Packing Trees in Communication Networks*

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Abstract. Given an undirected edge-capacitated graph and a collection of subsets of vertices, we consider the problem of selecting a maximum (weighted) set of Steiner trees, each spanning a given subset of vertices without violating the capacity constraints. We give an integer linear programming (ILP) formulation, and observe that its linear programming (LP-) relaxation is a fractional packing problem with exponentially many variables and with a block (sub-)problem that cannot be solved in polynomial time. To this end, we take an r-approximate block solver to develop a $(1-\varepsilon)/r$ approximation algorithm for the LP-relaxation. The algorithm has a polynomial coordination complexity for any $\varepsilon \in (0,1)$. To the best of our knowledge, this is the first approximation result for fractional packing problems with only approximate block solvers and a coordination complexity that is polynomial in the input size and ε^{-1}. This leads to an approximation algorithm for the underlying tree packing problem. Finally, we extend our results to an important multicast routing and wavelength assignment problem in optical networks, where each Steiner tree is also to be assigned one of a limited set of given wavelengths, so that trees crossing the same fiber are assigned different wavelengths.

1 Introduction

Multicast is an efficient approach to deliver data from a source to multiple destinations over a communication network. This approach is motivated by emerging telecommunication applications, e.g., video-conferencing, streaming video and distributed computing. In particular, a multicast session is established by finding a Steiner tree in the network that connects the multicast source with all the multicast destinations.

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In this paper we address the following *Steiner tree packing* problem, that is fundamental in multicast communications. We are given a communication network represented by an undirected graph, a capacity associated with every edge in the graph, and a set of multicast *requests* (each defined by a subset of vertices to be connected, called *terminals*). A feasible solution to this problem is a set of Steiner trees, each spanning a multicast request, such that the number of Steiner trees crossing the same edge is bounded by the capacity of that edge. The goal is to maximize the total *profit/throughput* (the weighted sum of the successfully routed requests). It is worth noting that some requests may not be successfully routed due to the edge capacity. This problem arises in the communication network that provides multicast communication service to multiple groups of users in order to realize the routing that attains the maximum global profit for the whole network with limited bandwidth resources.

A special case in which the same request is spanned by the maximum number of edge-disjoint Steiner trees was studied in [10]. The authors presented a $4/|S|$-asymptotic approximation algorithm, where S is the terminal set to be connected. This problem was further studied in [14] and an algorithm was proposed to find $\lfloor \lambda_S(G)/26 \rfloor$ edge-disjoint Steiner trees, where $\lambda_S(G)$ is the size of a minimum S-cut in G. Another generalization is to find the maximum collection of Steiner forests spanning different requests [15]. There are also many applications in this category (see [14]). A related problem of realizing all given multicast requests as to minimize the maximum edge congestion was studied from theoretical and experimental aspects in [2, 5, 6, 12, 16, 23]. This is essentially equivalent to a routing problem in VLSI design [19]. Another related problem is that of realizing all given multicast requests at the minimum cost. This problem was studied in [4] and [13] for the special case of all Steiner trees connecting the same set of vertices, and in [22] for the general case where for each Steiner tree a different set of vertices is given.

We show that the relaxation of the Steiner packing problem is a *fractional packing problem* in Section 2, which has attracted considerable attention in the literature [7, 17, 25]. In general, a block solver is called to play a similar role to the separation oracle in the ellipsoid methods in [9]. The approximation algorithm in [7] is only for the case that the block problem is polynomial time solvable. In addition, the approximation algorithms in [17, 25] have coordination complexity depending on the input data, and are thus not polynomial in the input size. A problem related to fractional packing is the convex min-max resource-sharing problem, which is studied in [8, 24]. If the block problem is $\mathcal{NP}$-hard, an approximation algorithm is designed in [11] with polynomial coordination complexity.

To date, we are not aware of any approximation results for the Steiner tree packing problem in its full generality (where for each Steiner tree is required to connect a set of vertices). Furthermore, we are not aware of any approximation algorithm for fractional packing problems with coordination complexity polynomial in the input size while the block problem is $\mathcal{NP}$-hard.

