Neuro-fuzzy Logic in Signal Processing for Communications: From Bits to Protocols

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Abstract. The present work shows how communication systems benefit from fuzzy logic. From signal processing applications, which process bits at the physical layer in order to face complicate problems of non-Gaussian noise, to practical and robust implementations of these systems and up to higher layers in the communication chain, which are engaged in the protocol design. The ability for modeling uncertainty with a reasonable trade-off between complexity and model accuracy, makes fuzzy logic a promising tool.

1 Introduction

Since the introduction of fuzzy logic in the engineering field, this discipline has been very successful in automatic control [1] with applications such as autonomous robot navigation, auto focus cameras, image analysis and diagnosis systems. A proof of this success can be found in the number of companies developing hardware and software for developing fuzzy systems (Accel Infotech Pte, Ltd., Adaptive Informations Systems, American NeuraLogix, Fujitsu, Oki Electronic, OMRON Corporation, Toshiba, SGS-Thomson, Siemens, etc.).

The present work shows how communication systems benefit also from fuzzy logic systems. From signal processing applications that process bits at the physical layer in order to face complicate problems of non-Gaussian noise, to practical and robust implementations of these systems and up to higher layers in the communication chain, which are engaged in the protocol design. The ability for modeling uncertainty with a reasonable trade-off between complexity and model accuracy makes fuzzy logic a promising tool.

In the 90's, Bart Kosko [2-3] and Jerry Mendel [4-5] began to study the application of fuzzy logic and set theory to the signal processing field. Since then various works have appeared focused on fuzzy logic under the intelligent signal processing framework [6-7]. Fuzzy systems are able to build up mathematical models from linguistic knowledge and do not require statistical knowledge, although they can incorporate it, offering an scalable design with the available information. Communication systems can

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benefit from these features to gain in robustness and in fast acquisition and tracking, as the present work shows in the application of interference canceling in CDMA (Code Division Multiplex Systems). Another important feature is that fuzzy systems offer also physical interpretability, this helps adjust the parameters of the system in an easy and friendly way. The second application presented in this work at bit level takes advantage of this feature in order to design a robust beamformer for communication signals, resulting in an easy implementable and tunable beamformer when compared with other existing techniques in the literature. These applications together with others developed by the authors in [8-9] are based on the capability of fuzzy logic to implement model-free function approximations. All these works aim to achieve the intelligent filtering that Lofti Zadeh in 1954 stated in his work "General filters for separation of signal and noise" [10]. We could summarize it saying that intelligent and robust filtering consist in decision making. Decision making that should operate not with an statistical model but with the available data: *"Since the extent to which a signal can be separated from noise is ultimately limited by the statistical data available to the designer, a more rational approach would be to start with all the available data about the signal and noise an try to design a filter that would use it fully and most effectively,"[10].*

However, we should talk about the benefits of fuzzy logic and systems with caution. If expert or linguistic knowledge is not available to carry out the decision making in an "intelligent" way, fuzzy logic systems can be used as filters or classifiers that generalize the ones designed on a statistical basis (e.g. minimum mean square error, maximum likelihood, maximum a posteriori probability); thus, offering much more flexibility and possibilities than the classical statistical systems, but presenting a greater complexity that results difficult to cope with. Concerning this aspect, the present work compares different fuzzy and non-fuzzy classifiers. In spite of the greater flexibility that fuzzy systems offer due to the fuzzy instead of crisp classification thresholds, the conclusion is that the fuzzy systems only stand out when expert knowledge and not only data is available in the design.

At the protocol level in the communication stack, known research applications are queuing and buffer management, distributed access control, hand-off control, load management, routing, call acceptance, policing congestion mitigation, bandwidth allocation, channel assignment, network management, etc [11-17]. All of them take advantage of: the flexibility of fuzzy logic, its ability to cope with different types of inputs and its decision making structure. Protocols are in fact controllers that have to make decisions based on many different variables; thus, the appropriateness of fuzzy logic. Part of the present chapter is dedicated to applications regarding hand-off algorithms [18-19], combining distance measurements with received signal strength to decide hand-off while keeping quality of Service. Although the work focuses on horizontal hand-off in WLAN (Wireless Local Area Network), we point out that in the emerging multimedia systems, hand-off is also considered vertically, as a switching between different systems covering the same area and service. As an example, consider switching between GPRS, UMTS and a satellite segment, as in [20].

Next some of the mentioned applications are presented: section 2 describes the interference canceller for CDMA, section 3 is dedicated to the robust beamformer, section 4 is devoted to fuzzy classification and finally, section 5 discusses on the use of fuzzy logic for hand-off control. Finally conclusions come. For a review of fuzzy logic and systems in signal processing we refer to the tutorial in [5].

2 Fuzzy Logic Canceller for CDMA

The new communication standards require high capacity to support the increasing demand of multimedia services. In order to achieve the high capacity, the standards propose to reduce the cell site and reutilize frequency or codes. However, this strategy ask for more sophisticated signal processing techniques that are able to cope with the increases level of interference. This section focus on a CDMA system, where K spread spectrum users are received at each single-user terminal. The signal model for the sample k of the received signal is

$$r_k = A_k s_k + i_k + n_k \tag{1}$$

where s_k represents the binary information $(\in \pm 1)$ of the desired user. The interference i_k can be either analog or digital and n_k models both the thermal noise (AWGN or additive white Gaussian noise) and the multiple access interference or MAI as (2) formulates

$$n_k = \sum_{l=1}^{K} \sqrt{P_l}\, s_k^l + \sqrt{P_N}\, w_k \tag{2}$$

where s_k^l is the binary sequence of the undesired user j, and w_k is the thermal noise. P_j and P_N represent the corresponding powers. Next section is devoted to the design of the fuzzy canceller.

2.1 Formulation of the Fuzzy Interference Canceller

The canceller subtracts the interference signal from the received one in (1). Therefore, it is necessary a non-linear filter able to estimate i_k in a non-Gaussian noise environment. When the interference is analog, the conditional mean estimator of (3) is the optimal one

$$\hat{i} = E\{i \mid r\} = \int_i i p(i \mid r) di \tag{3}$$

where p(i/r) represents the a posteriori probability of the interfering signal. Applying the Bayes theorem and the signal model in (1), p(i/r) can be equated as

$$p(i \mid r) = \frac{p(i)\sum_{p=1}^{Q} \lambda_p \exp\left(\frac{-\underline{z}_p^T \underline{z}_p}{2\sigma_w^2}\right)}{\int_i p(i)\sum_{p=1}^{Q} \lambda_p \exp\left(\frac{-\underline{z}_p^T \underline{z}_p}{2\sigma_w^2}\right)} \tag{4}$$

where $\mathbf{z}_p = \left[\mathbf{r}_p - \mathbf{m}_p - \mathbf{i}\right]^T$, $\mathbf{r}_p$ embraces all the received samples and $\mathbf{m}_p$ consists of all the possible noise and MAI states. Finally, p(i) is the a priori probability of the interference and λ_p is the a priori probability of the p noise state.

Combining (3) and (4) we get the following equation for the conditional mean of the interference

$$\hat{i} = \int_i ip(i \mid r)di \approx \sum_{l=1}^{L}\sum_{j=1}^{Q} i_l \frac{w_{j,l}\exp\left(\frac{-z_{j,l}^T z_{j,l}}{2\sigma_w^2}\right)}{\sum_{l=1}^{L}\sum_{o=0}^{K^N} w_{o,l}\exp\left(\frac{-z_{o,l}^T z_{o,l}}{2\sigma_w^2}\right)} \tag{5}$$

where $w_{j,l}$ is the product of p(j) and λ_l . Note that (5) can be seen as a fuzzy system with LxQ linguistic rules, exponential membership functions, Sugeno inference and centroid defuzzification. In other words, i_l represents the output centroid for the l-th rule, which is weighted by w_{jl}. Therefore, (5) can be reformulated as (6)

$$i_{fuzzy} = \sum_{j=1}^{M} i_j \Phi_j \tag{6}$$

where Φ_j is the fuzzy basis function of rule j. Under high Signal to Noise ratio conditions, the fuzzy interference estimator of (6) approximates the maximum a posteriori estimator as equated in (7)

$$i_{fuzzy} = \sum_{j=1}^{M} i_j \Phi_j \approx i_m \Phi_m \tag{6}$$

where Φ_m is the fuzzy basis function that presents a maximum value, which is close to one.

