

# An Energy Minimization Process for Extracting Eye Feature Based on Deformable Template

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**Abstract.** Eye feature extraction is of crucial importance for face recognition. Deformable template is an efficient model for this task. However, it usually suffers from the problem of local minima. To avoid local minima, in this paper, a new energy minimization process is proposed, which emphasizes on local properties of energy terms. The minimization process is divided into three steps. The iris is located firstly. Then the eye boundaries are adjusted. Finally, all energy terms are activated to tune the eye template. Each step needs not to be split to some sub-steps. Empirical comparison with other minimization processes shows the superiority of the proposed process in terms of both efficiency and accuracy.

## 1 Introduction

Eye feature extraction plays an important role in many applications, such as visual interpretation, recognition of human face [1], intelligent coding system and HCI (human-computer interface) [2, 3]. In the case of interpretation and recognition of human faces, most of attempts are made using geometrical features, where the relative positions and the shapes of the different features are measured. In HCI, the facial features, including the important part on face - eye, are extracted first, then these features are tracked to get the information of the facial expressions [3].

In eye feature extraction, the deformable template is an efficient model. Many methods [4-8] use deformable template to extract eye feature after the pioneer work of Yuille [9]. However, it always suffers from many problems, such as local minima, and low convergence speed. To improve the convergence speed, and to guarantee a good fit for avoiding local minima, minimization process has been extensively studied [4, 5, 7, 9] beside designing new energy function that can grasp the essence of eye feature. In this paper, we focus on the minimization process to improve the performance of eye feature extraction. The details of analyzing the problems about minimization process are described in section 2.

In this paper, a new minimization process is proposed to alleviate the problem of local minima. Considering the local properties of some energy terms, such as corner energy terms, the energy function of the deformable template is optimized in three epochs. The proposed method has been applied to real eye images.

The experiments show that the proposed method can balance the precise of eye feature localization and the time complexity.

The remainder of this paper is organized as follows. In Section 2, a review of some existing minimizing processes is provided. In Section 3, the geometric template and energy function used in our method is brief overviewed. The proposed minimization process is described in Section 4. The comparative experimental results for showing the superiority of the proposed method over some existing methods are presented in Section 5. Finally, the conclusions are given in Section 6.

## 2 Related Works

The existing minimization methods differ in both epochs and iteration methods. Summaries of three methods are given in Tables 1 to 3.

**Table 1.** Yuille *et al*'s method [9]

steps		Energy function	Parameters adjusted
Adjust iris	1	Valley	Iris location
	2	Valley, Intensity, Edge	Iris location and size
	3	Valley, Intensity, Edge	
Adjust eye Boundaries	4	Peak	Eye location and angle
	5	Peak, Intensity, Edge, Prior potential about eye boundaries	
Finely tune	6-8	All energy term	Eye location, size and angle All parameters

**Table 2.** Lam *et al*'s method [5]

steps		Energy function	Parameters adjusted
Adjust iris	1	Valley	Iris location and size
	2	Valley, Intensity, Edge	
Adjust eye Boundaries	3	Orientation, Boundary	Angle
	4	Edge, Intensity, Prior	Eye location and size
Finely tune	5	All energy term	All parameters

Some questions of these methods exist. First, is it necessary to divide each epoch into some sub-steps? In general, too many minimization steps would result in some parameters of the eye template being overly changed [4]. Some energy terms are conflicted, and only these conflicted terms reacted with the eye template would let the template deform correctly [4]. Second, whether the fewer epochs is the better? Some energy terms only work in small neighborhood. That is, only after the template is moved near the correct location, the energy

**Table 3.** Shan *et al*'s method [5]

steps	Energy function	Parameters adjusted
Adjust iris	1 Weighted edge map, Internal, Prior	Iris location and size
Adjust upper eyelid boundary	2 Weighted edge map, Internal, Prior	Upper eyelid boundary
Adjust lower eye boundary	3 Weighted edge map, Internal, Prior	Lower eyelid boundary

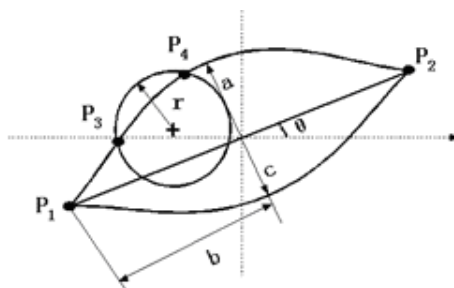
terms could interact with the input image to deform the template. For example, the corner energy term in [4] uses local information, which would interact with the input image only after the corners of the template near the right locations. If all energy terms are activated with the template in only one step, from our experiment results, the problem of local minima is serious and the convergence speed is slow in many cases.