The contribution of this paper can be summarized as follows. We formulate the Steiner tree packing problem in its full generality as an ILP, and observe that its

LP-relaxation is a fractional packing problem with exponentially many variables and an $\mathcal{NP}$-hard block problem. We thus develop a $(1-\varepsilon)/r$-approximation algorithm for fractional packing problems with polynomial coordination complexity, each iteration calling an r-approximate block solver, for $r \geq 1$ and any given $\varepsilon \in (0,1)$. This is the first result for fractional packing problems with only approximate block solvers and a coordination complexity strictly polynomial in input size and ε^{-1}. In fact, the coordination complexity of our algorithm is exactly the same as in [7] where the block problem is required to be polynomial time solvable. Then we present an algorithm for the Steiner tree packing problem and also apply our approximation algorithm for integer packing problems to establish a method to directly find a feasible solution. We extend our results to an important multicast routing and wavelength assignment problem in optical networks, where each Steiner tree is also to be assigned one of a limited set of given wavelengths, so that trees crossing the same fiber are assigned different wavelengths.

The remainder of this paper is organized as follows. In Section 2 we give an ILP formulation of the Steiner tree packing problem. Then we present and analyze the approximation algorithm for fractional packing problems in Section 3 and use it to develop an approximation algorithm for the integer Steiner tree packing problem in Section 4. The approach to directly find integer approximate solutions is discussed in Section 5. The multicast routing and wavelength assignment problem in optical networks is studied in Section 6. Finally, Section 7 concludes the paper. Due to the limit of space we do not give all proofs of our results in this version. We refer the readers to the full version of our paper [21] for details.

2 Mathematical Programming Formulation

We are given an undirected graph $G = (V, E)$ representing the input communication network, and a set of multicast requests $S_1, \ldots, S_K \subseteq V$ to be routed by Steiner trees. Each edge $e_i \in E$ is associated with a capacity c_i indicating the bandwidth of the corresponding cable. Denote by $\mathcal{T}_k$ the set of all Steiner trees spanning S_k, $k \in \{1, \ldots, K\}$. The number of trees $|\mathcal{T}_k|$ may be exponentially large. Furthermore, we define an indicator variable $x_k(T)$ for each tree as follows: $x_k(T) = 1$ if $T \in \mathcal{T}_k$ is selected for routing S_k; Otherwise $x_k(T) = 0$. In addition, each request S_k is associated with a weight w_k to measure its importance in the given multicast communication network. Therefore, the Steiner tree packing problem can be cast as the following ILP:

$$\begin{array}{lll} \max \sum_{k=1}^{K} w_k \sum_{T\in\mathcal{T}_k} x_k(T) & & \\ \text{s.t.} \sum_{k=1}^{K} \sum_{T\in\mathcal{T}_k \& e_i\in T} x_k(T) \leq c_i, & \forall e_i \in E; & \\ \sum_{T\in\mathcal{T}_k} x_k(T) \leq 1, & k = 1, \ldots, K; & \\ x_k(T) \in \{0, 1\}, & \forall T \,\&\, k = 1, \ldots, K. & \end{array} \qquad (1)$$

The first set of constraints in (1) means that the congestion of each edge is bounded by the edge capacity. The second set of constraints shows that at most one tree is selected to realize the routing for each request. It is possible that in a feasible solution, no tree is chosen for some requests, i.e., some requests may not be realized, due to the edge capacity constraints.

The special cases of the Steiner tree packing problem studied in this work have been shown $\mathcal{APX}$-hard [10, 14, 15], so is our underlying problem. There may be exponentially many variables in (1). Thus many exact algorithms such as standard interior point methods can not be applied to solve its LP-relaxation. The LP-relaxation of (1) may be solved by the volumetric-center [1] or the ellipsoid methods with separation oracle [9]. However, those approaches will lead to a large amount of running time.

As usual, we first solve the LP-relaxation of (1), and then apply rounding techniques to obtain a feasible solution. We call the linear relaxation of the Steiner tree packing problem as the *fractional Steiner tree packing problem*, and its solution as the *fractional solution* to the Steiner tree packing problem. The LP-relaxations of (1) is in fact a fractional packing problem [7, 17, 25]. However, the approximation algorithm in [7] is only for the case that the block problem is polynomial time solvable. Unfortunately, it is not the case for the Steiner tree packing problem as its block problem is the *minimum Steiner tree problem.* In addition, the approximation algorithms in [17, 25] both lead to complexity bounds that depend on the input data, and only result in pseudo polynomial time approximation algorithms. Thus, we need to study approximation algorithms for fractional packing problems with approximate block solvers and input data independent complexity.