2.2 Expert Knowledge in the Fuzzy Interference Canceller

The fuzzy system is designed based on 4 variables: $\hat{i}_{k-3}, \hat{i}_{k-2}, r_{k-1}, r_k$. In order to reduce the fuzzy rule base, the variable r_{k-1} has been taken as reference of the input universe of discourse. Therefore, the input vector $\mathbf{x}$ is $\mathbf{x} = \left[\hat{i}_{k-3} - r_{k-1}\ \hat{i}_{k-2} - r_{k-1}\ r_k - r_{k-1}\right]^T = \left[x_1\ x_2\ x_3\right]^T$. The fuzzy rule base has the following structure

$$If\ x_1\ is\ A_1^j \quad and \quad x_2\ is\ A_2^j \quad and \quad x_3\ is\ A_3^j\ THEN\ y\ is\ B_j\ with\ weight\ w_j$$

where A_i^j, B_j are the input and output fuzzy sets respectively for rule j. The input and output variables have been modeled with F=7 fuzzy sets. The mean of the fuzzy sets takes the values $\{-3,-2,-1,0,1,2,3\}$ and the variance equal to 0.5. The fuzzy rule base is initialized in order to model the slow evolution of the narrow band interference when compared with the sampling time [21]. Finally, the weights are tuned by the Least Mean Square (LMS) algorithm of (7).

$$\theta_{j,k} = \theta_{j,k-1} + e_k \cdot \phi_{j,k-1} \cdot \left[w_{j,k-1} - \hat{i}_{fuzzy_k} \right] / \theta_{j,k-1} \tag{7}$$

where the error e_k is

$$e_k = \left(r_k - \hat{i}_k \right) - sign\left(r_k - \hat{i}_k \right) \tag{8}$$

2.3 Simulations

Next the fuzzy system has been compared with the two-sided linear filter of [22-23] with 10 coefficients. This coefficients and the the centroids of the fuzzy filter have been adapted with a decision directed LMS. Fig. 1 shows the Signal to Noise and Interference Ratio improvement when the interference is modeled as an autoregressive process. The SNR is equal to 20 dB. 10 Monte Carlo runs have been conducted with 660 samples for training each and 8000 for evaluation. Note the better performance of the fuzzy filter. In Fig. 2 the Bit Error Rate (BER) has been evaluated for SNR=20 dB and a multiple access interference of K=3 users and spreading factor of SF=11. As the weighted fuzzy filter takes into account the states of the MAI, it outperforms both the linear and the non-weighted filter.

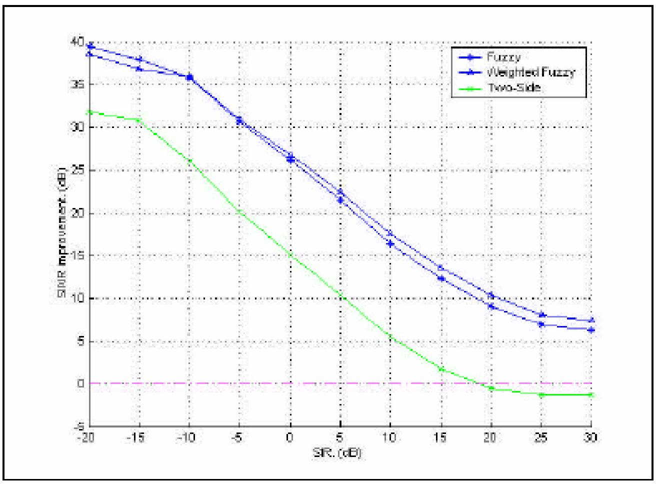

Fig. 1. Suppression of an AR interference. SNR= 20 dB.

Fig. 3 evaluates the performance of the fuzzy filter depending on the Signal to Noise ratio (SNR) for SIR equal to –20 dB. The only interference in the scenario has been modeled as an autoregressive process. The fuzzy filter has been compared with the optimum filter (DDK) and with the minimum mean square error one (MMSE). Note that for low power of Gaussian noise (i.e. in an interference limited scenario), the fuzzy filter outperforms the other ones. Finally, Fig. 4 shows the fast acquisition time of the fuzzy interference canceller due to the initial expert knowledge incorporated in the rule base.

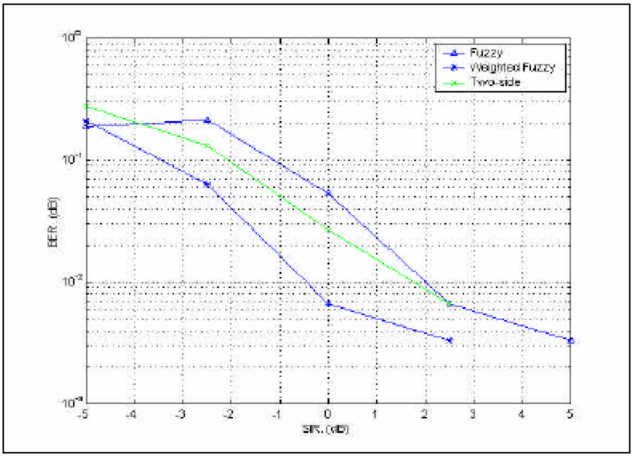

Fig. 2. Suppression of a digital interference. SNR= 20 dB and MAI of 3 users.

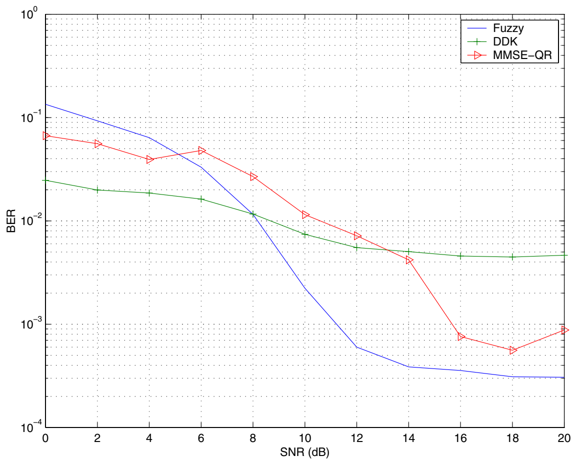

Fig. 3. Suppression of a digital interference. SIR= -20 dB and autoregressive interference.

2.4 Conclusions

The designed fuzzy filter is able to cope with both analog and digital interference even in the presence of MAI. On the other hand, due to the difficulty of statistical modeling, classical filters, which relay just on statistics, are not able to cope with this complex situation. The initial fuzzy rule base is built up from expert knowledge and can be trained with data whenever available; thus, approaching to the optimum MAP interference estimator. Therefore, the system is scalable with the available information and if only a short training is possible, the expert knowledge incorporated in the rule base guarantees a better performance than existing interference cancellers. In fuzzy systems optimality is pursued by emulating an expert operator. This is the best

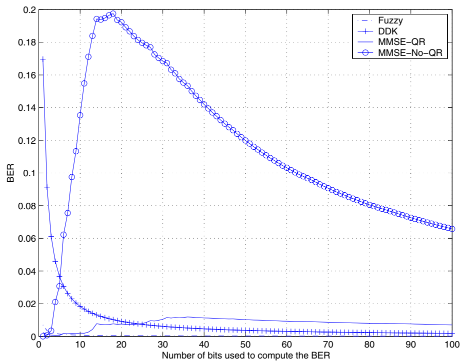

Fig. 4. Acquisition time

that can be done in the context of fuzzy logic. It is worth noting, however, that in all cases of queuing control where a mathematically optimal solution is known, as it is the case of interference estimation in CDMA, the fuzzy system yields precisely the same optimal solutions. In addition, in [24] the authors design a hierarchical rule base that reduces the computational complexity without degrading performance in most of the cases.

