Can "Bill-and-Keep" Peering Be Mutually Beneficial?

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Abstract. We analyze "Bill-and-Keep" peering between two providers, where no money exchanges hands. We assume that each provider incurs costs from its traffic traversing its as well as the peer's links, and compute the traffic levels in Nash equilibrium. We show that Nash strategies are not blind, i.e., they are neither pure hot-potato nor pure cold-potato strategies. Rather, the Nash strategies involve strategically splitting traffic between a provider's own links and its peer's. We derive necessary and sufficient conditions for both the providers to be better (or worse) off in Nash equilibrium compared to the blind strategies.[1] We also analyze society's performance as a whole and derive necessary and sufficient conditions for the society to be better (or worse) off. In particular we establish that, under Bill-and-Keep peering, while it is not possible for two asymmetric providers to be both worse off, it is certainly possible for both to be better off.

1 Introduction

Today's Internet is composed of many distinct networks, operated by independent network providers, also referred to as Internet Service Providers (ISPs). Each provider is interested in maximizing its own utility and the objectives of the providers are not necessarily aligned with any global performance objective. Most relationships between providers may be classified under one of two categories [7] : *transit* and *peer*. In a transit relationship, a traffic-originating provider pays a transit provider to carry traffic destined to nodes outside the originator's local network. On the other hand, in a peering relationship the providers agree to accept and carry traffic from each other.

In this paper, we focus primarily on peering relationships. In a peering arrangement, a pair of providers agree to install bi-directional links at multiple peering points to accept traffic from each other. In today's Internet, peering relationships are mostly "Bill-and-Keep" [1]. In this arrangement, the providers don't charge each other for the traffic accepted on the peering links. This arrangement is also referred to as "Zero-Dollar" peering or "Sender-Keep-All" (SKA) peering [3]. Under the peering relationship, since the ISPs are interested in minimizing their own costs, they predominantly use the nearest-exit or *hot-potato* routing [6], where outgoing traffic exits a provider's network as quickly as possible. In some cases, where the receiver is a bigger player and is able to exert its market power, the routing is farthest-exit or *cold-potato* [7].

Various aspects of ISP peering have been analyzed by [5], [1], [4], [9], [8]. [5] was the first paper to analyze ISP peering in depth from an economic perspective. It analyzed the impact of access charge on strategies of the providers and showed that, in a

[1] By better off we mean weakly better off, i.e., the cost in Nash equilibrium is less than or equal to the cost under blind strategies.

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broad range of environments, operators set prices for their customers as if their customers' traffic were entirely off-net. [8] extended the models in [5] to include the fact that the ISPs are geographically separated. It thus analyzed the local ISP interaction separately from the local and transit ISP interaction. It also analyzed the economics of private exchange points and showed that they could become far more wide spread. Both [5] and [8] used linear pricing schemes assuming fixed marginal costs. In addition, they assumed hot-potato routing. [9] extended the models in [5] to include customer delay costs, finding that they have a substantial effect on market structure. [4] used a different model of ISP peering. It assumed that customers are bound to ISPs, subject to general, non-linear marginal costs. It then looked at how ISPs could charge each other in response to the externality caused by their traffic.

All these models ignored one important aspect of peering: they did not consider the case when the provider incurs costs even when its traffic flows on its peers links. This would happen, for example, if the providers care about the end-to-end quality of service (QOS) for traffic originating within their networks. In this paper we focus on this situation, and show that this cost structure has a substantial effect on how ISPs route traffic. Moreover, we look at the case where peering happens in the absence of pricing, with the costs incurred on the peer's links serving as a proxy for a transfer price. Our results imply that Bill-and-Keep peering, currently used mainly due to ease of implementation, can be beneficial if combined with non-myopic routing.

We analyze peering decisions using non-cooperative game theory [2] in a simple two-provider model. Analyzing Nash equilibria, under mild assumptions on the cost structures, we show that it is not in the provider's interest to route traffic in a hot-potato or a cold-potato fashion in equilibrium. That is, the *blind* strategies are not Nash. Rather, the Nash strategy involves strategically splitting traffic among peering points. We also show that, in Nash equilibria, it is not possible for both ISPs to be worse off with respect to the blind strategies. We then show that it is possible to have the two remaining scenarios in equilibrium, where either one or both ISPs are better off. We derive necessary and sufficient conditions on the cost functions for each of the two cases to occur. In addition, under a specific pricing scheme, we show that peers who would peer and be both better off under Bill-and-Keep peering, will choose to use cold-potato peering and effectively not peer at all.

