A Qualitative Trajectory Calculus and the Composition of Its Relations

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Abstract. Continuously moving objects are prevalent in many domains. Although there have been attempts to combine both spatial and temporal relationships from a reasoning, a database, as well as from a logical perspective, the question remains how to describe motion adequately within a qualitative calculus. In this paper, a Qualitative Trajectory Calculus (QTC) for representing and reasoning about moving objects in two dimensions is presented. Specific attention is given to a central concept in qualitative reasoning, namely the composition of relations. The so-called composition-rule table is presented, which is a neat way of representing a composition table. The usefulness of QTC and the composition-rule table is illustrated by an example.

1 Introduction

In the last two decades, spatial and temporal information have received significant attention in various fields of research, ranging from geography and geometry to artificial intelligence and computer science. Qualitative calculi have been proposed, both in the temporal (e.g. [1],[2]) and the spatial (for an overview: see [3]) domain. The mentioned formalisms are especially suited to express relationships between entities. This type of formalism has gained wide acceptance as a useful way of abstracting from the real world. Only in recent years, the attention has extended to applications that involve spatio-temporal data. Nevertheless, both from the database area [e.g. 4-8] as from the research domain of qualitative reasoning [e.g. 9-13] movements of objects have been studied.

In the widely used Region Connection Calculus (RCC) [14] and 9-Intersection Model [15], topological relationships between two regions are defined. Apart from some limiting cases, such as a car accident and a predator catching a prey, where moving objects *meet*, mobile objects are represented by use of *disconnected from* (DC) in RCC and *disjoint* in the 9-Intersection Model. So, a limitation of these formalisms is that all DC relations are undifferentiated. This approach ignores some important aspects of reasoning about continuously moving physical objects. For example, given two trains on a railroad, it is of the utmost importance to know their movement with respect to each other in order to detect whether or not they would crash in the near future. Therefore, a challenging question remains: "how do we

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handle changes in movement between moving objects, if there is no change in their topological relationship?" With this in mind, the Qualitative Trajectory Calculus (QTC) is presented in [16]. QTC is a language for representing and reasoning about movements of objects in a qualitative framework, able to differentiate between groups of disconnected objects. In this paper, we specifically study the 81 relations of the so-called QTC Double-Cross, or QTC_C for short.[1] This calculus is partly based on the Double-Cross Calculus introduced by Freksa and Zimmermann [20]. We discuss the reasoning power of QTC_C and apply the important reasoning technique of composition tables, originating from the domain of temporal reasoning [1]. Since a composition table encodes all possible compositions of relations for a specific calculus, a simple table look-up operation can replace complex theorem proving [21]. This is why composition tables are very useful from a computational point of view [22,23]. Besides the simple look-up mechanisms, composition tables play an important role when working with incomplete information and larger inference mechanisms as exemplified in Section 4. It is not surprising that composition tables have found their way in the domain of qualitative spatial reasoning [24-27]. As composition table look-up forms an integral part of temporal and spatial reasoning calculi [28], it will have its importance in spatio-temporal reasoning, and thus when studying moving objects. In this paper, instead of the full composition tables for QTC_C, composition rules to generate composition tables are presented. These rules can be implemented in information systems in order to generate composition tables automatically, which is highly preferable due to the extent of the tables; the 81 QTC_C-relations generate a matrix composed of 6561 (81×81) entries.

This paper is organized as follows. In Section 2, QTC_C is defined. In Section 3, we discuss the composition-rule table. In Section 4, we show how both QTC_C and the composition-rule table can be used for reasoning with incomplete knowledge about moving objects. Note that we did not intend to present a formal background of the calculus, neither did we intend to make a comparison to other calculi and conceptual approaches dealing with orientation and/or motion, such as [4-13,29-43].[2]

2 QTC Double-Cross (QTC_C)

We assume continuous time for QTC. Depending on the level of detail and the number of spatial dimensions, different types of QTC are defined in [16]. In general, QTC makes comparisons between positions of two objects at different moments in time. The movement of the first object (called *k*) with respect to the second object (called *l*) is studied by comparing the distance between *l* at the current time point (denoted *t*) and *k* during the period immediately before the current time point (denoted t^-), with the distance between *l* at *t* and *k* during the period immediately after the current time point (denoted t^+). In addition, the movement of *l* with respect to *k* is studied by comparing the distance between *k* at *t* and *l* at t^-, with the distance between

[1] For a description and an illustration of how QTC can be extended to movements along (road) networks and how QTC_C has to be used during longer periods containing multiple QTC_C relations, we refer, respectively, to [17, 18], and [19].

