Optimal Starting Price in Online Auctions*

Hai Yu[1,2], Shouyang Wang[1,2], and Chuangyin Dang[3]

[1] Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100080, China

[2] Graduate School of the Chinese Academy of Sciences, Beijing 100039, China

{yuhai, sywang}@amss.ac.cn

[3] Department of Manufacturing Engineering and Engineering Management, City University of Hong Kong, Kowloon, Hong Kong

mecdang@cityu.edu.hk

Abstract. Reserve price auctions are one of hot research topics in the traditional auction theory. Here we study the starting price in an online auction, counterpart of the public reserve price in a traditional auction. By considering three features of online auctions, the stochastic entry of bidders (subject to Poisson process), the insertion fee proportional to the starting price, and time discount, we have analyzed the properties of extremum points of the starting price for maximizing seller's expected revenue, and found that, under certain conditions, the optimal starting price should be at the lowest allowable level, which is contrary to results from the classic auction theory and finds its optimality in reality. We have also developed a general extended model of multistage auction and carried out analysis on its properties. At last, some directions for further research are also put forward.

1 Introduction

It's well known that in an endogenous entry auction, the optimal auction should be zero reserve combined with optimal entry fee or optimal reserve combined with zero entry fee. That is they are alternative. In online auctions such as eBay auctions, seller can't charge entry fee from potential bidders, so they have to set optimal reserve to maximize their expected revenue.

In an online auction, seller can set some parameters to feature her auction. She may activate the Buy-It-Now option, choose among different durations and payment methods, initiate the starting price at high level or low level and set hidden reserve price and so on. The most important consideration of a seller in online auctions is how to set the starting price and hidden reserve price[1]. Because the hidden reserve price is seldom used by sellers in online auction[7][12], the starting price plays an important role on affecting the result of an auction. Lower starting price can attract more bidders while having risk in a the higher possibility of receiving lower final price thus lower expected revenue. On the other hand, if the starting price is too high, the item will not be auctioned off at all.

* Supported by NSFC and SRG: 7001719 of City University of Hong Kong.

X. Deng and Y. Ye (Eds.): WINE 2005, LNCS 3828, pp. 315–324, 2005.

How to trade-off? Is there any optimal starting price for maximizing the seller's ex ante expected revenue? Do sellers in online auction have some rationalities in setting their starting price optimally? The starting price, an alternative of public reserve price in traditional auction theory, is thus our main attention in this paper.

As it is well known, the online auction market is different from that of the traditional auction. Some features have changed the seller's expected revenue. The first one is the stochastic entry of bidders, which results in uncertainty in the number of participants in an online auction. While in the traditional auction theory, the number of bidders is critical to determine the seller's expected revenue. The second one is that in online auctions such as eBay auctions, a seller needs to pay the insertion fee (roughly proportionally to the starting price) to initiate her auction. This will affect the seller's decision making on how high the starting price should be. The last one is about the depreciation of item's valuation due to the duration of an online auction. In the case of products with a rapid depreciation like consumer electronics, computer equipment and fragile or perishable goods like flower and food, long auctions can reduce the value of the product and reduce the seller's revenue [10]. That is, the time discount factor is also needed to be considered when formulating the seller's expected revenue.

We intend to formulate the seller's expected revenue by considering above more realistic conditions, so as to analyze the optimal starting price policy the seller should adopt with the objective of maximizing her expected revenue. In addition, noting that in practice, online auction markets give sellers opportunities of re-auctioning their items with very cheap costs, we'll extend our model to a multistage auction and call for further research. It should be noted that all the discussions in this paper is under the IPV (Independent Private Value) assumption.

This paper is organized as follows. We give out the basic model and discuss several basic properties of extremum points of starting price for maximizing the seller's expected revenue in section 2. Subsequently in section 3, we analyze the optimality of the starting price in online auctions. The extended model for a multistage auction is suggested in section 4, and section 5 concludes the paper and puts forward several directions for further research.