3 Approximation Algorithm for Fractional Packing Problems

In this section, we develop an approximation algorithm for fractional packing problems based on the approach in [7]. Our algorithm allows that the block problem can only be approximately solved. Our complexity is still strictly polynomial in the input size and ε^{-1}, which is superior to the methods in [17, 25].

We consider the following fractional packing problem:

$$\max\{c^T x | Ax \le b, x \ge 0\}. \tag{2}$$

Here A is a $m \times n$ positive matrix, and $b \in \mathbb{R}^m$ and $c \in \mathbb{R}^n$ are positive vectors. In addition, we assume that the (i, j)-th entry $A_{i,j} \le b_i$ for all i and j. The corresponding dual program is:

$$\min\{b^T y | A^T y \ge c, y \ge 0\}. \tag{3}$$

Similar to the strategies in [7, 8, 11, 17, 24, 25], an (approximate) block solver is needed, which is similar to the separation oracle for the ellipsoid methods. For a given $y \in \mathbb{R}^m$, the block problem is to find a column index q that $(A_q)^T y/c_q = \min_j (A_j)^T y/c_j$. In our algorithm, we assume that we are given the following

Table 1. Approximation algorithm for fractional packing problems

$\delta = 1 - \sqrt{1-\varepsilon}$, $u = (1+\delta)((1+\delta)m)^{-1/\delta}$, $k = 0$, $x^k = 0$, $f^k = 0$, $y_i^k = u/b_i$, $D^k = um$;
while $D^k < 1$ **do** {*iteration*}
  $k = k + 1$;
  call $ABS(y^{k-1})$ to find a column index q;
  $p = \arg\min_i b_i/A_{i,q}$;
  $x_q^k = x_q^{k-1} + b_q/A_{p,q}$;
  $f^k = f^{k-1} + c_q b_p/A_{p,q}$;
  $y_i^k = y_i^{k-1}[1 + \delta(b_p/A_{p,q})/(b_i/A_{i,q})]$;
  $D^k = b^T y^k$;
end do

approximate block solver $ABS(y)$ that finds a column index q that $(A_q)^T y/c_q \leq r \min_j (A_j)^T y/c_j$, where $r \geq 1$ is the approximation ratio of the block solver.

Our algorithm is an iterative method. We first maintain a pair of a feasible primal solution x to the fractional packing problem (2) and an infeasible dual solution y. At each iteration, based on the current dual solution y, the algorithm calls the approximate block solver once. Then the algorithm increases the component of the primal solution x corresponding to the returned column index by a certain amount and multiples the dual solution y by a factor larger than 1. This iterative procedure does not stop until the dual objective value is more than 1 (though the dual solution may be still infeasible). The algorithm is shown in Table 1. In the algorithm, D^k is in fact the dual objective value for the dual vector y_k at the k-th iteration, though it can be infeasible. Let $\mathcal{OPT}$ denote the optimum dual value (also the optimum objective value of the primal program according to the duality relation). In addition, we assume that the algorithm stops at the t-th iteration. We have the following bound:

Lemma 1. *When the algorithm stops,* $\mathcal{OPT}/f^t \leq r\delta/\ln(um)^{-1}$.

The solution x_t delivered by the algorithm could be infeasible and some packing constraints may be violated. Thus we need to scale the solution by an appropriate amount to obtain a feasible solution.

Lemma 2. *The scaled solution* $x_S = x^t/\log_{1+\delta}((1+\delta)/u)$ *is feasible for (2) and the corresponding objective value is* $f^t/\log_{1+\delta}((1+\delta)/u)$.