Next section presents a fuzzy beamformer that takes advantage of the fuzzy rule base in order to obtain a design close to the physical properties of the scenario and, therefore, easy to implement and to adjust when compared to existing non-fuzzy beamformers.

3 Fuzzy-Inference-Based Robust Beamforming

Adaptive array beamformers are used to spatially discriminate a desired signal from the unwanted interferers. They usually operate with the spatial signature or steering vector associated with the signal source to be filtered out, and they typically produce a constant array response towards that direction while minimizing other contributions (see Fig. 5). Significant degradation appears when the desired steering vector is not known exactly [25]. It is specially noticeable when the number of snapshots is low (i.e. the so called sample support problem) and gets worse for high Signal to Noise plus Interference Ratio (SNIR). The phenomenon is that desired information is treated as interference and consequently nulled.

Numerous methods have been proposed. We classify them depending on the knowledge they require from the uncertain desired Direction of Arrival or DOA. Traditional approaches require a nominal DOA and its corresponding uncertainty range [26-28]. The approach in these techniques is to gain robustness related to DOA errors at the expenses of decreasing interference and noise suppression and in general the problem of interference within the uncertainty range for the DOA is not addressed. A different approach consists in computing a DOA estimate and proceed as if the DOA were already known [29], yielding the so-called Direction-based techniques. Finally, the third group of techniques in complexity order contains subspace techniques, as in

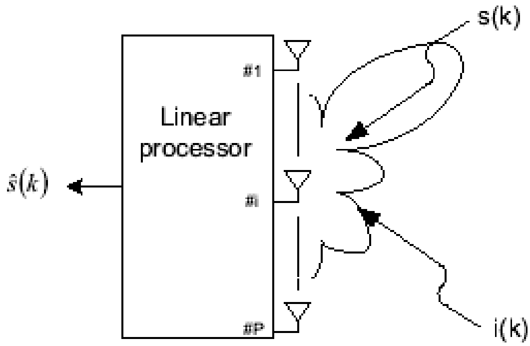

Fig. 5. An example of spatial linear processing to filter out a signal s(k) in presence of an interference i(k)

[30] or references therein, where the signal plus interference subspace is estimated in order to reduce mismatch. However, they also suffer from significant performance degradation when the available data doesn't provide good estimates. The above mentioned techniques resort to different robust signal processing schemes, as for instance: regularization, minimax/worst-case design or Bayesian approaches.

This paper uses fuzzy logic as another tool worth considering when imprecise a priori knowledge of input characteristics makes the sensitivity to deviations from assumed conditions an important factor in the design [31]. We derive a direction-finding based beamformer that describes DOA imprecision by means of fuzzy sets, which does not make statistical assumptions on interference and noise. An important issue in robust beamforming is the design or adjustment of parameters, whose values trade-off between robustness and array gain. In that sense, physical interpretability as in the proposed fuzzy techniques is always desirable.

This part is organized as follows. Section 3.1 states the problem. The fuzzy inference based beamformers are developed in Section 3.2, Section 3.3 parameter design, Section 3.4 presents performance examples and a summary is given in Section 3.5.

3.1 Problem Statement

The narrowband beamformer is a linear processor and consists of a set of P complex weights that combine the signals received at P sensors with the objective of filtering out a signal s(k) (see (9)) that impinges the array from a specific spatial direction (see Fig. 5)

$$\hat{s}(k) = \boldsymbol{w}^H \boldsymbol{x}(k) \tag{9}$$

where k is time index, $\mathbf{x}(k) = \left[x_1(k) \cdots x_P(k)\right]^T$ is the complex vector of array observations, $\mathbf{w}(k) = \left[w_1(k) \cdots w_P(k)\right]^T$ is the complex vector of beamformer weights, P is the number of array sensors. The base band observation data **x**(k) is

$$\begin{aligned} \boldsymbol{x}(k) &= \boldsymbol{s}(k) + \boldsymbol{i}(k) + \boldsymbol{n}(k) \\ &= s(k) \cdot \boldsymbol{a}_d + \boldsymbol{i}(k) + \boldsymbol{n}(k) \end{aligned} \tag{10}$$

where **s**(k) represents the desired signal contribution, **i**(k) the interference and **n**(k) is noise. Note that we have decomposed **s**(k) into desired signal waveform s(k) and desired signal steering vector $\mathbf{a}_d$, which contains the spatial information. It is easily modeled resorting to wave propagation theory and array geometry.

The weights are chosen accordingly to some optimum criterion, such as maximum SNIR, minimum mean square error or minimum power distortionless response (MPDR). All of them equate

$$\boldsymbol{w}_{opt} = \mu \boldsymbol{R}_x^{-1} \boldsymbol{a}_d \tag{11}$$

where μ is a scale factor and $\mathbf{R}_x = E\{\mathbf{x}\mathbf{x}^H\}$ is the data covariance matrix. A beamformer having this form is often referred to as the "optimum beamformer". Its performance strongly depends on both $\mathbf{a}_d$ and $\mathbf{R}_x$ in those common practical applications where they are obtained through estimation. In this paper we consider no knowledge of the desired DOA and that K snapshots are available for the estimation of both the covariance matrix and the DOA. The array is assumed to be calibrated, so errors in the spatial signature come form the DOA estimate.

The Sample Matrix Inversion (SMI) method is used for adaptive beamforming. The weights are updated every K snapshots using the K-sample covariance matrix of (12), as well as the DOA estimate, which is obtained with the Capon estimator; thus, obtaining the so-called Capon beamformer. Other estimates are possible, however, without loss of generality for the proposed techniques

$$\hat{\boldsymbol{R}}_x = \frac{1}{K} \sum_{k=1}^{K} \boldsymbol{x}\boldsymbol{x}^H \tag{12}$$

Direction-finding based beamformers suffer from significant performance degradation when the DOA estimates are not reliable, because of low number of snapshots or low SNR. Next section develops an adaptive beamformer which balances the user of observed data and approximate DOA knowledge.

3.2 Fuzzy Inference Based Beamformer

Assuming partial knowledge about the desired DOA (i.e. a nominal DOA and uncertainty region), we aim to use the capability of fuzzy systems to approximate any continuous function on a compact (closed and bounded) domain to obtain a reliable estimate of s(k). Fuzzy theory states that it is always possible to find a finite number of rule patches (which describe the system) to cover the graph of *f* while keeping the distance $|f(x) - F(x)|$ as small as we please, being F(x) the fuzzy approximation function.

We consider one input variable to the system, the DOA candidate u_d that we extract from input data (Capon estimator). In order to take into account DOA errors, we define an interval of possible DOA values, the prior and L fuzzy sets $\{A_i, i=1...L\}$ are placed equispaced along it. They describe the imprecise DOA estimate. As far as we are also concerned about practicality and implementation issues, we choose input fuzzy sets to be triangular. Their membership degree over the variable u can be expressed as

$$\mu_{A_i} = \begin{cases} 1 - \frac{|u-u_i|}{amp \cdot d_p} & if\ |u-u_i| \leq amp \cdot d_p \\ 0 & if\ |u-u_i| > amp \cdot d_p \end{cases} \tag{13}$$

with $d_p = \frac{u_{d_{min}} - u_{d_{max}}}{L}$, where u_{dmin}-u_{dmax} stands for the prior width. Widths are set using amp and $u_i = \frac{d}{2}(2(i-1)-(L-1))$.

