NONBLOCKER: Parameterized Algorithmics for MINIMUM DOMINATING SET

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Abstract. We provide parameterized algorithms for NONBLOCKER, the parametric dual of the well known DOMINATING SET problem. We exemplify three methodologies for deriving parameterized algorithms that can be used in other circumstances as well, including the (*i*) use of extremal combinatorics (known results from graph theory) in order to obtain very small kernels, (*ii*) use of known exact algorithms for the (nonparameterized) MINIMUM DOMINATING SET problem, and (*iii*) use of exponential space. Parameterized by the size k_d of the non-blocking set, we obtain an algorithm that runs in time $\mathcal{O}^*(1.4123^{k_d})$ when allowing exponential space.

1 Introduction

The *minimum dominating set* of a graph $G = (V, E)$ is a subset $V' \subseteq V$ of minimum cardinality such that for all $u \in V - V'$ there exists a $v \in V'$ for which $(u, v) \in E$. The problem of finding a minimum dominating set in a graph is arguably one of the most important combinatorial problems on graphs, having, together with its variants, numerous applications and offering various lines of research [11]. The problem of finding a set of at most k vertices dominating the whole n-vertex graph is not only $\mathcal{NP}$-complete but also hard to approximate [2], [10]. Moreover, this problem is also intractable when viewed as a parameterized problem [5]. The status is different if the problem is to find a set of at most $k = n - k_d$ vertices dominating a given n-vertex graph, where k_d ($k-$dual) is considered the parameter. Our focus in this paper is to present a new $\mathcal{O}^*(2.0226^{k_d})$-algorithm for this dual problem which we will henceforth call the NONBLOCKER problem. (We will make use of the $\mathcal{O}^*$-notation that has now become standard in exact algorithmics: in contrast to the better known $\mathcal{O}$-notation, it not only suppresses constants but also polynomial-time parts.)

Interesting relationships are known for the optimum value $\mathrm{nb}(G)$ of k_d for a graph G: Nieminen [16] has shown that, for a non-trivial connected graph, $\mathrm{nb}(G)$ equals the maximum number of *pendant edges* among all spanning forests for G (an edge $\{u, v\}$ in a forest F is pendant iff either u or v have degree one

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in F), and therefore $\mathrm{nb}(G)$ is again equal to the size of a maximum minimal edge cover of G due to a result of Hedetniemi [12]. How to algorithmically relate minimum dominating sets and maximum minimal edge covers is shown in [14].

On graphs of degree at least one, Ore [17] has shown (using different terminology) that the NONBLOCKER problem admits a kernel of size $2k_d$. Ore's result was improved by McCuaig and Shepherd [15] for graphs with minimum degree two; in fact, their result was a corollary to the classification of graphs that satisfy a certain inequality stated by Ore with equality. Independently, the result had been discovered by the Russian mathematician Blank [3] more than fifteen years ago, as noticed by Reed in [19]. More precisely, they have shown:

Theorem 1. *If a connected graph $G = (V, E)$ has minimum degree two and is not one of seven exceptional graphs (each of them having at most seven vertices), then the size of its minimum dominating set is at most $2/5 \cdot |V|$.*

The algorithms we present are easy to implement, addressing an important need of professional programmers. They essentially consist only of exhaustively applying simple data reduction (preprocessing) rules and then doing some search in the reduced problem space. (The mathematical analysis of our simple algorithm is quite involved and non-trivial, however.)

Our data reduction rules make use of several novel technical features. We introduce a special annotated *catalytic vertex*, a vertex which is forced to be in the dominating set we are going to construct. The catalytic vertex is introduced by a *catalyzation rule* which is applied only once. The graph is reduced and when no further reduction rules are applicable, a special *de-catalyzation rule* is applied. The *de-catalyzation rule* also is applied only once. We believe that the use of (de-)-catalyzation rules that might also *increase* the parameter size (since they are only applied once) is a technique that might find more widespread use when developing kernelization algorithms.

2 Definitions

We first describe the setting in which we will discuss MINIMUM DOMINATING SET in the guise of NONBLOCKER.

A *parameterized problem* $\mathcal{P}$ is a subset of $\Sigma^* \times \mathbb{N}$, where Σ is a fixed alphabet and $\mathbb{N}$ is the set of all non-negative integers. Therefore, each instance of the parameterized problem $\mathcal{P}$ is a pair (I, k), where the second component k is called the *parameter*. The language $L(\mathcal{P})$ is the set of all YES-instances of $\mathcal{P}$. We say that the parameterized problem $\mathcal{P}$ is *fixed-parameter tractable* [5] if there is an algorithm that decides whether an input (I, k) is a member of $L(\mathcal{P})$ in time $f(k)|I|^c$, where c is a fixed constant and $f(k)$ is a recursive function independent of the overall input length $|I|$. The class of all fixed-parameter tractable problems is denoted by $\mathcal{FPT}$.

