingent on the presence of life. However, even in an abiotic model (Gerard et al. 1990), assuming MEP associated with heat transport tends to lower the luminosity threshold for a snowball Earth. If the MEP principle and this tentative link to climate regulation can be generalized it should make us more optimistic about finding life on potentially habitable extra-solar planets.

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