The choice for the output fuzzy sets follows from the conditional mean beamfomer w_{CM} in (14)

$$\begin{aligned} \hat{s}(k) &= E\{\hat{s}(k)/\boldsymbol{X}\} \\ &= \sum_{i=1}^{L} p(u_i/\boldsymbol{X})E\{\hat{s}(k)/\boldsymbol{X}, u_i\} \\ &= \sum_{i=1}^{L} p(u_i/\boldsymbol{X})\boldsymbol{w}_{opt}^{H}(u_i)\boldsymbol{x} = \boldsymbol{w}_{CM}^{H}\boldsymbol{x} \end{aligned} \tag{14}$$

where X represents the available data set and $p(u_i/\mathbf{X})$ is the a posteriori probability density function (pdf). The global estimation problem is divided into L smaller problems that assume fixed input parameters u_i and that are gain controlled by the probability of each possible signal incoming direction u_i given the data set.

In light of (14), it is clear that the conditional mean beamformer is optimal as far as L is big enough and the a posteriori pdf is assumed perfect. In that asymptotic case, the conditional mean beamformer is the optimum beamfomer in the minimum mean square error sense. When a finite (and low) number of optimum beamformers are available, a fuzzy approximator that does not rely on statistics but on expert knowledge can be derived. The output fuzzy sets B_i are designed accordingly as singletons placed at $\mathbf{w}_{opt}^{H}(u_i)\mathbf{x}$, which, in other words, is the output of the "optimum beamfomer" pointing at u_i. Due to philosophy similarities, we use the Bayesinan beamformer [28] as a benchmark for comparison. Indeed, it is derived from (14) and uses a parametric model for the pdf.

To completely describe the system, assuming we implement additive fuzzy systems with singleton fuzzification, product inference and centroid defuzzification [5], we establish the rules Ri, i=1…L that relate inputs with outputs:

R_i: IF the DOA is A_i THEN the desired signal is B_i

It can be shown that the final expression of the spatial filter is the one given in (15)

$$w_F = \sum_{i=1}^{L} \frac{\mu_{A_i}(\hat{\mu}_d)}{\sum_{j=1}^{L} \mu_{A_i}(\hat{\mu}_d)} \, w_{opt}(u_i) \tag{15}$$

Fig. 6 shows an operation example of the beamformer for the case L=3. Note that although statistics play an important role in computing both the output fuzzy sets and the input to the system, the rules are transparent to statistics and rely to a knowledge based approach. Thus, the system is not so model-dependent and consequently model-limited as for example the Bayesian one. Another important aspect is how we tune parameters to incorporate expert knowledge. This is discussed in the next section.

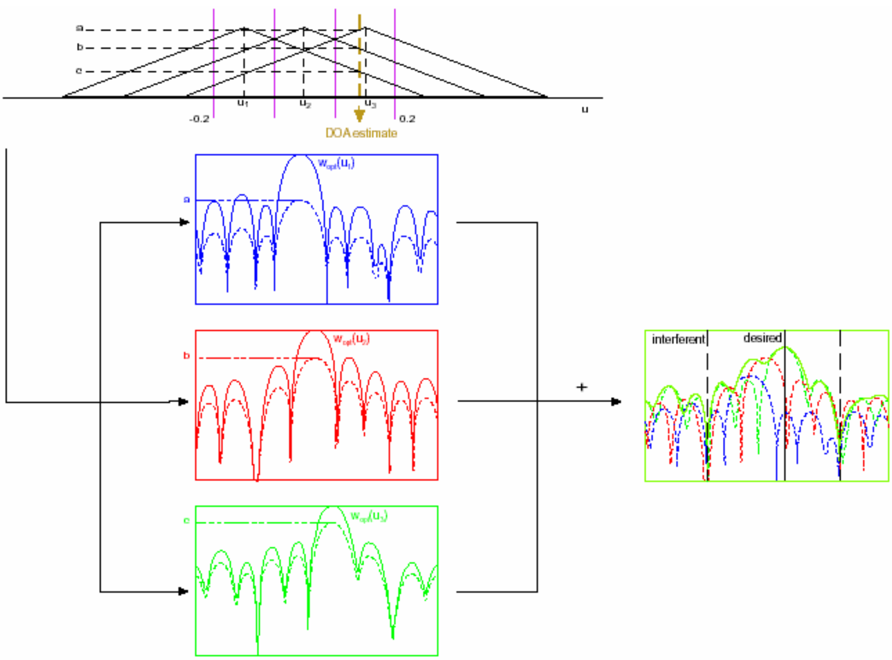

Fig. 6. Beamforming example for the case L=3

3.3 Parameter Design

The fitting quality of the designed fuzzy beamformer will strongly depend on the election of the parameters related to the input fuzzy sets (width, mean and number) and the number of snapshots K. The number of fuzzy sets L is established on complexity criteria. For the means and widths, u_i and *amp* respectively, it is possible to tune them with learning algorithms such as the stochastic gradient descent on the squared error [31]. In that case a training sequence is necessary. For practicality reasons, this work only considers the set up of the fuzzy sets widths and keeps fixed

Table 1. Heuristic rules for tuning the fuzzy sets widths function of SNR and in-prior SIR

SNR	In-pr. SIR	Width
$\geq 0dB$	$\geq 2dB$	$\frac{1}{6}L$
$0dB \geq \text{SNR} > -10dB$	$\geq 2dB$	$\frac{1}{2}L$
$\leq -10dB$	$\geq 2dB$	L
$\forall$	$< 2dB$	$1.5\ L$

means. It is done heuristically as shown in Table1 in order to make the beamformer easy tuning and user friendly. Note that the scenario is described by the SNR and the in-prior Signal to Interference ratio (SIR).

The DOA description by means of fuzzy sets is less critical; actually, good empirical results are achieved when a low number of fuzzy sets (L=6) is used.

If information reaches the sensors spatially spread or distributed over some known angular region, it is possible to incorporate this information into the fuzzy system thanks to non-singleton fuzzification.

Finally, the last parameter to fix is the number of snapshots K. It is generally chosen as large as possible to get a good estimate of the data correlation matrix and DOA but small enough so that temporal fluctuations may be tracked. Randomness in the covariance matrix causes the cancellation of the desired signal in the $\mathbf{w}_{opt}$ beamformer of (15). One way of diminishing these problems is using diagonal loading [26] at the expenses of less interference suppression.

Next section presents some significative results.

3.4 Simulations

We assume a uniform linear array with 10 omnidirectional sensors spaced half a wavelength apart. The uncertainty in the DOA of the desired signal is over the region [-0.2, 0.2] and it has been equally divided into L=6 intervals. We are mostly interested in evaluating the performance of the proposed fuzzy beamformer from the point of view of array gain. This figure of merit is defined as

$$G = \frac{SNIR_{out}}{SNIR_{in}} = \frac{|w^H a_d|^2}{w^H \rho_n \, w} \tag{16}$$

where $\mathbf{w}$ stands for the weights of the beamfomer, $\mathbf{a}_d$ for the desired signal steering vector and $\boldsymbol{\rho}_n$ represents noise plus interferers normalized covariance matrix.

Just to make the understanding of the proposed beamformer easier, the results of the fuzzy beamformer formulated in (15) are presented together with those obtained by the Minimum Power Distortionless Response (MPDR) beamformer of (11) and the Bayesian beamformer of [28]. Note that the fuzzy inference based beamformer combines both philosophies: it is a direction-finding based beamformer, such as the MPDR one, and is able to cope with DOA uncertainties such as the Bayesian beamformer. This fact motivates the election of these two techniques, although they imply less computational complexity. We note in addition that there are many other approaches, each one having its own advantages, problems and applications, where neither is absolutely better than the other. In [28], the Bayesian beamformer is extensively compared with the linearly constrained minimum variance beamformer using directional constraints and a subspace beamformer. Thus, for the sake of clarity we do not include them in this paper and refer to the conclusions in [28]. The main goal of this simulation section is to show that the fuzzy inference beamformer is an alternative technique easy to implement and worth considering in scenarios with specific features such as different interference conditions.