Now we are ready to show the performance bound of the solution:

Theorem 1. *When the algorithm stops, the scaled solution* x_S *is a* $(1-\varepsilon)/r$*-approximate solution to the fractional packing problem (2).*

Proof. According to the duality relation, the optimum dual value $\mathcal{OPT}$ is also the optimum objective value of the primal problem (2). Thus we need to examine the objective value corresponding to the feasible solution x_S. According to the definition $u = (1+\delta)((1+\delta)m)^{-1/\delta}$, we have $\ln(um)^{-1} = (1-\delta)\ln[m(1+\delta)]/\delta$

and $\ln((1+\delta)/u) = \ln[m(1+\delta)]/\delta$. Denote by $\mathcal{ALG}$ the objective value of the solution delivered by our algorithm. From the above relations and Lemma 1, the following bound holds:

$$\frac{\mathcal{ALG}}{\mathcal{OPT}} = \frac{f^t}{\mathcal{OPT}\log_{1+\delta}((1+\delta)/u)} \geq \frac{\ln(um)^{-1}}{r\delta}\frac{\ln(1+\delta)}{\ln((1+\delta)/u)} = \frac{(1-\delta)\ln(1+\delta)}{r\delta}$$

According to the elementary inequality $\ln(1+z) \geq z - z^2/2$ for any $0 \leq z \leq 1$, we have $r_{ALG} = \inf(\mathcal{ALG}/\mathcal{OPT}) \geq (1-\delta)(\delta - \delta^2/2)/(r\delta) \geq (1-\delta)^2/r = (1-\varepsilon)/r$.

Theorem 2. *There exists a $(1-\varepsilon)/r$-approximation algorithm for the fractional packing problem (2) that performs $O(m\varepsilon^{-2}\ln m)$ iterations, calling an r-approximate block solver once per iteration, for any $\varepsilon \in (0,1]$.*

Thus we have developed the first algorithm that find a $(1-\varepsilon)/r$-approximate solution to fractional packing problems (2) with a complexity polynomial in the input sizes and ε^{-1}, provided an approximate block solver. It is a generalization of the approximation algorithm in [7].

4 Approximation Algorithm for Steiner Tree Packing

We first study the LP-relaxation of (1).

Theorem 3. *There is a $(1-\varepsilon)/r$-approximation algorithm for the fractional Steiner tree packing problem with complexity $O((m+K)K\varepsilon^{-2}\beta\ln(m+K))$, where r and β are the approximation ratio and the complexity of the minimum Steiner tree solver called as an oracle, respectively.*

Proof. To use our approximation algorithm for fractional packing problems, the only problems are to identify the block problem and to find an (approximate) solver. Notice that the dual vector $y = (y_1, \ldots, y_m, y_{m+1}, \ldots, y_{m+K})^T$ consists of two types of components. The first $m = |E|$ components $y_1, \ldots, y_m$ corresponds to the edges $e_1, \ldots, e_m$. The remaining K components $y_{m+1}, \ldots, y_{m+K}$ in y corresponds to the second set of constraints in (1). It is easy to verify that the block problem is as follows: to find a tree T that $\min_k \min_{T\in\mathcal{T}_k}(\sum_{e_i\in T} y_i + y_{m+k}\delta_{k,T})/w_k$. Here the indicator $\delta_{k,T} = 1$ if $T \in \mathcal{T}_k$, and otherwise $\delta_{k,T} = 0$. To solve the block problem, one can search for K trees corresponding to the K requests separately, such that each tree routes a request with the minimum of $\sum_{e_i\in T} y_i$. Afterwards, for each of these K trees, the additional term y_{m+k} is added, and the sums are divided by w_k respectively. Thus the tree with the minimum value of $(\sum_{e_i\in T} y_i + y_{m+k}\delta_{k,T})/w_k$ over all K trees is selected, which is the optimum solution to the block problem. Since the value y_{m+k} is fixed for a fixed request k at each iteration, the block problem is in fact equivalent to finding a tree spanning the request S_k that $\min_{T\in\mathcal{T}_k}\sum_{e_i\in T} y_i$, for $k = 1, \ldots, K$. Regarding y_i the length of edge e_i for $i = 1, \ldots, m$, the block problem is in fact the minimum Steiner tree problem in graphs. Thus, we can use the approximation algorithm developed in Section 3 with an approximate minimum Steiner tree solver to obtain a feasible solution to the LP-relaxation of (1), and the theorem follows.