We are also interested in the performance of the society as a whole. Therefore, we compare society's performance in Nash equilibrium versus the blind strategies. We derive necessary and sufficient conditions for the society to be better or worse off. In this process, we show that society is always better off under Bill-and-Keep peering if the costs incurred are linear.

2 The Model and the Nash Strategies

We look at a two ISP peering model. A realistic model would include the actual topologies of the two ISPs. However, most of the insights can be gained from the following simple model (figure 1); analysis of a generalized model is work in progress. We have two ISPs, S and R, with two peering points, $P1$ and $P2$. The ISPs have nodes S_i and R_i, respectively, located right next to the peering point $P_i, i \in \{1, 2\}$. We assume that

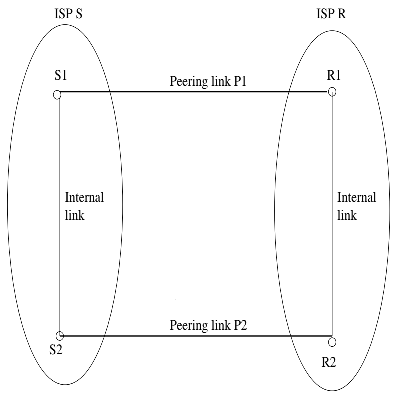

Fig. 1. The Peering Model

ISP S sends one unit of traffic from node S_1 to node R_2, and similarly, ISP R sends one unit of traffic from node R_1 to S_2. We also assume that the cost to send traffic across the peering links is zero.

The ISPs have the choice to split these flows between the two peering points. We look at the two components for ISP S first. The hot potato component, f_S^R, goes over to ISP R at the closest peering point P_1, and then travels to R_2 on the internal link R_1R_2. The cold potato component, f_S^S, first travels on the internal link S_1S_2, and then crosses over to ISP R at the farthest peering point P_2. The hot potato and cold potato components for ISP R, i.e., f_R^S and f_R^R, can be described in a similar way. Overall, ISP S carries flows f_S^S and and f_R^S on its internal link, whereas ISP R carries flows f_S^R and f_R^R on its internal link. The cost incurred per unit of traffic on ISP S's links is given by $C_S(f_S^S + f_R^S)$ and that on ISP R's link is given by $C_R(f_S^R + f_R^R)$.

We assume that these per unit cost functions are strictly increasing, convex and twice differentiable. We also assume that the per unit cost of one ISP carrying all the traffic is more than the per unit cost of the other ISP carrying zero traffic, i.e.,

$$C_S(2) \geq C_R(0) \tag{1a}$$
$$C_R(2) \geq C_S(0) \tag{1b}$$

In Nash equilibrium, given f_R^S, ISP S solves (by choosing $0 \leq f_S^R \leq 1$)

$$\begin{aligned} &\text{minimize } J_S(f_S^R, f_R^S) = C_S(f_S^S + f_R^S)f_S^S + C_R(f_S^R + f_R^R)f_S^R \\ &\text{subject to } f_S^S + f_S^R = 1 \text{ and } f_R^S + f_R^R = 1, \end{aligned}$$

and ISP R solves (by choosing $0 \leq f_R^S \leq 1$)

$$\begin{aligned} &\text{minimize } J_R(f_S^R, f_R^S) = C_S(f_S^S + f_R^S)f_R^S + C_R(f_S^R + f_R^R)f_R^R \\ &\text{subject to } f_S^S + f_S^R = 1 \text{ and } f_R^S + f_R^R = 1, \end{aligned}$$

given f_S^R.

The first order conditions are given by

$$C'_S(f_S^S + f_R^S)f_S^S + C_S(f_S^S + f_R^S) = C'_R(f_S^R + f_R^R)f_S^R + C_R(f_S^R + f_R^R) \quad (2a)$$

$$C'_S(f_S^S + f_R^S)f_R^S + C_S(f_S^S + f_R^S) = C'_R(f_S^R + f_R^R)f_R^R + C_R(f_S^R + f_R^R). \quad (2b)$$

Note that the second order conditions will be satisfied automatically since the per unit cost functions are strictly increasing and convex, especially at interior point solutions.

We next show that the Nash strategies don't include blind strategies.