2 Basic Model

We consider a risk-neutral seller who chooses to auction one item on an eBay-like online auction site. She doesn't set any hidden reserve price, and initiates her auction by paying an insertion fees proportional to the starting price r, which is prescribed by her. we assume this insertion fee rate is c_0, the intrinsic value of the item to the seller is v_s with $v_s \geq 0$, and the number of bidders is fixed at n. Suppose that bidders have independent private values (assessed at the end of the auction) to the item being auctioned, and the common cumulative distribution function (c.d.f.) of a bidder's valuation is $F(v), v \in [\underline{v}, \overline{v}]$, which is twice differentiable. The probability density function (p.d.f.) is $f(v)$.

Because online auctions, such as eBay auctions, are essentially second-price auctions in which the bidders' equilibrium bidding behaviors are the same with or without ambiguity about the number of bidders [6], we can use the similar method of [11] to construct the seller's expected revenue providing there are fixed n bidders participate:

$$E(\pi_s, n) = v_s F^n(r) + n\int_r^{\bar{v}} (vF'(v) + F(v) - 1)F^{n-1}(v)dv - c_0 r. \quad (1)$$

The expressions for $E(\pi_s, n)$ is composed of three parts. The first is that when no bid surpasses the starting price, i.e. the highest valuation of bidders is less than the starting price, in which case the item doesn't sell at the auction. The auction revenue increasing from this outcome is v_s and occurs with probability $F^n(r)$. The second is that the highest bid is at least equal to the starting price, so the item sells in the auction. This outcome's contribution to $E(\pi_s, n)$ is

$$n\int_r^{\bar{v}} (vF'(v) + F(v) - 1)F^{n-1}(v)dv$$

(from Prop.1 of [11]). At last, the seller pays the insertion fee $p_0 = c_0 r$ definitely to initiate her auction.

In online auctions, bidders can place their bids at any point in the duration. We assume that the bid or bidder arrivals follow a Poisson process, i.e. the probability of n bidders participating in the auction is:

$$p_n = \frac{\lambda e^{-\lambda}}{n!}, n = 0, 1, 2, \cdots \quad (2)$$

where $\lambda > 0$ is the average number of bidders during the duration of auction.

As discussed above, we consider the effect of duration on item's valuation and denote the time discount factor as $\delta \in [0, 1)$. Therefore, the seller can only receive $\frac{v_s}{1+\delta}$ by retaining the item in case of the failed auction.

By further considering the above two assumptions resulting from the characteristics of online auctions, the seller's expected revenue should be:

$$E(\pi_s) = \sum_{n=0}^{\infty} p_n [n\int_r^{\bar{v}} (vF'(v) + F(v) - 1)F^{n-1}(v)dv + \frac{v_s}{1+\delta}F^n(r)] - c_0 r. \quad (3)$$

To get the extremum point of r for the seller, consider the first-order necessary optimality condition for $E(\pi_s)$, i.e.:

$$\frac{\partial E(\pi_s)}{\partial r} = \sum_{n=0}^{\infty} p_n [n(1 - rf(r) - F(r))F^{n-1}(r) + \frac{nv_s}{1+\delta}F^{n-1}(r)f(r)]|_{r=r^*} - c_0 = 0.$$

Combined with equation (2), solving and simplifying it gives:

$$r^* = \frac{1 - F(r^*) - \frac{c_0}{\lambda} e^{\lambda(1-F(r^*))}}{f(r^*)} + \frac{v_s}{1+\delta}. \quad (4)$$

Following We'll discuss the properties of the extremum point r^*.

Lemma 1. *Denote*

$$H(v) = f^2(v)(2 - c_0 e^{\lambda(1-F(v))}) + f^{'}(v)(1 - F(v) - \frac{c_0}{\lambda} e^{\lambda(1-F(v))}),$$

and

$$M(v) = v - \frac{1 - F(v) - \frac{c_0}{\lambda} e^{\lambda(1-F(v))}}{f(v)}, v \in (\underline{v}, \overline{v}),$$

if $H(v) \geq 0$*, then* $M(v)$ *increases in* v*.*

Proof. Taking the derivative of $M(v)$ w.r.t. v gives

$$\frac{dM(v)}{dv} = 1 - \frac{f^2(v)[c_0 e^{\lambda(1-F(v))} - 1] - f^{'}(v)[1 - F(v) - \frac{c_0}{\lambda} e^{\lambda(1-F(v))}]}{f^2(v)} = \frac{H(v)}{f^2(v)},$$

that is $\frac{dM(v)}{dv} \geq 0 \Leftrightarrow H(v) \geq 0$. □

Proposition 1. *The extremum point* r^* *for* $E(\pi_s)$ *may be unique, non-unique or non-existent at all.*