Using too many epochs (or steps) in minimization process will result in parameters being overly changed, and using too few epochs is also harmful to the minimizing process. A minimization process with suitable epochs should be designed to minimize the energy function.

### 3 Geometric Model and Energy Function

#### 3.1 Geometric Model

The eye template used in our system is parameterized as in Fig. 1 [8]. The circle is centered at  $X_c$  with radius  $r$ . The two half parabolas are centered at  $X_e$ , both with width  $b$ . The upper and lower parabolic curves have the heights  $a$  and  $c$ , respectively. These curves intersect at the four points  $P_1$ ,  $P_2$ ,  $P_3$ ,  $P_4$ . Therefore, the eye template could be represented by  $(X_c, X_e, r, a, b, c, \theta)$ . All parameters are allowed to change.

**Fig. 1.** The eye template in our system

### 3.2 Energy Function

The energy function used in our system is the same as [8] except using the corner strength function in SUSAN (Smallest Unvalue Segment Assimilating Nucleus) [9] detector to define the corner energy term. The corner energy term is defined as

$$E_c = 1 - \frac{1}{n} \sum_{i=1}^n Corstr_i . \quad (1)$$

where  $Corstr_i$  is the value of corner strength at the  $i$ -th corner,  $n$  is the possible number of corners in the eye template.

The corner strength in pixel  $(x, y)$ , where  $x$  and  $y$  are the coordinate values in  $X$  and  $Y$  direction respectively, is defined as

$$Corstr_i = \begin{cases} g - n(x, y) & \text{if } n(x, y) < g \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where  $n(x, y)$  is just the number of pixels in the USAN (Unvalue Segment Assimilating Nucleus). The fixed threshold  $g$  (the "geometric threshold"), which is set to  $n_{max}/2$  or even smaller to detect sharper corners, where  $n_{max}$  is the maximum value which  $n$  can take. To be consistent with other energy terms, which are normalized to  $(0, 1)$ , the corner strength is also normalized. The details of SUSAN detector can refer to [9].

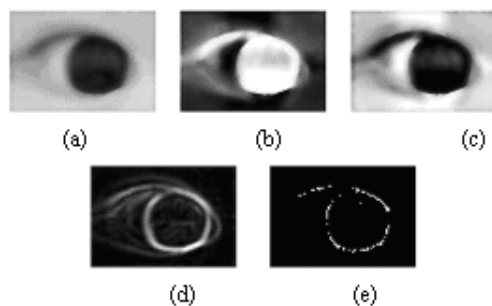
Then the energy function is defined in terms of the deformation of the template based on these fields. The details of other energy terms can refer to [4, 8].

## 4 Minimization Process

To avoid local minima, and to improve the convergence speed, improvement on minimization process for energy function is required. The new method is proposed based on the properties of energy function and our following observations. Fig.2 shows the valley, peak, edge fields and the corner strength field. It can be found that,

- The iris can be located rather well by only using the energy terms about the iris.
- Some energy terms, such as that on valley field, peak field and edge field, can interact with the template in big regions. These energy terms are the main forces that drag the template in a large region to the correct position and correct scale.
- Some energy terms, such as corner energy term, can only interact with the eye template in small regions. When the whole template is located in its neighborhood, the template can be refined by these energy terms. However, when the whole template is far from its location, the forces associated with these energy terms may drag the template to local minima.

Based on the observations mentioned above, and the discussion in Section 2, the following rules that should be considered in the minimization process are proposed:



**Fig. 2.** The original image, valley, peak, edge fields and corner strength field of an eye image from (a) to (e)

- The iris should be located firstly. Then, the eye boundaries can be modified according to the location of iris.
- The energy terms that interact with the template in far ranges would be active prior to those in small ranges. The energy terms that interact with the template in far ranges can drag the template to its location, and avoid to local minima, while the energy terms that interact with the template in small ranges can adjust the template finely.
- Using as few as possible steps for minimization process to reduce the problem of over-changing parameters.