[2] For a formal axiomatization of QTC and a confrontation with several other calculi, see [16].

k at *t* and *l* at t^+. Each object can move away from or towards the other, or can be stable with respect to the other. These three possibilities result respectively in the qualitative values of +, − and 0. In QTC Basic or QTC_B, only this changing of distance is of importance. The calculus QTC_C considers additionally the direction in which an object is moving with respect to the line segment between the two objects. QTC_C is partly based on the Double-Cross Calculus introduced by Freksa and Zimmermann [20,26,44,45]. Their central research question was: "*Consider a person walking from some point a to point b. On his way, he is observing point c. He wants to relate point c to the vector ab*" [45,p.51]. Freksa and Zimmermann propose a double-cross induced by two reference points: the positions of the observer at t_1 (point *a* in Fig. 1a) and the point where the observer is walking towards (point *b* in Fig. 1a). Through these pinpoints, the reference line (*RL*) is defined. Also through these pinpoints and perpendicular to *RL*, the first perpendicular reference line ($RL\perp 1$) and the second perpendicular reference line ($RL\perp 2$) are defined. The three lines (*RL*, $RL\perp 1$, and $RL\perp 2$) form a double-cross and distinguish six 2D regions, six 1D infinite half lines, one 1D line segment between the two reference points, and the two 0D reference points themselves (Fig. 1b). This way, they define a set of fifteen basic relations that can be utilized to navigate using qualitative spatial information. Based on the front/back dichotomy and the left/right dichotomy, the position of the observed point *c* can be described in terms of these fifteen relations. For example, in Fig. 1c, *c* is localized in zone 14. A major goal of this calculus was to find a natural and efficient way to deal with incomplete knowledge, e.g. if it is not possible to decide whether the third point is behind or in front of the second point.

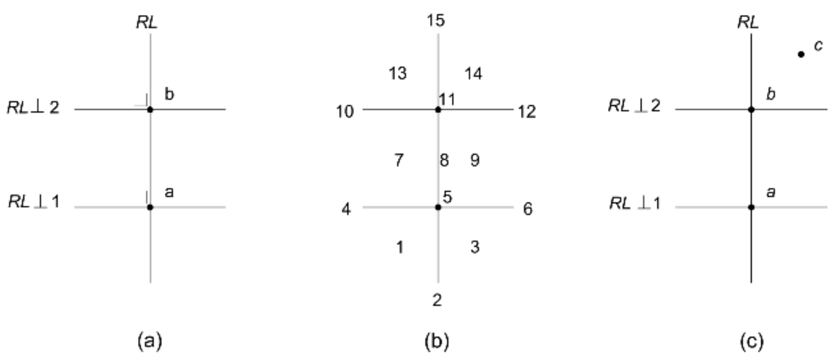

Fig. 1. The Double-Cross Calculus (Source: [44])

Worth mentioning is the difference between the approach of Freksa-Zimmermann and the QTC calculi. In the Double-Cross Calculus (Fig. 2a), the location of a moving point from t_1 ($k|t_1$) to t_2 ($k|t_2$) with respect to a static point ($l|t_1 = l|t_2$) is described. The movement of *k* results in a vector of which the beginning and the end serve as pinpoints for the double-cross. The double-cross forms the reference frame for the calculus. However, QTC_C (Fig. 2b$_1$ and b$_2$) examines the movement of two objects *k* and *l* with respect to each other, between t_1 and t_2. Both movements are represented via a vector

(Fig. 2b$_2$), degenerated to a point if an object is not moving (Fig. 2b$_1$). The origins of these vectors serve as pinpoints for the double-cross, being the reference frame for the calculus. The Double-Cross Calculus only considers a single movement (Fig. 2a), in which one of both objects is moving. QTC$_C$ supports single movements (Fig. 2b$_1$) as well as dual movements in which both objects move (Fig. 2b$_2$).