Proof. By definition, $H(v)$ can be negative or positive. From Lemma 1, $\frac{dM(v)}{dv}$ has the same signal as that of $H(v)$. So $M(v)$ may be non-monotone in $[\underline{v}, \overline{v}]$. On the other hand, from Equation (4), the extremum point r^* should satisfy: $M(r^*) = \frac{v_s}{1+\delta}$. So, r^* may be unique, non-unique or non-existent at all. □

Consider the boundary points of the interval of r. Because at $r = \overline{v}$, no bidders will participate, the expected revenue can't be maximized at $r = \overline{v}$.

Proposition 2. *Fixing* λ*,* δ*,* v_s *and* c_0*, the expected revenue* $E(\pi_s)$ *can achieve its global maximum at either* r^* *or* $r = \underline{v}$*, where* r^* *meets the Equation (4).*

Theorem 1. *If* $H(v) \geq 0$*,* $v \in [\underline{v}, \overline{v}]$*, then the expected revenue* $E(\pi_s)$ *can achieve its global maximum at* r^**, which is determined in Equation (4).*

Proof. From equation (4), we have $M(r^*) = \frac{v_s}{1+\delta}$. Because $H(v) \geq 0$, from Lemma 1, we know $M(v) = v - \frac{1-F(v)-\frac{c_0}{\lambda}e^{\lambda(1-F(v))}}{f(v)}$ is increasing in v. Thus for $\underline{v} < r < r^*$, $M(r) \leq \frac{v_s}{1+\delta}$, i.e. $1 - rf(r) - F(r) \geq -\frac{v_s}{1+\delta} f(r) + \frac{c_0}{\lambda} e^{\lambda(1-F(r))}$. Thus, $\frac{\partial E(\pi_s)}{\partial r} \geq 0$, for $\underline{v} < r < r^*$. Similarly, $\frac{\partial E(\pi_s)}{\partial r} \leq 0$, for $r^* < r < \overline{v}$. Therefore, $E(\pi_s)$ can achieve its maximum at r^*. □

Remark 1. Theorem 1 has indicated that if the parameter combinations of c_0, λ and $F(\cdot)$ make $H(v)$ satisfy the strong condition $H(v) \geq 0$ within the whole interval $[\underline{v}, \overline{v}]$, then the extremum point r^* is unique. Furthermore, the seller's expected revenue can achieve global maximum at it.

Because the insertion fee rate c_0 is fixed at each auction site, following we'll only discuss the relations between r^* and λ, δ. Denote

$$K(r^*, \lambda, \delta) = r^* - \frac{1 - F(r^*) - \frac{c_0}{\lambda} e^{\lambda(1-F(r^*))}}{f(r^*)} - \frac{v_s}{1+\delta} = 0.$$

Then

$$\frac{dr^*}{d\lambda} = -\frac{\frac{\partial K}{\partial r^*}}{\frac{\partial K}{\partial \lambda}} = -\frac{\frac{dM}{dr^*}}{\frac{c_0 e^{\lambda(1-F(r^*))}}{f(r^*)\lambda^2}[\lambda(1-F(r^*))-1]}.$$

Similarly,

$$\frac{dr^*}{d\delta} = -\frac{\frac{\partial K}{\partial r^*}}{\frac{\partial K}{\partial \delta}} = -\frac{\frac{dM}{dr^*}}{\frac{v_s}{(1+\delta)^2}}.$$

Intuitively, the higher the discount factor δ is, the lower the starting price should be, i.e. $\frac{dr^*}{d\delta}$ should be less than zero, which requires $\frac{dM}{dr^*} \geq 0$. On the other hand, because r^* can't be too high generally, $\lambda(1-F(r^*))-1 \geq 0$ is easy to be held. Thus, generally, $\frac{dr^*}{d\lambda} \leq 0$. That is when the average number of bidders λ reach a certain level (satisfying $\lambda(1-F(r^*))-1 \geq 0$), the larger expected number of bidders on average, the lower the optimal starting price should be. This is consistent with the intuition. The more the bidders, the higher probability the bidders with higher valuation will arrive, resulting in more furious competition among higher valuator, lower starting price can play an important role in attracting these high level bidders.