Then, a three-steps minimization process is proposed to optimize the energy function that would avoid the problem of local minima, that is,

Step 1. The iris is located in this step. The image intensity and edge forces for the circle are allowed to act on the template. In this step,  $(X_e, r)$  are updated and other parameters remain unchanged.

Step 2. The eyelids are adjusted to the correct location. The eyelids can be rotated, translated and resized by using the exterior forces, i.e., the forces except the interior force that only adjust the shape of the eye template. In this stage, the corner strength image force would react with the input image,  $(X_c, a, b, c, \theta)$  are tuned in this stage and the parameters about the iris,  $(X_e, r)$ , remain unchanged.

Step 3. In this step, all parameters are finely tuned by considering all the energy terms.

**Table 4.** Proposed minimization process

steps		Energy function	Parameters adjusted
Locating iris	1	Valley, Intensity, Edge	The parameters about iris
Adjust eye boundaries	2	Edge, Intensity, Peak, Corner, Prior	The parameters about eye boundaries
Finely tune	3	All energy term	All parameters

In the processing of minimization, each step needs not to be split into some sub-steps. This proposed method is summarized in Table 4.

## 5 Experimental Results

### 5.1 Measurement

In order to evaluate the performances of locating eye features quantitatively, the statistic errors of corner location is used in this paper because the location of eye corners can reflect the structure of eye feature effectively. The error of corner location is defined as the average Euclidean distance between the ground truth and the detected location, i.e.

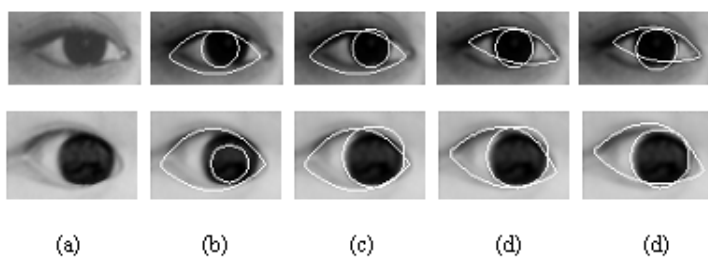
$$Errcor_j = \frac{1}{N} \sum_{i=1}^N \|X_{truth_{i,j}} - X_{extracted_{i,j}}\|. \quad (3)$$

where  $X_{truth_{i,j}}$  represents the location of ground truth of  $j$ -th eye corner in the  $i$ -th eye image, and  $X_{extracted_{i,j}}$  represents the location of  $j$ -th eye corner extracted from the  $i$ -th eye image.  $N$  is the number of examined eye images.  $Errcor_j$  means the error value of  $j$ -th corner.

### 5.2 Experimental Results

The proposed algorithm has been implemented using Matlab and applied to real images. In the experiments, totally 120 eye images, in which 110 images selected from the Pitt-CMU Facial Expression AU Coded Database [11] and 10 images downloaded from Internet, are used. The typical image size is 101x56, the radius of the circle for computing corner strength is 3. The initial parameters of eye feature are determined manually. Firstly, six points from the original image are selected manually. Then the ground truth of the eye template is calculated. Finally, the ground truth is displaced by a random variable as initial parameter to simulate the real situations. Based on the initial parameters, the eye feature is extracted through minimization process. Fig. 3 shows a sequence of eye templates at the end of each step. In each step, the parameters obtained from the previous step are taken as the initial parameters for the current step.

The performances of extracting eye feature using proposed minimization process are compared with those of other minimization processes while using same energy function proposed in [8]. Both one-step minimization process that likes Xie *et al*'s method [4], and 5-step minimization that likes Lam *et al*'s method [5] are taken into account here. To compare the behavior, these methods are all use the same pre-process and initial parameters. The statistic results are reported in Table 5. The first two columns show the distances of inner eye corner and outer eye corner extracted from the ground truth respectively (unit is pixel). The third column is the mean of the value in the first two columns. Some results are shown in Fig. 4. For convenience, we only give the final extraction results.



**Fig. 3.** Eye templates at the end of each step. (a) Original image (b) Initial template (c) Result of step 1 (d) Result of step 2 (e) Result of step 3.