Unfortunately, the minimum Steiner tree problem is $\mathcal{APX}$-hard [3]. Thus, the approximation algorithm in [7] is not applicable in this case. We apply randomized rounding [18, 19] to find a feasible (integer) solution. As indicated in [18, 19], to guarantee non-zero probability that no constraint is violated, a scaling technique is necessary to be employed. Denote by c the minimum edge capacity. Suppose that there exists a scalar v satisfying $(ve^{1-v})^c < 1/(m+1)$. From [18], we can immediately obtain the following bound:

Theorem 4. *There is an approximation algorithms for the Steiner tree packing problem such that the objective value delivered is at least*

$$\begin{cases} (1-\varepsilon)vOPT/r - (\exp(1)-1)(1-\varepsilon)v\sqrt{OPT\ln(m+1)/r}, & \text{if } OPT > r\ln(m+1); \\ (1-\varepsilon)vOPT/r - \dfrac{\exp(1)(1-\varepsilon)v\ln(m+1)}{1+\ln(r\ln(m+1)/OPT)}, & \text{otherwise,} \end{cases}$$

where OPT is the optimal objective value of (1).

In (1) there are exponential number of variables. However, by applying our approximation algorithm for fractional packing problems in Section 3, we just need to generate K approximate minimum Steiner trees for the K requests at each iteration corresponding to the current dual vector. Thus there are only $O((m+K)K\varepsilon^{-2}\ln(m+K))$ Steiner trees generated in total. This is similar to the column generation technique for LPs, and the hardness due to exponential number of variables in (1) is overcome.

5 Integrality

A solution to the fractional packing problem (2) has *integrality* w if each components in the solution is a non-negative integer multiple of w. In this case we modify the approximation algorithm in Table 1 as follows: At the k-th iteration, after calling the approximate block solver, the increments of x and y are $x_k(q) = x_{k-1}(q) + w$ and $y_k(i) = y_{k-1}(i)\{1 + \delta[w]/[b(i)/A(i,q)]\}$, and the following result follows:

Theorem 5. *If $w \leq \min_{i,j} b_i/A_{i,j}$ in the fractional packing problem (2), then there exists an algorithm that finds a $(1-\varepsilon)/r$-approximate solution to (2) with integrality $w\delta/(1+\log_{1+\delta} m)$ within $O(m\varepsilon^{-2}\rho\ln m)$ iterations, where $\rho = \max_{i,j} b_i/A_{i,j}$.*

Corollary 1. *If $b_i/A_{i,j} \geq (1+\log_{1+\delta} m)/\delta$ for all i and j, then there exists an algorithm that finds a $(1-\varepsilon)/r$-approximate solution to integer packing problems within $O(m\varepsilon^{-2}\rho\ln m)$ iterations.*

Corollary 2. *If all edge capacities are at least $(1+\log_{1+\delta}(m+K))/\delta$, then there exists an algorithm that finds a $(1-\varepsilon)/r$-approximate integer solution to the Steiner tree packing problem (1) within $O((m+K)K\varepsilon^{-2}c_{\max}\beta\ln(m+K))$ time, where r and β are the approximation ratio and the complexity of the minimum Steiner tree solver called as the oracle, and $c_{\max}$ is the maximum edge capacity.*

We have presented a pseudo polynomial time approximation algorithm for integer packing problems. However, this approach is still useful, as it can directly lead to an integer solution and can avoid the rounding stage. We believe that there exist instances in practice that our approximation algorithm for integer packing problem works efficiently.

6 Multicast Routing and Wavelength Assignment in Optical Networks

In the multicast routing and wavelength assignment problem in optical networks, we are given an undirected graph $G = (V, E)$, a set of multicast requests $S_1, \ldots, S_K \subseteq V$, and a set $\mathcal{L} = \{1, \ldots, L\}$ of wavelengths. It is assumed that every edge represents a bundle containing multiple fibers in parallel. In particular, we let $c_{i,l}$ denote the number of fibers of edge $e_i \in E$ that have wavelength $l \in \mathcal{L}$. Note that wavelengths that are not available in a fiber are assumed to be pre-occupied by existing connections in the network. The goal is to find routing trees with the maximum total profit/throughput, such that every selected request is realized by a Steiner tree and assigned one of the given wavelengths, and that trees crossing the same fiber are assigned different wavelengths.