The problems DOMINATING SET and NONBLOCKER are defined as follows: An instance of DOMINATING SET (DS) is given by a graph $G = (V, E)$, and the parameter, a positive integer k. The question is: Is there a *dominating set*

$D \subseteq V$ with $|D| \le k$? An instance of NONBLOCKER (NB) is given by a graph $G = (V, E)$, and the parameter, a positive integer k_d. The question is: Is there a *non-blocking set* $N \subseteq V$ with $|N| \ge k_d$?

A subset of vertices V' such that every vertex in V' has a neighbor in $V \setminus V'$ is called a *non-blocking set*. Observe that the complement of a non-blocking set is a dominating set and vice versa. Hence, $G = (V, E)$ has a dominating set of size at most k if and only if G has a non-blocking set of size at least $k_d = n - k$. Hence, DOMINATING SET and NONBLOCKER are called *parametric duals*.

Let $\mathcal{P}$ be a parameterized problem. A *kernelization* is a function K that is computable in polynomial time and maps an instance (I, k) of $\mathcal{P}$ onto an instance (I', k') of $\mathcal{P}$ such that (I, k) is a YES-instance of $\mathcal{P}$ if and only if (I', k') is a YES-instance of $\mathcal{P}$, $|I'| \le f(k)$, and $k' \le g(k)$ for arbitrary functions f and g. The instance (I', k') is called the *kernel* (of I). The importance of these notions for parameterized complexity is due to the following characterization.

Theorem 2. *A parameterized problem is in $\mathcal{FPT}$ iff it is kernelizable.*

Hence, in order to develop $\mathcal{FPT}$-algorithms, finding kernelizations can be seen as the basic methodology. The search for a small kernel often begins with finding local reduction rules. The reduction rules reduce the size of the instance to which they are applied; they are exhaustively applied and finally yield the kernelization function. In this paper we introduce a small variation of this method; namely, we introduce a catalyzation and a de-catalyzation rule, both of which are applied only once. Contrary to our usual reduction rules, these two special rules might increase the instance size.

We use this approach to solve the following *Catalytic Conversion* form of the problem. An instance of NONBLOCKER WITH CATALYTIC VERTEX (NBCAT) is given by a graph $G = (V, E)$, a catalytic vertex c, and the parameter, a positive integer k_d. The question is: Is there a *non-blocking set* $N \subseteq V$ with $|N| \ge k_d$ such that $c \notin N$? The special annotated catalytic vertex is assumed to be in the dominating set (not the non-blocking set).

3 Catalytic Conversion: $\mathcal{FPT}$ Agorithm for NONBLOCKER

Our kernelization algorithm for solving NONBLOCKER uses two special rules 1 and 2 to introduce and then finally to delete the catalytic vertex. The actually preprocessing then uses five more rules that work on an instance of NBcat.

Reduction rule 1 (Catalyzation rule). *If (G, k_d) is a* NONBLOCKER-*instance with $G = (V, E)$, then (G', c, k_d) is an equivalent instance of* NONBLOCKER WITH CATALYTIC VERTEX, *where $c \notin V$ is a new vertex, and $G' = (V \cup \{c\}, E)$.*

Reduction rule 2 (De-catalyzation rule). *Let (G, c, k_d) be an instance of* NONBLOCKER WITH CATALYTIC VERTEX. *Then, perform the following surgery to obtain a new instance (G', k'_d) of* NONBLOCKER *(i.e., without a catalytic vertex):*

Add three new vertices u, v, and w and introduce new edges cu, cv, cw, uv and vw. All other vertices and edge relations in G stay the same. This describes the new graph G'. Set $k'_d = k_d + 3$.

Reduction rule 3 (The Isolated Vertex Rule). *Let (G, c, k_d) be an instance of* NBCAT. *If C is a complete graph component (complete subgraph) of G that does not contain c, then reduce to $(G - C, c, k_d - (|C| - 1))$.*

Observe that Rule 3 applies to isolated vertices. It also applies to instances that do *not* contain a catalytic vertex. A formal proof of the soundness of the rule is contained in [18]. Notice that this rule alone gives a $2k_d$ kernel for general graphs with the mentioned result of Ore (details are shown below). By getting rid of vertices of degree one, we can improve on the kernel size due to Theorem 1.