Next we make a comparative study between the Bayesian and fuzzy beamformers. The objective is to see how easy is to adjust parameters in both beamformers. In the Bayesian technique, we have to adjust the γ parameter, that establishes the confidence given to the calculated a posteriori probabilities. The basic parameter for the fuzzy beamformer is the width of the fuzzy sets. In this study, we consider all other variables without error, i.e. no DOA error, perfect covariance matrix, etc. . . , although the presence of interference inside the prior interval plays an important role for deciding the signal to be focused. Bayesian beamformer computes itself an estimate of the pdf of the DOA from the data, while fuzzy beamformer departs from a given estimate (therefore we study it both focusing desired or interferent fonts). The different choices for the parameters establish a trade-off between robustness and performance.

Of key importance is how sensitive the performance of the beamformer is to the setting of its design parameters. Thus, high sensitivity implies low practicality of the beamformer. Figures 7 and 8 depict the array gain variations along with the fuzzy sets widths (fuzzy) and γ parameter (Bayesian), respectively. Three different SNR are considered, and an in-prior interference is simulated. Note that there is in general a trade-off between array gain at high SNR and acceptable performance at low SNR (for the fuzzy beamformer we assume that at SIR < 0dB, the DOA estimate points towards the interference). The reader can appreciate the smoother evolution that fuzzy beamformer provides. Finally, Fig. 9 shows the robustness or less sensitivity of the fuzzy system for different values of DOA misadjustment when compared with the Bayesian system. The presence of interference within the uncertainty range of the desired signal is not taken into account in the statistical model of the Bayesian beamformer; thus, its worst performance.

3.5 Conclusions

This work makes use of fuzzy logic systems as universal model-free function approximators and proposes a fuzzy inference based beamformer. The obtained beamformer is a direction-finding based technique that offers a robust approximation of the conditional mean estimate of the desired signal. The term robust is quite wide and this work focuses on the problem of DOA uncertainty in scenarios where interference signals are present. We note that no constraints have been imposed on the nature of the sources (i.e.point or spread). As expected, due to the soft DOA quantization, the fuzzy approach presents a “graceful" degradation when the working conditions are different from those expected. Additionally, the robustness of the presented beamformer applies also when adjusting its design parameters (fuzzy sets widths and number of beams). Because of the fuzzy systems interpretability, the parameters are easy to set once the scenario is known, thus demonstrating its practicality.

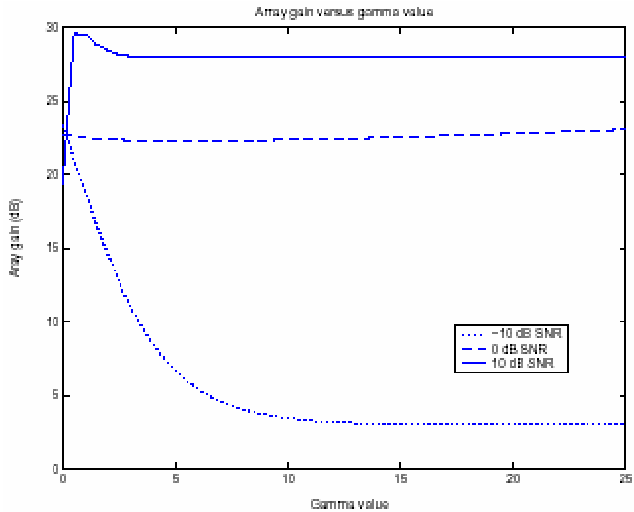

Fig. 7. Bayesian beamformer array gain versus γ value at different SNR, u_d=0.14, interferent directions u_{int}= [-0.5, 0.6, -0.07] and INR=[20, 20, 0] dB

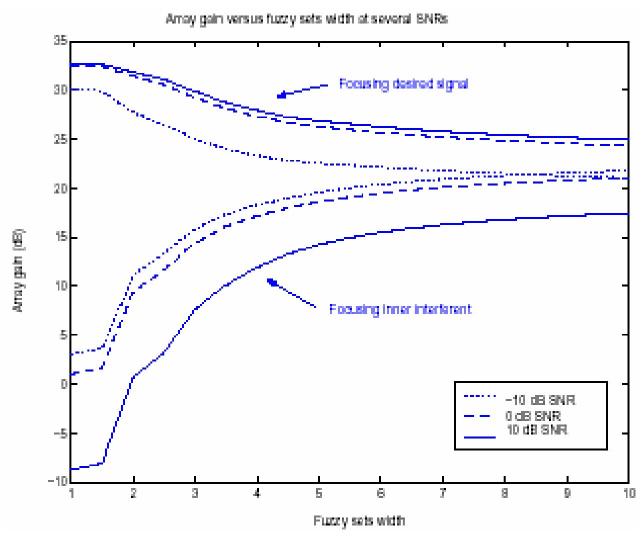

Fig. 8. Fuzzy beamformer array gain versus fuzzy sets width at different SNR focusing either desired or in-prior interference signal. Same scenario as Fig. 7.

Next section 4 is devoted to fuzzy classification for signal separation in 2-Dimensional spaces. It shows the greater flexibility that fuzzy systems offer in front of classical classifiers. Better results are then obtained in most of the cases, however, in order to take advantage of the great potential of fuzzy systems, expert knowledge would be needed.

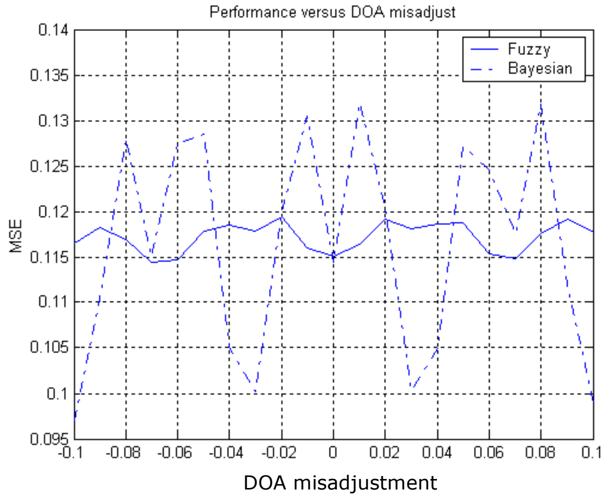

Fig. 9. MMSE versus DOA misadjustment, u_d=0, SNR = 0dB, u_{int}={-0.5, 0.6},INR={20, 20} dB

4 Fuzzy Logic for Signal Classification

This work addresses the problem of signal separation for 2-Dimensional spaces. Instead of resorting to statistical properties of the signals, this work treats the problem as one of image segmentation. Variants of known fuzzy classifiers are studied and compared with existing techniques, as the Unsupervised Maximum Likelihood (MLU) classifier or the watershed technique. The goal is the separation of seismic waves collected from experimental data.

Whenever there is uncertainty in the statistical model, fuzzy logic can be useful. This is maybe the case of supervised classification problems when the number of training data is low, or when there is lack of knowledge in the underlying parametric model as it is the case of geophysical signals, such as the ones addressed in this paper. This work aims at seismic wave separation by means of signal classification. Section 4.1 reviews a selection of the existing techniques [1,2] and studies how to treat fuzziness in order to better manage uncertainty. Section 4.2 applies these techniques to image segmentation for seismic wave separation or identification. Finally conclusions come in Section 4.3.

4.1 Fuzzy Unsupervised Classifiers

We focus on unsupervised classifiers that are going to be applied to 2-Dimensional signal separation, also called image segmentation. The algorithms like clustering or fuzzy C-means (FCM) [32-33], unsupervised Maximum Likelihood (MLU) [34] and watershed (W) [35] are the ones to be studied. From the simulations that we have carried out, we can conclude that a variant of the FCM, the so-called FACM (proposed by Gustafson and Kessel [32-33]), is the one that on average gives a better performance.