Denote by $\mathcal{T}_k$ the set of all trees spanning the request S_k, for all $k = 1, \ldots, K$. Here $|\mathcal{T}_k|$ could be exponentially large. Then we define an indicator variable $x_k(T, l)$ as follows: $x_k(T, l) = 1$ if $T \in \mathcal{T}_k$ is selected for routing S_k and is assigned wavelength l; Otherwise $x_k(T, l) = 0$. In addition, each request S_K is associated with a weight w_k indicating its importance in the given multicast optical network. Thus the ILP of the problem is as follows:

$$\begin{array}{rll} \max & \sum_{k=1}^{K} w_k \sum_{l=1}^{L} \sum_{T \in \mathcal{T}_k} x_k(T, l) & \\ \text{s.t.} & \sum_{k=1}^{K} \sum_{T \in \mathcal{T}_k \& e_i \in T} x_k(T, l) \le c_{i,l}, & \forall e_i \in E \,\&\, l \in \mathcal{L}; \\ & \sum_{l=1}^{L} \sum_{T \in \mathcal{T}_k} x_k(T, l) \le 1, & k = 1, \ldots, K; \\ & x_k(T, l) \in \{0, 1\}, & \forall T,\, l \in \mathcal{L} \,\&\, k = 1, \ldots, K. \end{array} \tag{4}$$

The first set of constraints ensures that each of these trees can be routed through a separate fiber. The second set of constraints indicate that the we just need to route each request by at most one tree and assign at most one wavelength to it.

First, for the fractional multicast routing and wavelength assignment problem, we have the following result (see [21] for proof):

Theorem 6. *There is a $(1 - \varepsilon)/r$-approximation algorithm for the fractional multicast routing and wavelength assignment problem in optical networks with complexity $O((mL + K)KL\varepsilon^{-2}\beta \ln(mL + K))$, where r and β are the approximation ratio and the complexity of the minimum Steiner tree solver called as the oracle, respectively.*

Similar to Section 4, for any real number v satisfying $(ve^{1-v})^c < 1/(m + 1)$, where $c = \min_{i,l} c_{i,l}$ is the minimal capacity, we can obtain a bound for the integer solution by randomized rounding [18, 19]:

Theorem 7. *There is an approximation algorithms for the multicast routing and wavelength assignment problem in optical networks such that the objective value delivered has the same bound as in Theorem 4, where OPT is the optimal objective value of (4).*

Furthermore, we can apply our approximation algorithm for integer packing problems described in Section 5 to (4) and directly obtain an integer solution:

Theorem 8. *If all edge capacities are at least* $(1 + \log_{1+\delta}(mL + K))/\delta$*, then there exists an algorithm that finds a* $(1 - \varepsilon)/r$*-approximate integer solution to the multicast routing and wavelength assignment problem in optical networks (4) within* $O((mL + K)LK\varepsilon^{-2}c_{\max}\beta \ln(mL + K))$ *time, where* r *and* β *are the approximation ratio and the complexity of the minimum Steiner tree solver called as the oracle, and* $c_{\max}$ *is the maximum capacity.*

7 Conclusions and Future Research

in this paper, we have addressed the problem of maximizing a Steiner tree packing, such that each tree connects a subset of required vertices without violating the edge capacity constraints. We have developed a $(1 - \varepsilon)/r$ approximation algorithm to solve the LP-relaxation provided an r-approximate block solver. This is the first approximation result for fractional packing problems with only approximate block solvers and a coordination complexity that is polynomial in the input size and ε^{-1}. This generalizes the well-known result in [7] while the complexity is the same, and it is superior to many other approximation algorithms for fractional packing problems, e.g., [17, 25]. In this way we have designed approximation algorithms for the Steiner tree packing problem. Finally, we have studied an important multicast routing and wavelength assignment problem in optical networks. We are further interested in both theoretical and practical extensions of this work. An interesting problem is to develop approximation algorithms for fractional/integer packing problems with other properties. From the practical point of view, we aim to develop more realistic models for routing problems arising in communication networks and design strategies to (approximately) solve them efficiently. We have applied our approximation algorithm for fractional packing problems to the global routing problem in VLSI design [20]. In addition, we are working on implementation of our approximation algorithms with challenging benchmarks to explore their power in computational practice.

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