Proposition 1. *In Nash equilibrium, no ISP uses a blind strategy.*

Proof. We prove this in two parts. We first prove that, in Nash equilibrium, no ISP routes pure cold potato. We then prove that no ISP routes pure hot potato. The proofs are very similar and we have omitted the second part.

We now show that it is not possible for ISP R to do pure cold potato routing (a similar argument can be applied to ISP S). This is done in three steps. First, it is not possible for both ISPs to do pure cold potato routing. This would require $f_S^R = f_R^S = 0$, and[2]

$$\frac{\partial J_S}{\partial f_S^R}(0,0) = -C'_S(1) - C_S(1) + C_R(1) \geq 0 \quad (3a)$$

$$\frac{\partial J_R}{\partial f_R^S}(0,0) = -C'_R(1) - C_R(1) + C_S(1) \geq 0. \quad (3b)$$

This requires

$$C_S(1) + C'_S(1) \leq C_R(1) \leq C_S(1) - C'_R(1), \quad (4)$$

which is a contradiction since $C_S(x)$ and $C_R(x)$ are strictly increasing.

Second, it is not possible for ISP S and R to do hold potato and cold potato routing, respectively. This would require $f_S^R = 1$, $f_R^S = 0$, and

$$\frac{\partial J_S}{\partial f_S^R}(1,0) = -C_S(0) + C'_R(2) + C_R(2) \leq 0 \quad (5a)$$

$$\frac{\partial J_R}{\partial f_R^S}(1,0) = C_S(0) - C'_R(2) - C_R(2) \geq 0. \quad (5b)$$

This requires

$$C_S(0) \geq C_R(2) + C'_R(2), \quad (6)$$

which is a contradiction from (1) and the fact that $C_R(x)$ is strictly increasing.

Finally, it is not possible for ISP R to do cold potato routing and ISP S to send some nonzero amount, but not all, off its traffic to ISP R. This would require $0 < f_S^R < 1$, $f_R^S = 0$, and

$$\frac{\partial J_S}{\partial f_S^R}(f_S^R,0) = -C_S(f_S^S) - C'_S(f_S^S)f_S^S + C'_R(f_S^R + 1)f_S^R + C_R(f_S^R + 1) = 0 \quad (7a)$$

$$\frac{\partial J_R}{\partial f_R^S}(f_S^R,0) = C_S(f_S^S) - C'_R(f_S^R + 1) - C_R(f_S^R + 1) \geq 0. \quad (7b)$$

[2] The partial derivatives are evaluated at (f_S^R, f_R^S).

This requires

$$C'_S(f_S^S)f_S^S \leq C'_R(f_S^R+1)(f_S^R-1) \leq 0, \tag{8}$$

which is a contradiction since $0 < f_S^R < 1$ and $C_S(x)$ and $C_R(x)$ are strictly increasing. □

3 Individual Performance

In this section we provide necessary and sufficient conditions for the individual players to be better or worse off in Nash equilibrium compared to the blind strategies. We also provide examples showing that both ISPs can be better off in Nash equilibrium. In addition, we illustrate potential problems with introduction of pricing.

3.1 Preliminaries

We start by writing (2) in an alternate form. Using $f_S^S + f_S^R = 1$, $f_R^S + f_R^R = 1$, and denoting $f_S^R - f_R^S = f_d$, $f_S^R + f_R^S = f_a$, we get

$$C'_S(1-f_d)(1-f_d) + 2C_S(1-f_d) = C'_R(1+f_d)(1+f_d) + 2C_R(1+f_d) \tag{9a}$$

$$f_a = 1. \tag{9b}$$

In Nash equilibrium, we denote the solution to (9) as f_d^{Nash}. We also denote the Nash equilibrium costs of ISP S and R as J_S^{Nash} and J_R^{Nash}, respectively. We then get the following lemma.