4.1.1 Fuzzy a C-Means

This algorithm minimizes the following distance of samples $\mathbf{x}_k$ to the cluster centers $\mathbf{v}_i$

$$J_m = \sum_{i=1}^{C}\sum_{k=1}^{N}(\mu_{ik})^m (\mathbf{x}_k - \mathbf{v}_i)^T \mathbf{A}_i (\mathbf{x}_k - \mathbf{v}_i) \tag{17}$$

where

$$\underset{1\le i\le C}{\forall} A_i = [\rho_i \det C_{Fi}]^{\frac{1}{D}} C_{Fi}^{-1} \qquad \rho_i = \det(\mathbf{A}_i) \tag{18}$$

where D is the dimension of the feature space (e.g. D=3) and

$$C_{Fi} = \left[\sum_{k=1}^{N}(\mu_{ik})^m (\mathbf{x}_k - \mathbf{v}_i)(\mathbf{x}_k - \mathbf{v}_i)^T\right]\left[\sum_{k=1}^{N}(\mu_{ik})^m\right]^{-1} \tag{19}$$

where the fuzziness is controlled by factor “m”.

Along the study that we have carried out we have observed a better performance when matrix Ai (the Mahalanobis distance) is introduced to the Fuzzy c-means. In this way the Mahalanobis distance defines an ellipsoid with an specific volume centered at each cluster (that we can refer to an image). Also better performance is obtained if the membership function is designed as

$$\underset{\substack{1\le i\le C\\1\le k\le N}}{\forall} \mu_{ik} = \left(\frac{1}{d_{ik}^{md}}\right)^{\frac{1}{m-1}}\left[\sum_{j=1}^{C}\left(\frac{1}{d_{jk}^{md}}\right)^{\frac{2}{m-1}}\right] \tag{20}$$

where

$$d_{ik} = (\mathbf{x}_k - \mathbf{v}_i)^T \mathbf{A}_i (\mathbf{x}_k - \mathbf{v}_i) \tag{21}$$

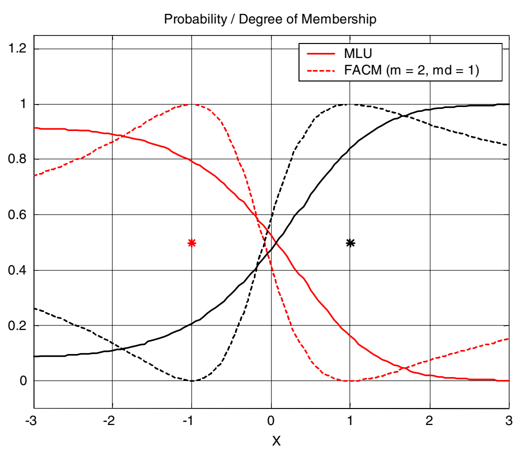

Fig. 10. Comparison of membership functions between MLU and FACM (classes with different variance show different borders)

Note that with respect to the Fuzzy c-means we have incorporated the parameter "md", which helps to better tune the membership function. For md>1, the membership degree of those points close to the cluster center are emphasized with respect to those more far apart.

A more closed mathematical analysis [36] reveals that FACM is like a version of the MLU, that uses a more generic kernel function as the Gaussian one used by MLU (we can see an example in Fig. 10). Therefore, FACM has more degrees of freedom to adapt to the data; thus, offering better results if it is properly tuned.

4.1.2 Fuzzy Watershed

The conventional morphological segmentation technique is the watershed transform [35]. The idea of watershed is drawn from a topographic analogy. Consider the gray-level intensity as a topographic relief. Find the minima and "pierce" them. Immerse the whole relief into water and let the water flood the areas adjacent to the piercing points. As the relief goes down some of the flooded areas will tend to merge; prevent this happening by raising infinitely tall dams along the watershed lines. When finished, the resulting network of dams defines the watershed of the image. Each of the lakes that have been formed are called catchment basins and correspond to the resulting classes of the classifier. Fig. 11 shows an example of watershed in a section of topographic surface.

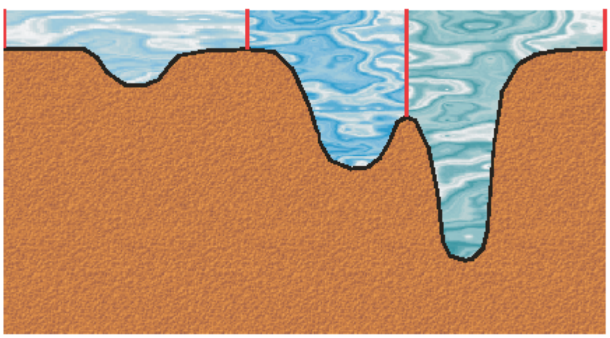

Fig. 11. Example of watershed in a section of topographic surface

Note that in the watershed segmentation there is no intersection between regions. As there is a great deal of ambiguity in the segmentation process, we studied the possibility of fuzzy membership degrees to allow the different catchment basins to intersect (in an algorithm that we call fuzzy Watershed or FW). However, the lack of a clear feature or knowledge to design the fuzzy membership functions, makes this extra degree of freedom in general useless as we show next.

4.1.3 Simulations

We have generated 12 different 2-Dimensional data of 100,000 samples each in order to evaluate the performance of the studied classifiers. Table 2 shows the probability of misclassification and Table 3 the mean error when reconstructing the image from the

Table 2. Misclassification error

Classifier	**Gaussian**	**NonGaussian**
CM	0.2404	0.3505
FCM	0.2436	0.3536
ACM	0.1226	0.1693
FACM	0.1175	0.1446
MLU	0.1071	0.1455

Table 3. Reconstruction error

Classifier	**Gaussian**	**NonGaussian**
CM	$7.447\ 10^{-8}$	$3.056\ 10^{-8}$
FCM	$3.256\ 10^{-8}$	$1.755\ 10^{-8}$
ACM	$2.839\ 10^{-8}$	$2.240\ 10^{-8}$
FACM	$6.707\ 10^{-9}$	$9.397\ 10^{-9}$
MLU	$1.131\ 10^{-11}$	$1.305\ 10^{-8}$

classified data. Although the FACM, the ACM and the UML are very similar in performance, note the better behavior of the FACM in front of the ACM, that is the c-means with the Mahalanobis distance. Note also the similar performance of the FACM and the unsupervised ML, although the UML presents a worst behavior in front of non-Gaussian shapes.

Next section considers in addition the watershed technique and fuzzy variants for the seismic image segmentation.

4.2 Separation of Seismic Signals

This application departs from a series of temporal signals measured in geological prospecting. The aim is to separate the different component waves.

4.2.1 Introduction to Seismic Prospecting

Seismic prospecting allows to know the structure of the earth underneath. A small explosive detonates on the surface and an array of sensors measures the generated waves in the subsoil. There are as many seismic waves as layers (between 6 and 10 Km of depth), see Fig.12. As the transmission speed of the waves in the different materials is known, the subsoil composition can be studied by analyzing the amplitude variations of each wave if the terrain of the surface is known.

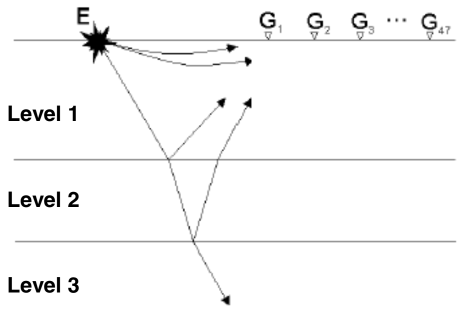

Fig. 12. Seismic profile

The generated waves belong to three different classes: i) waves P (primary), ii) waves S (secondary), and iii) waves L (long). Waves P are internal and longitudinal and fastest than waves S, which are internal but transversal. Waves L are superficial and of big amplitude and they cause the damages during the earthquakes. The explosions during seismic prospection cause mainly P and S waves.

4.2.2 Experimental Data

The experimental data that has been used in this work consists of a sequence of 47 temporal signals of 512 samples each, which, after an explosion, have been captured by each of the 47 seismic sensors. Fig. 13 shows the data, where we can see 4 different waves that separate as they propagate along the array because of the different propagation speeds.