3 Optimal Starting Price

In the traditional auction theory (e.g., [5][9][11]), the optimal starting price r^* maximizing the seller's expected revenue should satisfy: $r^* = v_s + \frac{1-F(r^*)}{f(r^*)}$, which implies $r^* > v_s$, so there is a positive probability the seller refuses the entry of bidders with valuations exceeding the valuation to the seller. However, in online auction market, there are nontrivial percentage of sellers set the starting price at the lowest allowable level. An obvious discrepancy between theory and practice motivates us to explore the possibility of setting the starting price equal to the lowest allowable level other than above the true value to the seller. We open up our study by an example as follows.

Example 1. Suppose the valuations of bidders are distributed uniformly on unit interval, i.e. $F(v) = v$, where $v \in [0,1]$, the discount factor $\delta = 0.05$, insertion fee rate $c_0 = 0.03$, and true value to the seller $v_s = 0.2$. We derive the condition under which should seller start the auction from the lowest price, zero. Given the above assumption, and from Equation (4), the extremum points should satisfy:

$$0.691 - r^* = \frac{0.015}{\lambda} e^{\lambda(1-r^*)}. \tag{5}$$

It's easy to verify that the right hand side of Equation (5) is increasing in λ and the left hand side is less than 0.691. So there exists λ^*, when $\lambda \geq \lambda^*$, no r^* satisfies Equation (5). That is, the maximum of the expected revenue is achieved at the starting price $r = 0$. For this example, the optimal starting price should

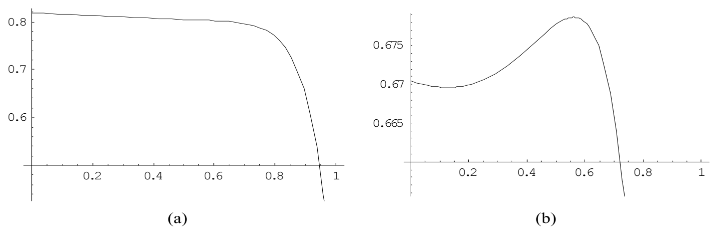

Fig. 1. The expected revenue varied with starting price, (a)$\lambda = 11$, (b)$\lambda = 6$

be zero for all $\lambda \geq 11$. Figure 1(a) gives the expected revenue varied with the starting price from 0 to 1.

While, if the average number of bidders λ is less than 11, say at 6, then there exist two extremum points, one is the global minimum $r_1^* = 0.1284$ and the other is the global maximum $r_2^* = 0.5603$, which are plotted in Figure 1(b).

The implication behind example 1 is very intuitive. In fact, it originates from the following theorem.

Theorem 2. *For fixed c_0, $F(v), v \in [\underline{v}, \overline{v}]$, δ and v_s, there exists such threshold λ^* that:*

(1)for $\lambda < \lambda^*$, there exist one or two extremum points for $E(\pi_s)$;
(2)for $\lambda \geq \lambda^*$, the maximum of $E(\pi_s)$ is achieved by letting starting price equal to the lowest allowable level $\underline{v}$.

Proof. Define $T(v, \lambda) = -\frac{c_0}{\lambda} e^{\lambda(1-F(v))} + [1 - F(v) - f(v)(v - \frac{v_s}{1+\delta})]$, from Equation (4) we know that the extremum point of $E(\pi_s)$ must be the root of $T(v, \lambda) = 0$, for fixed λ. On the other hand, according to L'Hospital's Rule,

$$\lim_{\lambda \to \infty} -\frac{c_0}{\lambda} e^{\lambda(1-F(v))} = \lim_{\lambda \to \infty} -c_0(1 - F(v)) e^{\lambda(1-F(v))} = -\infty;$$

on the other hand, for fixed c_0, $F(v), v \in [\underline{v}, \overline{v}]$, δ and v_s, $1 - F(v) - f(v)(v - \frac{v_s}{1+\delta})$ must be bounded. So there must exist a threshold λ^*, for all $\lambda > \lambda^*$, $T(v, \lambda) < 0$, which induces that $T(v, \lambda) = 0$ has no root. That is no available extremum point determined by equation (4). Because we have excluded the possibility of maximum point at $r = \overline{v}$, it is clear immediately that the expected revenue for seller must achieve its maximum at the lowest starting price $\underline{v}$. This has proved the second claim.