**Table 5.** Average error of corner of some minimization process (unit is pixel)

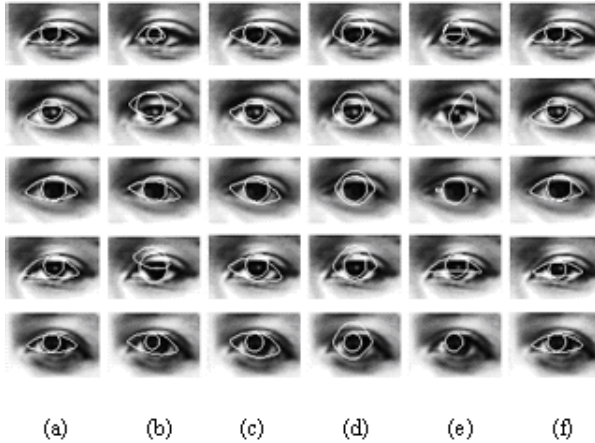
	Inner corner	Outer corner	Mean
1-step	2.3	2.0	2.2
5-step	3.8	3.7	3.8
Proposed	1.5	2.0	1.8



**Fig. 4.** Results of comparison using proposed energy function. (a) 3-steps (b) one-step (c) 5-steps.

From the experiments, the proposed method using 3-steps minimization process gets the best results.

From Fig. 4, it could be found if too many steps were applied, some parameters would be over changed. For example, in Fig. 4(c), the eye boundaries always tilt in the same direction, which maybe caused by overly changed angle parameter. If only one step is used to minimize the energy function, the eye template also fall into local minima due to the disturbance of the energy terms that only interact



**Fig. 5.** Results of comparison (a) Proposed (b) Xie *et al* [4] (c) Lam *et al* [5] (d) Shan *et al* [7] (e) One step (f) Tan *et al* [8]

**Table 6.** Average error of corner of some methods(unit is pixel)

	Inner corner	Outer corner	Mean
Xie <i>et al</i> [4]	6.2	5.8	6.0
Lam <i>et al</i> [5]	4.4	4.4	4.4
Shan <i>et al</i> [7]	5.1	5.2	5.1
Tan <i>et al</i> [8]	3.3	3.8	3.5
Proposed method	1.5	2.0	1.8

with eye template in small range, such as corner energy terms. However, the proposed method can extract the eye template more robustly.

We also compared our proposed method with that of other eye feature extraction methods using the same eye images. The compared method are Xie *et al* [4], Lam *et al* [5], Shan *et al* [7], Tan *et al* [8] and proposed method. Fig. 5 shows some examples of the comparisons. The statistic value about corner location from the ground truth is given in Table 6.

Xie's method also fails in extracting eye boundaries in many cases. The reason is also the local property of corner energy term. The method proposed by Lam *et al* [5] could extract the eye boundaries but fail to extract the right corners. This may be caused by the energy function. Because only edge and prior potential are used by Shan's method, and not having a finely tuned process, it is sensitive to initial parameters and always falls into local minima. Both Tan's method [8] and proposed method use the same minimization process, their performances outperform others. And the proposed method is the best because the corner energy function using SUSAN corner strength function can represent the properties of corner more efficiently.



**Table 7.** Comparison of process time (unit is second)

Method	Xie <i>et al</i> [4]	Lam <i>et al</i> [5]	Shan <i>et al</i> [7]	One step	Tan <i>et al</i> [8]	Proposed
Time	57.8	66.6	5.1	52.6	54.1	33.8

In addition, we have compared the convergence time of these methods. Table 7 shows the mean time used in optimization process. The speed of Shan's method is far faster than others are. The reason is the simplification of energy function. However, the accuracy of Shan's method is poor. In the remaining methods, our method is faster than others are. This is because only fewer parameters require modifications in the first two steps, and the parameters change only within a very small region in the third step in the proposed method.

## 6 Conclusion

In this paper, a three-steps minimization process for energy functions has been proposed to alleviate the problem of local minima by considering the local properties of energy terms. Experimental results show that our minimization method works well, and justify its superiorities over the existing methods.

Comparing with previous minimization process using the same energy function, such as 1-step and 5-steps minimization process, the proposed method improved the accuracy of locating eye corners about 22% and 111%, respectively. Comparing with other eye feature methods using different energy function and minimization process, such as the methods proposed by Xie *et al* [4], Lam *et al* [5], Shan *et al* [7] and Tan *et al* [8], the mean error of proposed method is reduced to 30%, 41%, 35% and 51%, respectively, while the speed was faster than Xie *et al*'s [4] and Lam *et al*'s [5] methods.

Currently, there are still a number of coefficients in the different energy terms that should be determined before minimization. This is our future work.

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