Reduction rule 4 (The Catalytic Rule). *Let (G, c, k_d) be an instance of* NONBLOCKER WITH CATALYTIC VERTEX. *Let $v \neq c$ be a vertex of degree one in G with $N(v) = u$ (where $N(v)$ refers to the set of neighbor vertices of v). Transform (G, c, k_d) into $(G', c', k_d - 1)$, where:*

- *If $u \neq c$ then $G' = G_{[c \leftrightarrow u]} \setminus v$, i.e., G' is the graph obtained by deleting v and merging u and c into a new catalytic vertex $c' = \langle c \leftrightarrow u \rangle$.*
- *If $u = c$ then $G' = G \setminus v$ and $c' = c$.*

Lemma 1. *Rule 4 is sound.*

Proof. "Only if:" Let (G, c, k_d) be an instance of NBCAT. Let $V' \subset V(G)$ be a non-blocking set in G with $|V'| = k_d$. The vertex v is a vertex of degree one in G. Let u be the neighbor of v in G. Two cases arise:

1. If $v \in V'$ then it must have a neighbor in $V(G) \setminus V'$ and thus $u \in V(G) \setminus V'$. Deleting v will decrease the size of V' by one. If $u = c$, then $(G', c', k_d - 1)$ is a YES-instance of NBCAT. If $u \neq c$, merging u and c will not affect the size of V' as both vertices are now in $V(G') \setminus V'$. Thus, $(G', c', k_d - 1)$ is a YES-instance of NBCAT.
2. If $v \in V(G) \setminus V'$, then two cases arise:
   - 2.1. If u is also in $V(G) \setminus V'$ then deleting v does not affect the size of V'. Note that this argument is valid whether $u = c$ or $u \neq c$.
   - 2.2. If $u \in V'$ then $u \neq c$. If we make $v \in V'$ and $u \in V(G) \setminus V'$, the size of V' remains unchanged. Since u did not dominate any vertices in the graph, this change does not affect $N(u) \setminus v$, and Case 1 now applies.

"If:" Conversely, assume that $(G', c', k_d - 1)$ is a YES-instance of NBCAT.

1. If $u = c$, then we can always place v in V' and thus (G, c, k_d) is a YES-instance for NONBLOCKER WITH CATALYTIC VERTEX.
2. If $u \neq c$, getting from G' to G can be seen as (1) splitting the catalytic vertex c' into two vertices c and u, (2) taking c as the new catalytic vertex, and (3) attaching a pendant vertex v to u. As the vertex u is in

$V(G) \setminus V'$, v can always be placed in V', increasing the size of this set by one. Thus (G, c, k_d) is a YES-instance for NBCAT, concluding the proof of Lemma 4. ∎

Reduction Rule 3 can be generalized as follows:

Reduction rule 5 (The Small Degree Rule). *Let (G, c, k_d) be an instance of* NONBLOCKER WITH CATALYTIC VERTEX. *Whenever you have a vertex $x \in V(G)$ whose neighborhood contains a non-empty subset $U \subseteq N(x)$ such that $N(U) \subseteq U \cup \{x\}$ and $c \notin U$ (where $N(U)$ is the set of vertices that are neighbors to at least one vertex in U), then you can merge x with the catalytic vertex c and delete U (and reduce the parameter by $|U|$).*

Without further discussion, we now state those reduction rules that can be used to get rid of all consecutive degree-2-vertices in a graph:

Reduction rule 6 (The Degree Two Rule). *Let (G, c, k_d) be an instance of* NBCAT. *Let u, v be two vertices of degree two in G such that $u \in N(v)$ and $|N(u) \cup N(v)| = 4$, i.e., $N(u) = \{u', v\}$ and $N(v) = \{v', u\}$ for some $u' \neq v'$. If $c \notin \{u, v\}$, then merge u' and v' and delete u and v to get $(G', c', k_d - 2)$. If u' or v' happens to be c, then c' is the merger of u' and v'; otherwise, $c' = c$.*

Reduction rule 7 (The Degree Two, Catalytic Vertex Rule). *Let (G, c, k_d) be an instance of* NBCAT, *where $G = (V, E)$. Assume that c has degree two and a neighboring vertex v of degree two, i.e., $N(v) = \{v', c\}$. Then, delete the edge vv'. Hence, we get the new instance $((V, E \setminus \{vv'\}), c, k_d)$.*

Notice that all cases of two subsequent vertices u, v of degree two are covered in this way: If u or v is the catalytic vertex, then Rule 7 applies. Otherwise, if u and v have a common neighbor x, then Rule 5 is applicable; x will be merged with the catalytic vertex. Otherwise, Rule 6 will apply. This allows us to eliminate all of the exceptional graphs of Theorem 1 (since all of them have two consecutive vertices of degree two).