Lemma 1. *The ISPs costs in Nash equilibrium are equal.*

Proof. From (9), in Nash equilibrium, we have $f_S^S = f_R^S = \frac{(1-f_d^{Nash})}{2}$ as well as $f_S^S = f_R^S = \frac{(1-f_d^{Nash})}{2}$. This ensures that the costs at equilibrium are equal, i.e.,

$$J_S^{Nash} = J_R^{Nash} = J_{common}^{Nash} = \frac{J_{total}(f_d^{Nash})}{2}, \tag{10}$$

where we define the sum of costs as

$$J_{total}(f_d) = [C_S(1-f_d)(1-f_d) + C_R(1+f_d)(1+f_d)]. \tag{11}$$

□

We next define the following three differences

$$\Delta_1(f_d) = C_S(1-f_d) - C_R(1+f_d) \tag{12a}$$

$$\Delta_2(f_d) = C_S(1-f_d) + C'_S(1-f_d)(1-f_d) - C_R(1+f_d) - C'_R(1+f_d)(1+f_d) \tag{12b}$$

$$\Delta_3(f_d) = \Delta_1(f_d) + \Delta_2(f_d). \tag{12c}$$

Then, in Nash equilibrium, from (9), we get

$$\Delta_3(f_d^{Nash}) = 0 \tag{13a}$$

$$\Delta_1(f_d^{Nash}) = -\Delta_2(f_d^{Nash}) \tag{13b}$$

as well as

Lemma 2. *In Nash equilibrium, $\Delta_3(0) f_d^{Nash} \geq 0$.*

Proof. First consider $f_d^{Nash} \geq 0$. Since, from (13), $\Delta_3(f_d^{Nash}) = 0$, and $\Delta_3(f_d)$ is non-increasing in f_d, the result follows. The argument for $f_d^{Nash} < 0$ is similar □

Finally, we derive some useful inequalities, as follows. Since $C_S(x)$ and $C_R(x)$ are twice differentiable, strictly increasing, and convex, $C_S(x)x$ and $C_R(x)x$ are twice differentiable, strictly increasing, and strictly convex. Using Jensen's inequality, we get

$$C_S(1-f_d)(1-f_d) \geq C_S(1) - [C'_S(1) + C_S(1)]f_d \tag{14a}$$

$$C_R(1+f_d)(1+f_d) \geq C_R(1) + [C'_R(1) + C_R(1)]f_d, \tag{14b}$$

and

$$C_S(1) \geq C_S(1-f_d)(1-f_d) + [C'_S(1-f_d)(1-f_d) + C_S(1-f_d)]f_d \tag{15a}$$

$$C_R(1) \geq C_R(1+f_d)(1+f_d) - [C'_R(1+f_d)(1+f_d) + C_R(1+f_d)]f_d. \tag{15b}$$

Now, using (11), and (12) we get

$$J_{total}(f_d) \geq J_{total}(0) - \Delta_2(0)f_d \tag{16}$$

as well as

$$J_{total}(0) \geq J_{total}(f_d) + \Delta_2(f_d)f_d. \tag{17}$$

3.2 Necessary and Sufficient Conditions

We first show that, under our assumptions, it is not possible for both the ISPs to be worse off.

Lemma 3. *In Nash equilibrium, both ISPs are worse off only if $\Delta_1(0)\Delta_3(0) \geq 0$.*

Proof. For both to be worse off, from lemma 1, this requires

$$J_{common}^{Nash} = \frac{J_{total}(f_d^{Nash})}{2} \geq max(J_S^{blind}, J_R^{blind}), \tag{18}$$

where $J_S^{blind} = C_S(1)$ and $J_R^{blind} = C_R(1)$ when both ISPs are doing pure cold potato routing.[3] Using (11), (17) and (13), we get

$$\begin{aligned} \frac{J_{total}(f_d^{Nash})}{2} &\geq max(C_S(1), C_R(1)) \geq \frac{J_{total}(0)}{2} \\ &\geq \frac{J_{total}(f_d^{Nash})}{2} - \frac{\Delta_1(f_d^{Nash})f_d^{Nash}}{2}. \end{aligned} \tag{19}$$

This requires

$$\Delta_1(f_d^{Nash})f_d^{Nash} \geq 0, \tag{20}$$

which necessitates $\Delta_1(0)f_d^{Nash} \geq 0$. This, from lemma 2, is the same as $\Delta_1(0)$ $\Delta_3(0) \geq 0$. □

[3] Similarly, $J_S^{blind} = C_R(1)$ and $J_R^{blind} = C_S(1)$ when both ISPs are doing hot potato routing. In both cases, the min, max and avg operations give the same results.

Proposition 2. *In Nash equilibrium, both ISPs cannot be worse off.*

Proof. The case $f_d^{Nash} = 0$ is straightforward. From (11) and (10), we get $J_{common}^{Nash} = \frac{C_S(1)+C_R(1)}{2}$. Since $max(C_S(1), C_R(1)) \geq \frac{C_S(1)+C_R(1)}{2} \geq min(C_S(1), C_R(1))$, we must have one ISP better off and the other worse off. Thus, both ISPs can't be worse off.