Similarly, we can easily verify that

$$\lim_{\lambda \to 0} -\frac{c_0}{\lambda} e^{\lambda(1-F(v))} = -\infty$$

and note $\frac{\partial T(v,\lambda)}{\partial \lambda} = -\frac{c_0}{\lambda^2} e^{\lambda(1-F(v))}[\lambda(1 - F(v)) - 1]$. So $T(v, \lambda)$ is increasing with λ when $\lambda(1 - F(v)) \leq 1$ and decreasing with λ when $\lambda(1 - F(v)) > 1$. Combined with the fact that

$$\lim_{\lambda \to 0} T(v, \lambda) < 0, \lim_{\lambda \to \infty} T(v, \lambda) < 0,$$

if $T(v, \lambda)$ crosses the axis from below once, it must cross the axis again from above; if it just reaches the axis and decreases subsequently, unique root is found.

At last, we'll indicate that, for $\lambda < \lambda^*$, there always exist at least one solution for extremum condition (4). We rearrange it and find that r^* must satisfy:

$$e^{\lambda(1-F(v))} = \frac{\lambda}{c_0}[1 - F(v) - f(v)(v - \frac{v_s}{1+\delta})]. \tag{6}$$

For fixed v, denote the left hand side of (6) is $A(\lambda)$ and the right hand side is $B(\lambda)$. For certain λ^*, when $\lambda < \lambda^*$, there exists $v \in [\underline{v}, \overline{v}]$ that brings $A(\lambda)$ crosses $B(\lambda)$ one or two times. That is the first claim of Theorem 2 comes into exist. □

Remark 2. The implication behind Theorem 2 is that the average number of bidders λ is the critical determinant of optimal starting price. There exists a threshold λ^*, when $\lambda > \lambda^*$, the optimal starting price should be at the lowest allowable level. This enlightens the seller to pay more attention on attracting the participants into her auction, so as to achieve the maximal expected revenue at the lowest starting price. However, for those less popular items or short auctions, the optimal starting price should be other than the lowest one.

4 Model Extension

Now we consider the possibility of re-auction. In reality, some sellers prefer higher final price to higher probability of auctioning off the item. Then they usually choose very high starting price regardless of possibility of attracting no bidders at all. Were the item not auctioned off, the seller will re-auction it just after the end of current auction with same or lower starting price, by paying another insertion fee. This process will continue until the item is auctioned off or the seller decides to never auction it again. We describe this auction process as a multistage auction. Suppose the seller's belief of the longest stage is m, the c.d.f. of valuations in each auction is $F_i(v), v \in [\underline{v}_i, \overline{v}_i], i = 1, 2, \cdots, m$, then the seller's problem should be:

$$\max_{r_i, i \in \{1,2,\cdots m\}} E(\pi_1) \tag{7}$$

s.t.

$$\begin{cases} E(\pi_1) = \sum_{n=0}^{\infty} p_n[n\int_{r_1}^{\overline{v}_1}(vF_1'(v) + F_1(v) - 1)F_1^{n-1}(v)dv + \frac{E(\pi_2)}{1+\delta}F_1^n(r_1)] - c_0 r_1 \\ E(\pi_2) = \sum_{n=0}^{\infty} p_n[n\int_{r_2}^{\overline{v}_2}(vF_2'(v) + F_2(v) - 1)F_2^{n-1}(v)dv + \frac{E(\pi_3)}{1+\delta}F_2^n(r_2)] - c_0 r_2 \\ \cdots\cdots \\ E(\pi_m) = \sum_{n=0}^{\infty} p_n[n\int_{r_m}^{\overline{v}_m}(vF_m'(v) + F_m(v) - 1)F_m^{n-1}(v)dv + \frac{v_s}{1+\delta}F_m^n(r_m)] - c_0 r_m. \end{cases}$$

That is the seller should choose a series of optimal starting price $r_i^*, i \in \{1, 2, \cdots m\}$ for each stage of auction so as to maximize the total expected revenue $E(\pi_1)$.