Algorithm 1. A kernelization algorithm for NONBLOCKER

Input(s): an instance (G, k_d) of NONBLOCKER
Output(s): an equivalent instance (G', k_d') of NONBLOCKER with $V(G') \subseteq V(G)$, $|V(G')| \leq 5/3 \cdot k_d'$ and $k_d' \leq k_d$ OR YES

Apply the catalyzation rule.
Exhaustively apply Rules 3 to 7. In the case of Reduction Rule 5, do so only for neighborhoods U up to size two.
Apply the de-catalyzation rule.
{This leaves us with a reduced instance (G', k_d').}
if $|V(G')| > 5/3 \cdot k_d'$ **then**
  return YES
else
  return (G', k_d')
end if

Corollary 1. *Alg. 1 provides a kernel of size upperbounded by* $5/3 \cdot k_d + 3$ *for any* NONBLOCKER-*instance* (G, k_d)*, where the problem size is measured in terms of the number of vertices.*

4 Searching the Space

4.1 Brute Force

With a very small kernel, the remaining reduced NONBLOCKER-instance can be solved by brute-force search. Hence, we have to test all subsets of size k_d within the set of vertices of size at most $5/3 \cdot k_d$. Stirling's formula gives:

Lemma 2. *For any* $a > 1$, $\binom{ak}{k} \approx a^k \left(\frac{a}{a-1}\right)^{(a-1)k}$.

Corollary 2. *By testing all subsets of size* k_d *of a reduced instance* (G, k_d) *of* NONBLOCKER*, the* NONBLOCKER *problem can be solved in time* $\mathcal{O}^*(3.0701^{k_d})$.

4.2 Using Nonparameterized Exact Algorithmics

The above corollary can be considerably improved by making use of the following recent result of F. Fomin, F. Grandoni, and D. Kratsch [7] on general graphs:

Theorem 3. MINIMUM DOMINATING SET *can be solved in time* $\mathcal{O}^*(1.5260^n)$ *with polynomial space on arbitrary* n*-vertex graphs.*

The corresponding algorithm is quite a simple one for HITTING SET, considering the open neighborhoods of vertices as hyperedges in a hypergraph; the quite astonishing running time is produced by an intricate analysis of that algorithm. Due to the $5/3 \cdot k_d$-kernel for NONBLOCKER, we conclude:

Corollary 3. *By applying the algorithm of Fomin, Grandoni, and Kratsch [7] to solve* MINIMUM DOMINATING SET *on a reduced instance* (G, k_d) *of* NONBLOCKER*, the* NONBLOCKER *problem can be solved in time* $\mathcal{O}^*(2.0226^{k_d})$ *with polynomial space.*

4.3 Trading Time and Space

Due to the fact that the kernel we obtained for NONBLOCKER is very small, it may be worthwhile looking for an algorithm that uses exponential space. According trade-off computations are contained in [7], so that we may conclude:

Corollary 4. *By using exponential space,* NONBLOCKER *can be solved in time (and space)* $\mathcal{O}^*(1.4123^{k_d})$.

5 Discussion: Further Results and Open Questions

Questions on general graphs. We have presented two efficient parameterized algorithms for the NONBLOCKER problem, the parametric dual of DOMINATING SET. With the help of known (non-trivial) graph-theoretic results and new exact algorithms for MINIMUM DOMINATING SET, we were able to further reduce the involved constants.

It would be possible to use the result of Reed [19] to obtain a smaller kernel for NONBLOCKER if rules could be found to reduce vertices of degree two. Perhaps such rules may be possible only for restricted graph classes, e.g., NONBLOCKER restricted to bipartite graphs.

Finally, notice that our reduction rules get rid of all degree-two vertices that have another degree-two vertex as a neighbor. Is there an "intermediate" kernel size theorem (that somehow interpolates between the result of Blank, McCuaig and Shepherd and that of Reed)? Our use of the additional structural properties of the reduced graphs was to cope with the exceptional graphs from [15].

Planar graphs. Since the rules that merge the catalyst with other vertices may destroy planarity, we may only claim the $2k_d$ kernel in the case of planar graphs.