Next, we look at $f_d^{Nash} \neq 0$. We first consider $f_d^{Nash} > 0$. Both ISPs worse off requires

$$J_{common}^{Nash} \geq C_S(1). \tag{21}$$

Since C_S is convex, using $C_S(1) \geq C_S(1 - f_d^{Nash}) + C_S'(1 - f_d^{Nash})f_d^{Nash}$, we get

$$J_{common}^{Nash} \geq C_S(1 - f_d^{Nash}) + C_S'(1 - f_d^{Nash})f_d^{Nash}, \tag{22}$$

which, using (10) and (11), simplifies to

$$(1 + f_d^{Nash})[C_R(1 + f_d^{Nash}) - C_S(1 - f_d^{Nash})] \geq 2C_S'(1 - f_d^{Nash})f_d^{Nash}. \tag{23}$$

Now, since $C_S(x)$ is strictly increasing, using (12) we get, $\Delta_1(f_d^{Nash}) < 0$ as well as $\Delta_1(f_d^{Nash})f_d^{Nash} < 0$.

The argument for $f_d^{Nash} < 0$ is similar. Thus, in both cases, both ISPs worse off necessitates

$$\Delta_1(f_d^{Nash})f_d^{Nash} < 0. \tag{24}$$

This contradicts (20) of lemma 3. □

Now, we look for necessary conditions for both ISPs to be better off.

Proposition 3. *In Nash equilibrium, both ISPs are better off only if $\Delta_2(0)\Delta_3(0) \geq 0$.*

Proof. We first consider $f_d^{Nash} \leq 0$. Both ISPs better off requires

$$J_{common}^{Nash} \leq C_S(1). \tag{25}$$

Since C_S is convex, using $C_S(1) \leq C_S(1 - f_d^{Nash}) + C_S'(1)f_d^{Nash}$, we get

$$J_{common}^{Nash} \leq C_S(1 - f_d^{Nash}) + C_S'(1)f_d^{Nash}, \tag{26}$$

which, using (10) and (11), simplifies to

$$(1 + f_d^{Nash})[C_R(1 + f_d^{Nash}) - C_S(1 - f_d^{Nash})] \leq 2C_S'(1)f_d^{Nash}. \tag{27}$$

Now, since $C_S(x)$ is strictly increasing, using (12) we get, $\Delta_1(f_d^{Nash}) \geq 0$ as well as $\Delta_1(f_d^{Nash})f_d^{Nash} \leq 0$. From (13), this requires

$$\Delta_2(f_d^{Nash})f_d^{Nash} \geq 0, \tag{28}$$

which necessiates $\Delta_2(0)f_d^{Nash} \geq 0$. This, from lemma 2, is the same as $\Delta_2(0)\Delta_3(0) \geq 0$.

The argument for $f_d^{Nash} \geq 0$ is similar. □

Finally, we provide a sufficient condition for both ISPs to be better off.

Proposition 4. *If the costs under blind strategies are equal then both the ISPs are better off in Nash equilibrium.*

Proof. We start by noting that, when $C_S(1) = C_R(1)$, we have $J_S^{blind} = J_R^{blind}$. Also, from (10), we have $J_S^{Nash} = J_R^{Nash}$. This rules out the possibility that one ISP is strictly better off and the other one is strictly worse off. In addition, propostion 2 rules out the possibility that both are strictly worse off. Thus, the only remaining possibility is that they are both better off. □

3.3 Examples

We provide two examples that show that both ISPs can be better off. We look for functions $C_S(x)$ and $C_R(x)$ satisfying the following properties. First, they satisfy $C_S(1) = C_R(1)$ which, from (10), gives $J_S^{blind} = J_R^{blind} = J_{common}^{blind} = \frac{J_{total}(0)}{2}$, where J_{common}^{blind} is defined to be the common cost under blind strategies. Second, they satisfy $J_{common}^{Nash} \leq J_{common}^{blind}$, which is the same as $J_{total}(f_d^{Nash}) \leq J_{total}(0)$ - this basically requires one of the internal link costs, i.e., $C_S(x)x$ and $C_R(x)x$, to be more convex at $x = 1$. Now, both ISPs benefit by choosing an $f_d^{Nash} \approx 0$ such that the link with the more convex cost function carries less traffic.