4.2.3 Wave Separation

Before initiating the separation process, the data are pre-processed by means of the wavelet transform, which extracts the most relevant features in order to help the classifier in the separation process.

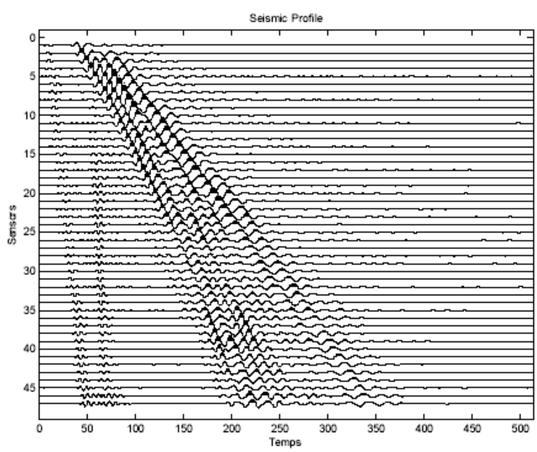

Fig. 13. Experimental seismic data: 47 sensors and 512 temporal samples at each sensor

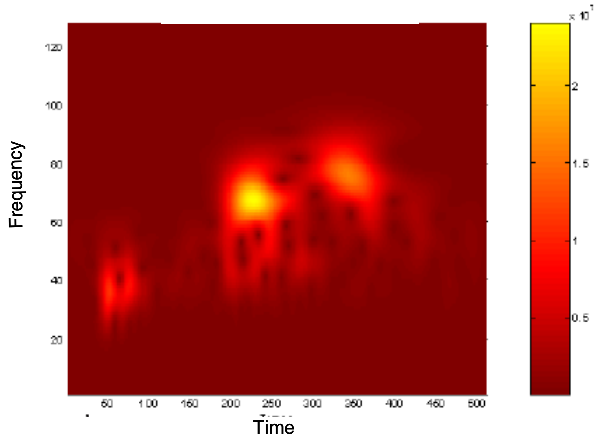

Fig. 14. Modulus of the time-frequency representation of sensor 47

The wavelet transform [6] obtains a representation in time and frequency for each of the signals that are measured at each sensor. Fig. 14 shows the modulus (scalogram) of the wavelet transform for sensor number 47. Note that 4 energy centers can be observed, which correspond to the 4 different temporal waves that propagate along the sensors. The separation is carried out by considering this energy distribution: each energy concentration is considered a different class.

Once the scalogram has been properly divided into segments by the appropriate classifier, the inverse wavelet transform is applied in order to obtain the separated signal in the time domain.

Before working with the experimental data, test or synthetic data has been used in order to evaluate the different methods described in section 4.1. 15 test signals have been generated by mixing different waveforms: sinusoids, wavelet kernels as Morlet type, Mexican hat, and Gaussian.

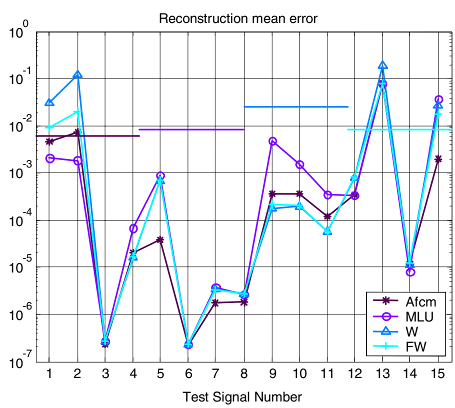

Fig. 15. Mean error when recovering each of the 15 temporal signals

Fig. 15 shows the results of the unsupervised classifiers: FACM, ML, W and FW for each of the 15 temporal mixtures. Note that although the FACM behaves well, it is not the best option for all the signals. In general, for low level of superposition in the scalogram, the FACM is the best, for medium level, the ML is to be chosen and, finally, for high superposition level, the W presents the best results.

When applied to seismic data, the FACM presents an additional advantage when compared to the other techniques. FACM does not need to look for the scalogram

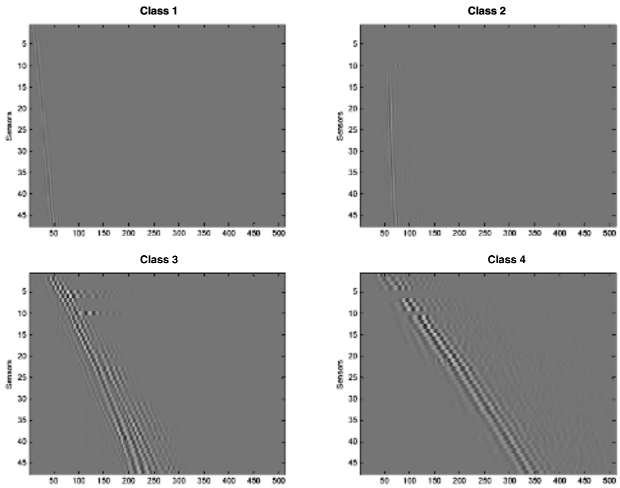

Fig. 16. Spatial-temporal profile of the seismic signal after separation via watershed

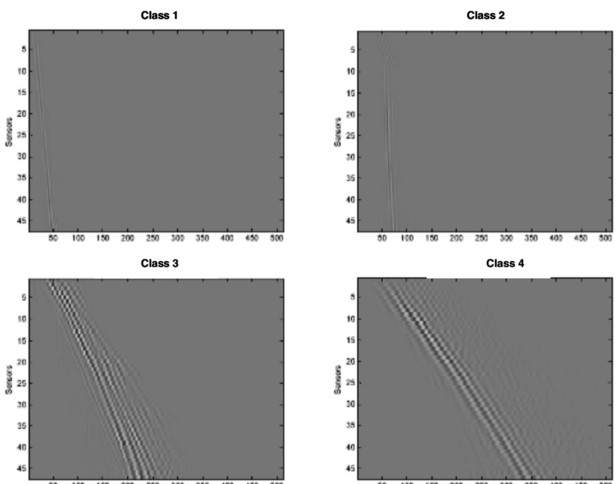

Fig. 17. Spatial-temporal profile of the seismic signal after separation via FACM (m=3, md=4 and m=2 for background class)

maxima image by image in a “manual” way. FACM can be initialized (i.e. cluster centers and matrix norm) with an image of well-separated clusters, as for instance the image obtained from the last sensor 47, and use the final parameters of one classification for initializing the classification of the next image. Thus taking advantage of the smooth evolution of the signal from sensor to sensor. We can also add one extra class used for background separation, leaving apart all the data points that doesn’t bring any energy to the classes.

After extensive simulations, we can conclude that there are not substantial differences among the methods, although the FACM behaves in general better than the others. As the watershed is the most used technique for image segmentation, in Fig. 16 we compare it against FACM in Fig. 17.

Note in Fig. 16 that from sensor 15, the classes become too close together for the watershed to separate them properly. In Fig. 17 these problems disappear because the FACM is able to follow the classes thanks to the initialization with signal from sensor number 47. Finally, Fig. 18 shows the 4 temporal waves in this last sensor after separation with FACM.