How to solve problem (7) is still a challenge. While, from some simple analyses and numerical study can we get some properties of this extended model.

Proposition 3. *The optimal starting price of current stage, 1) isn't affected by that of any previous stage, and 2) affects the previous optimal starting price.*

Remark 3. This property is very straightforward: the effect of the optimal starting price of current stage on that of previous stage is through its effect on current optimal expected revenue because of the expected revenues' recursion in problem (7). This proposition enlightens the seller to determine the optimal starting price series r_i^* backward.

Theorem 3. *If $H(v)$ (defined in Lemma 1) is larger than zero and assume $F_i(v) = F_j(v) = F(v)$, $v \in [\underline{v}, \overline{v}]$, then the optimal starting price series r_i^* will be non-increasing in $i, i \in \{1, 2, \cdots m\}$.*

Proof. According to previous analysis, the extremum point r_i^* in each stage should satisfy:

$$M(r_i^*) = \begin{cases} \frac{v_s}{1+\delta}, & i = m, \\ \frac{E(\pi_{i+1})}{1+\delta}, & \text{otherwise.} \end{cases}$$

Lemma 1 has shown that if $H(v)$ is lager than zero, then $M(v)$ is increasing in v. Obviously, $E(\pi_m) \geq v_s$, which results to $r_{m-1}^* \geq r_m^*$. Noting further that the model also needs $E(\pi_1) \geq E(\pi_2) \geq \cdots \geq E(\pi_m)$, for otherwise, the maximum of $E(\pi_1)$ can't be achieved (the auction may be continued infinitely). So we get $r_1^* \geq r_2^* \geq \cdots \geq r_m^*$. □

Remark 4. The result in Theorem 3 may be intuitive. When the seller anticipates multi-stage of auction, she may set a very high starting price at the first stage to look for the higher valuator. If she successes, she can get a higher revenue; otherwise, she can set a lower starting price in the next stage of auction so as to improve the probability of auctioning off the item. Once she decides the last opportunity of re-auction (i.e. the stage m), she can set the optimal starting price according to the discussion in section 3.

Example 2. Recall Example 1 in section 3, and assume $m = 4$. Considering different level of λ, the optimal starting price and corresponding expected revenue for each stage are shown in table 1.

Table 1. Optimal starting price and resulted expected revenue for each stage

Stage	$\lambda = 4$		$\lambda = 6$		$\lambda = 11$	
i	r_i^*	$E(\pi_i^*)$	r_i^*	$E(\pi_i^*)$	r_i^*	$E(\pi_i^*)$
4	0.574685	0.581486	0.560260	0.678683	0.000000	0.818205
3	0.767389	0.681668	0.815625	0.757464	0.884778	0.845557
2	0.816800	0.723277	0.854720	0.785441	0.898481	0.853474
1	0.837226	0.743121	0.868517	0.797060	0.902428	0.855996
Increasing rate	45.68%	27.80%	55.02%	17.44%	–	4.62%

From table 1 we can find that:

(1) Both the optimal starting price and corresponding expected revenue are decreasing in the stage of auction regardless of average number of arrived bidders. That is, when there is anticipation of re-auction, to maximize her expected revenue, the seller should set decreasing optimal starting prices, from higher to lower. In practical online auction market, there is a kind of seller characterized by following behavior: setting very high starting price when she auctioned her item first time. Fortunately enough, she can get excessive profit; if the item is not auctioned off in the first stage, she can re-auction it with a little lower starting price just after the end of the first stage. This process continues until she sets a suddenly lower starting price to maximize the expected revenue of last stage.
(2) The expected revenue is also increasing in the average number of bidders under each stage. This is consistent with the intuition and previous study[8]. This finding implies that attracting bidders as many as possible to participate the auction is the main source of revenue for the seller.
(3) The multi-stage auction can provide the seller more expected revenue than just one-shot auction. The difference is just $E(\pi_1) - E(\pi_m)$. As in table 1, all these difference is positive. However, the effect may be weakened as the average number of arrived bidders λ increases. As in this example, the increasing rate of expected revenue is the highest 27.80% at $\lambda = 4$, then decreases to 17.44% at $\lambda = 6$, finally to the lowest level of only 4.62% at $\lambda = 11$.
(4) The increasing rate of optimal starting price is increasing in λ.