Example 1. When $C_S(x) = x$ and $C_R(x) = x^2$, we get $J_{common}^{Nash} = 0.97 \leq 1.0 = J_{common}^{blind}$.

Example 2. This example uses more realistic per unit cost functions, given by $C_S(x) = \frac{\theta_S^N}{(\theta_S^D - x)}$ and $C_R(x) = \frac{\theta_R^N}{(\theta_R^D - x)}$ (in an $M/G/1$ queue, θ^N would be proportional to the variance of service times, whereas θ^D would be the capacity of the link). Using $\theta_S^N = 1.00, \theta_S^D = 1.10, \theta_R^N = 2.00, \theta_R^D = 1.20$, we get $J_{common}^{Nash} = 9.75 \leq 10.0 = J_{common}^{blind}$.

3.4 Can Pricing Be Bad?

In this section we illustrate potential problems with moving away from Bill-and-Keep peering toward a situation where ISP S charges ISP R an amount $p_S f_R^S$ and ISP R charges ISP S an amount $p_R f_S^R$, for some prices p_S and p_R. We consider the sequential game where the ISPs first pick prices, having committed to optimally choosing traffic splits thereafter. That is, ISP S solves (by choosing $0 \leq f_S^R \leq 1$)

$$\begin{aligned} &\text{minimize } J_S(f_S^R, f_R^S) = C_S(f_S^S + f_R^S)f_S^S + C_R(f_S^R + f_R^R)f_S^R \\ &\text{subject to } f_S^S + f_S^R = 1 \text{ and } f_R^S + f_R^R = 1, \end{aligned}$$

and ISP R solves (by choosing $0 \leq f_R^S \leq 1$)

$$\begin{aligned} &\text{minimize } J_R(f_S^R, f_R^S) = C_S(f_S^S + f_R^S)f_R^S + C_R(f_S^R + f_R^R)f_R^R \\ &\text{subject to } f_S^S + f_S^R = 1 \text{ and } f_R^S + f_R^R = 1 \end{aligned}$$

for fixed p_S and p_R, and this is used, in turn, to calculate the optimal p_S and p_R.

Using $C_S(x) = x$ and $C_R(x) = x^2$ (the cost functions from section 3.3), and solving in the above manner, we get $f_S^R = f_R^S = 0$. This is the same as the situation when the ISPs are routing blindly. In this case both the ISPs are worse off compared to peering without pricing.

4 Society's Performance

The necessary conditions for the society to be better (or worse) off turn out to be the same as the ones stated in propostion 3 and lemma 3. Intuitively, this makes sense since the society must be better (or worse) off when both the ISPs are better (or worse) off.

Now we look at sufficient conditions for the society to be better off or worse off. The following propositions summarize our results.

Proposition 5. *If* $\Delta_1(0)\Delta_3(0) \leq 0$ *then the society is better off in Nash equilibrium.*

Proof. This implies, from lemma 3, that society can't be strictly worse off. Thus, it must be better off. □

Proposition 6. *If* $\Delta_2(0)\Delta_3(0) \leq 0$ *then the society is worse off in Nash equilibrium. In addition, one ISP is better off and the other worse off.*

Proof. This implies, from proposition 3, that society can't be strictly better off. Thus, it must be worse off. Also, since proposition 2 rules out he possibility that both are worse off, it must be that one ISP is worse off and the other one is better off. □

We finish the paper with the following specific result

Proposition 7. *When the per unit cost functions are linear, society is better off in Nash equilibrium.*

Proof. To do so, we use the linear cost functions $C_S(x) = \theta_S x$ and $C_R(x) = \theta_R x$, where $\theta_S > 0$ and $\theta_R > 0$. In Nash equilibrium, we get

$$f_S^S = f_R^S = \frac{\theta_R}{(\theta_S + \theta_R)} \tag{29a}$$

$$f_S^R = f_R^R = \frac{\theta_S}{(\theta_S + \theta_R)}. \tag{29b}$$

Next, we assume that the society is strictly worse off. This gives

$$J_{total}^{Nash} = \frac{4\theta_S\theta_R}{(\theta_S + \theta_R)} > (\theta_S + \theta_R) = J_{total}^{blind}, \tag{30}$$

which reduces to

$$0 > (\theta_S - \theta_R)^2, \tag{31}$$

which is always false. Thus, we have a contradiction. □

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