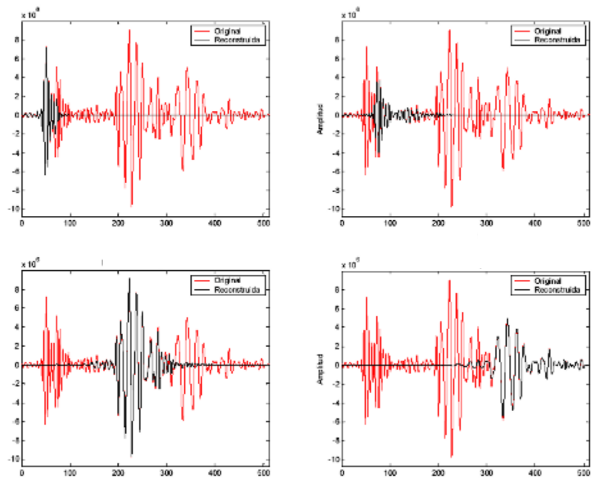

Fig. 18. Four waves after separation with FACM, comparison with the temporal signal of sensor 47

4.3 Conclusions

In this work fuzzy classifiers appear as a good alternative for image segmentation with the aim of seismic wave separation. With a proper tuning of their parameters their flexibility, when compared to other classifiers, allows them to adapt to many classification problems. Specifically, for the seismic wave classification problem the devised FACM techniques results in a good trade off between performance and complexity. This paper does not take into account expert knowledge of the seismic signal when designing the fuzzy system; thus, leaving open this point, which, to the authors believe can give promising results.

In the communication field, there is now a growing interest in reconfigurable receivers and cognitive radio. The study carried out in this work can be directed to blind recognition of the communication system in use (i.e. 3G or 3G systems); more specifically, to blind recognition or classification of the spectrum: channel bandwidth and shape, in order to reconfigure the entire architecture of the terminal with the appropriate software.

Up to now, the present work has applied fuzzy logic for signal filtering or separation at the bit or sample level. However the decision making of the rule base is also very useful for upper communication layers in the protocol stack that take into account input variables of different nature: congestion state, available load and total interference among others.

5 Fuzzy Logic at the Protocol Level: Horizontal Hand-Off

We have discussed on the application of fuzzy systems at the bit level so far. However, one of the main features of fuzzy systems is their explicit decision making. This feature is useful to carry out an intelligent filtering, as shown in the previous sections, but also to help design communication protocols. A protocol can be viewed as a control system, and control systems were one of the main applications of fuzzy logic. Protocols have to provide the users with Quality of Service (QoS) and this implies to cope with subjective variables, which are imprecise and difficult to quantify, as they depend on the user requirements. Analytical solutions do not exist many times for communications protocols; therefore, fuzzy control is a promising approach to the problem.

In this section we focus on the hand-off problem. Hand-off takes place when a movil terminal changes its cell or access point. When the change is done within the same communication system, the hand-off is horizontal and takes place when the QoS of the terminal diminishes and can be initiated either by the terminal or by the base station. Depending on the size of the area where the movil moves we can talk of micromobility or macromobility (see Fig. 19). The first one requires a hand-off at layer 2 or link layer, and the second one at layer 3 or network layer. We are concerned with the layer 2 hand-off initiated at the terminal when the received signal power falls below a threshold (see Fig. 20). When this occur, the terminal looks for an access point that offers it more signal strength. In order to reduce the so-called ping-pong effect among cells, the threshold has a hystereris. The hysteresis margin introduces a tolerance level above the received signal level from the current BS. In addition, there is a delay in the hand-off due to hysteresis. Thus a major obstacle facing the traditional hand-off with its hysteresis margin is the speed with which a hand-off can be made. It is for this reason that a better solution to hand-off is required, one that provides a fast yet stable performance. We propose to use fuzzy logic to design the thresholds so as to reduce the delay that the hand-off introduces in the signal transmission.

The thresholds are going to be design depending on the terminal profile, as for instance, its speed. Fig. 21 shows the proposed fuzzy controller for the hand-off threshold. Three fuzzy sets describe the universe of discourse for the speed and previous threshold is described with 5 fuzzy sets. Triangular fuzzy sets, min-max inference and centroide defuzzification are chosen for simplicity. The aim of the rule base is to optimize the threshold depending on the terminal speed. For high speed we would like to reduce the number of hand-offs because the terminal may go through many cells in a short time, thus the threshold should increase. On the other hand, for a slow terminal movement we would decrease the threshold level.

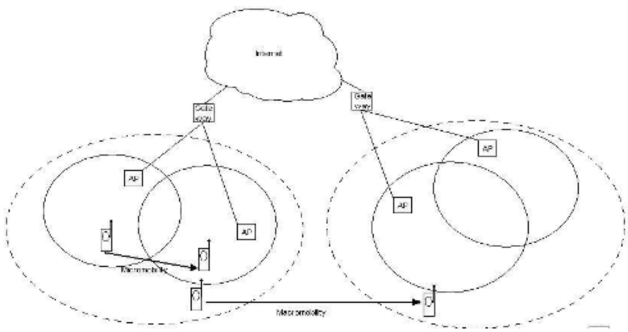

Fig. 19. Micromobility vs. macromobility

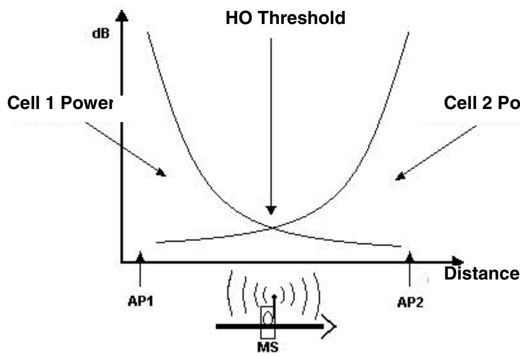

Fig. 20. Hand-off threshold at layer 2

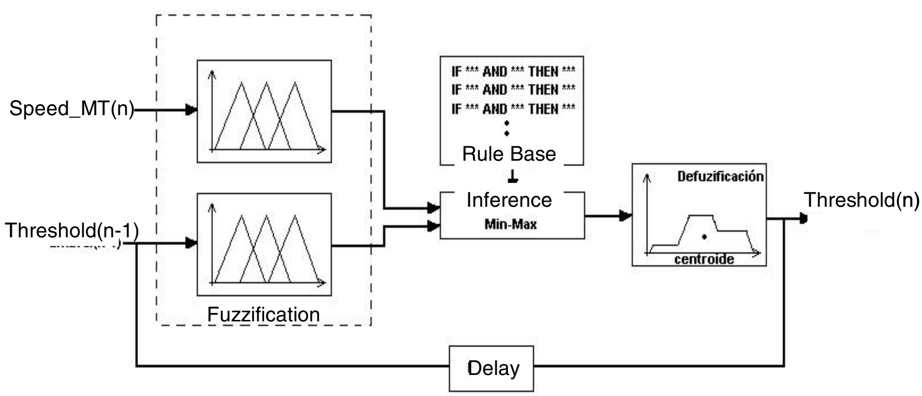

Fig. 21. Fuzzy controller for the hand-off threshold

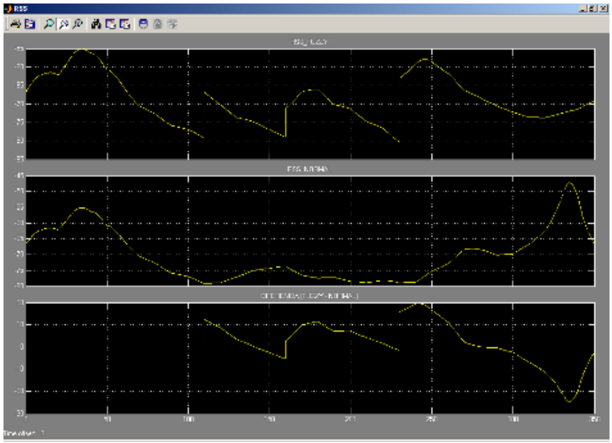

Fig. 22. Final received strength for the fuzzy controlled hand-off and for the conventional one

Fig. 22 plots the final power received by the access point for each of the strategies. In average, the fuzzy system offers more power, thus, better quality. However, depending on the application, voice for instance, the abrupt changes in the power might not be desirable. In case of file transfer, they are irrelevant. This fact motivates to incorporate the service type into the design of the final system. Future work is to incorporate more QoS variables in order to take the hand-off decision. Another aspect is the so-called soft hand-off, where two base stations or access points are received simultaneously during the hand-off. Fuzzy logic can then been used as an access point fusion technique.

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