If we consider the "damaged good effect" of re-auction: the bidders generally regard the re-auctioned items to be damaged and lower their offers[2], then they will value the item decreasingly as the stages go on. Consequently, $F_i(v)$ should first order stochastic dominate $F_j(v)$, for every $i < j$. i.e. $F_i(v) < F_j(v)$, for every $i < j$. Under this circumstance, it is further difficult to extract the analytical solution to problem (7).

5 Conclusion Remark

How to set appropriate starting price may be the most important problem for sellers in online auctions. In this paper, we make some efforts on this topic. By considering three features of online auctions—the stochastic entry of bidders (subject to Poisson process), the insertion fee proportional to the starting price, and time discount, we have investigated the properties of extremum points of the starting price for maximizing seller's expected revenue. Through some examples and analyses, we found that under certain conditions, the optimal starting price should be at the lowest allowable level, which is contrary to results from the classic auction theory and finds its optimality in reality. We have also developed a general extended model of multistage auction, given out analysis on its properties, and call for further research on its analytical solution.

It should be noted that setting the starting price at the lowest allowable level may be served as a kind of fraud. Kauffman and Wood[4] illustrated the prevalence of reserve price shilling, which is motivated by the avoidance of auction site's insertion fees. By setting a low starting price, and then secretly bidding that amount up by pretending to be a bidder, sellers can save money when selling items in online auction. This behavior should be investigated in further research.

The objectives of sellers in online auctions may not always be maximizing their expected revenue. For firm who is trying to dump its excess inventory, seller who clear her attic full of used tools, furniture, toys etc., the goal of using online auction is just to sell out the items as soon as possible. Many firms have found that online auctions are also a tool for managing inventory and marketing new products [3][10]. Under all above circumstances, the sellers are more inclined to let bidders compete with each other and determine the final price themselves without imposing any reserve. That is starting the auction from the lowest level finds its optimality in reality. So empirical study should proceed to investigate whether items with above characteristics are more likely be auctioned with the lowest allowable starting price in online auction sites.

References

1. Bajari, P., Hortacsu, A.: The Winner's Curse, Reserve Prices and Endogenous Entry: Empirical Insights from eBay Auctions, Rand J. Econom. 3(2003) 329-355
2. Greenleaf, E.A., Rao, A.G., Sinha, A.R.: Guarantees in Auctions: The Auction House as Negotiator and Managerial Decision Maker. Management Sci. 39(1993) 1130-1145
3. Heid, J., Malina, T.: Sell It on eBay: TechTV's Guide to Successful Online Auctions. Berkeley, CA : Peachpit Press. (2004)
4. Kauffman, R.J., Wood, C.A.: Running up the Bid: an Empirical Study of Seller Behavior in Internet Auctions. Proceedings of the 2000 Americas Conference on Information System, Long Beach, CA. (2000) 929-935
5. Krishna, V.: Auction Theory. Academic Press, San Diego, CA. (2002)
6. Levin, D., Ozdenoren, E.: Auctions with Uncertain Numbers of Bidders. J. Econom. Theory. 118(2004) 229-251
7. Lucking-Reiley, D.: Acutions on the Internet: What's Being Auctioned and How? J. Indust. Econom. 48(2000) 227-252
8. Lucking-Reiley, D., Bryan, D., Prasad, N., Reeves, D.: Pennies from eBay: the Determinants of Price in Online Auctions. Working Paper (2000)
9. Myerson, R.: Optimal Auction Design. Mathematics of Ope. Res. 6(1981) 58-73
10. Pinker, E.J., Seidmann, A., Vakrat, Y.: Managing Online Auctions: Current Business and Research Issues. Management Sci. 49(2003) 1457-1484
11. Riley, J.G., Samuelson, W.F.: Optimal Auctions. Amer. Econom. Rev. 71(1981) 381-392
12. Yu, H., Wang, S.Y., Dang, C.Y.: A Case Study of Online Auctions in eBay-eachnet: Dimension, Characters and Performance. Competitive Bidding and Auctions II, Global-Link Publisher, Hong Kong, London, Tokyo. (2005) 126-151