We now use the following result on planar graphs by Fomin and Thilikos [9]:

Theorem 4. *Every planar n-vertex graph has treewidth at most $9/\sqrt{8} \cdot \sqrt{n}$.*

Together with the treewidth-based algorithm for MINIMUM DOMINATING SET as developed in [1], we can conclude:

Corollary 5. *The* NONBLOCKER *problem, restricted to planar graphs, can be solved in time $\mathcal{O}^*(2^{9\sqrt{k_d}})$.*

Is it possible to find a better kernelization in the planar case? This would be interesting in view of lower bound results of J. Chen, H. Fernau, I. A. Kanj, and G. Xia [4] who have shown there is no kernel smaller than $(67/66 - \epsilon)k_d$. Such a result would immediately entail better running times for algorithms dealing with the planar case. Observe that the kernelization of Ore also applies to planar cubic graphs. Since NONBLOCKER is also $\mathcal{NP}$-complete for that graph class (see [13]) and since DOMINATING SET has a $4k$-kernel in that case, we know that there is no $(4/3 - \epsilon)k_d$-kernel for NONBLOCKER on planar cubic graphs.

Graphs of bounded degree. Interestingly, there are better algorithm for solving MINIMUM DOMINATING SET on cubic graphs (graphs whose degree is bounded by three). More precisely, in [8] it is shown that this restricted problem can be solved in time $\mathcal{O}^*(3^{n/6}) = \mathcal{O}^*(1.2010^n)$ based on pathwidth decomposition techniques. As in the planar case, we cannot make use of the catalyst rule, since its application may increase the degree of a vertex.

Due to the $2k_d$-kernel for NONBLOCKER based on Ore's result [17], we conclude:

Corollary 6. *By applying the algorithm of Fomin, Grandoni, and Kratsch [8] to solve* MINIMUM DOMINATING SET *on a reduced instance* (G, k_d) *of* NONBLOCKER*, the* NONBLOCKER *problem, restricted to instances of maximum degree three, can be solved in time* $\mathcal{O}^*(3^{k_d/3}) = \mathcal{O}^*(1.4423^{k_d})$ *with polynomial space.*

Notice however that we can even do better in this case. Namely, by applying all of our reduction rules but the decatalyzation rule, at most one vertex (namely the catalyst) will have a degree higher than three, when starting with a graph of maximum degree of three. Now, we can incorporate the information that all neighbors of the catalyst are already dominated in the pathdecomposition based algorithm for MINIMUM DOMINATING SET run on the graph G obtained from the reduced graph by deleting the catalyst. Since G has maximum degree three, the pathwidth bound of Fomin, Grandoni, and Kratsch [8] applies, so that we can conclude:

Corollary 7. *By applying the algorithm of Fomin, Grandoni, and Kratsch [8] to solve* MINIMUM DOMINATING SET *on a reduced instance* (G, k_d) *of* NONBLOCKER *(that is modified as described), the* NONBLOCKER *problem, restricted to instances of maximum degree three, can be solved in time* $\mathcal{O}^*(3^{5k_d/18}) = \mathcal{O}^*(1.3569^{k_d})$ *with polynomial space.*

Moreover, the kernelization primal/dual game can be played, since there is a trivial $4k$ kernel for MINIMUM DOMINATING SET on cubic graphs (each vertex in a dominating set can dominate at most three vertices). The lower bound results of J. Chen, H. Fernau, I. A. Kanj, and G. Xia [4] on kernel sizes yield a $2k$ kernel size lower bound for MINIMUM DOMINATING SET on cubic graphs. So, in that case, upper and lower bound are not far off each other, at least when compared to the planar case.

Related problems. Our approach seem to be transferrable to similar problems, although then several additional technical hurdles appear. For example, for a suitable definition of "parametric dual", we were able to derive similar kernel results as given in this paper for MINIMUM ROMAN DOMINATION, see [6].

In view of the fact that the MINIMUM DOMINATING SET algorithm only makes use of MINIMUM HITTING SET in its analysis, the same time bounds are also valid for the variant of MINIMUM TOTAL DOMINATING SET, where each vertex is required to be dominated by a neighbor (also the ones in the dominating set). However, our catalyzator technique only works for vertices that are in the dominating set and that are already dominated; vertices that are in the dominating set (e.g., since they are neighbors of a vertex of degree one) but not yet dominated themselves cannot be merged (only if their open neighborhoods are comparable with respect to inclusion). There exist results similar to Blank, McCuaig and Shepard's that might provide kernelizations for TOTAL NONBLOCKER, see [20].

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