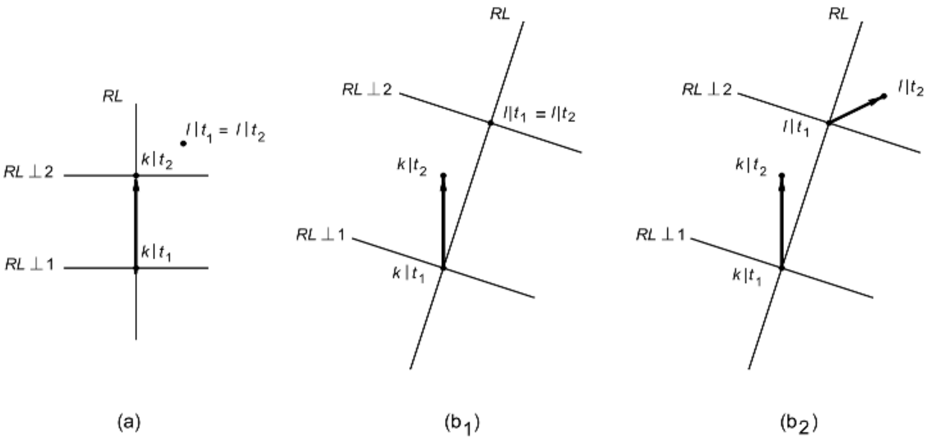

Fig. 2. Double-Cross Calculus versus QTC$_C$

Hence, a two-dimensional movement is presented in QTC$_C$ using the following conditions (C):

Assume: objects k and l

RL_{kl}: the directed reference line from k to l

C1. Movement of the first object, with respect to the first perpendicular reference line at time point t (distance constraint):

–[3]: k is moving towards l:

$$\exists t_1\ (t_1 < t \wedge \forall\, t^-\ (t_1 < t^- < t \rightarrow d(k|t^-, l|t) > d(k|t, l|t))) \wedge$$
$$\exists t_2\ (t < t_2 \wedge \forall\, t^+\ (t < t^+ < t_2 \rightarrow d(k|t, l|t) > d(k|t^+, l|t)))$$

+: k is moving away from l:

$$\exists t_1\ (t_1 < t \wedge \forall\, t^-\ (t_1 < t^- < t \rightarrow d(k|t^-, l|t) < d(k|t, l|t))) \wedge$$
$$\exists t_2\ (t < t_2 \wedge \forall\, t^+\ (t < t^+ < t_2 \rightarrow d(k|t, l|t) < d(k|t^+, l|t)))$$

0: k is stable with respect to l:

all other cases

C2. The movement of the second object wrt the second perpendicular reference line at time point t can be described as in condition 1 (C1) with k and l interchanged.

C3. Movement of the first object with respect to the directed reference line from k to l at time point t (side constraint):

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[3] We write – here, because there is a decrease in distance between both objects. If there is an increase in distance, we write +. If the distance remains the same, we will write 0.

−: k is moving to the left side of RL_{kl}:

$\exists t_1\ (t_1 < t \wedge \forall\, t^-\ (t_1 < t^- < t \rightarrow k$ is on the right side of RL_{kl} at $t)) \wedge$

$\exists t_2\ (t < t_2 \wedge \forall\, t^+\ (t < t^+ < t_2 \rightarrow k$ is on the left side of RL_{kl} at $t))$

+: k is moving to the right side of RL_{kl}:

$\exists t_1\ (t_1 < t \wedge \forall\, t^-\ (t_1 < t^- < t \rightarrow k$ is on the left side of RL_{kl} at $t)) \wedge$

$\exists t_2\ (t < t_2 \wedge \forall\, t^+\ (t < t^+ < t_2 \rightarrow k$ is on the right side of RL_{kl} at $t))$

0: k is moving along RL_{kl}:

all other cases

C4. The movement of the second object wrt the directed reference line from l to k at time point t can be described as in condition 3 (C3) with k and l interchanged.

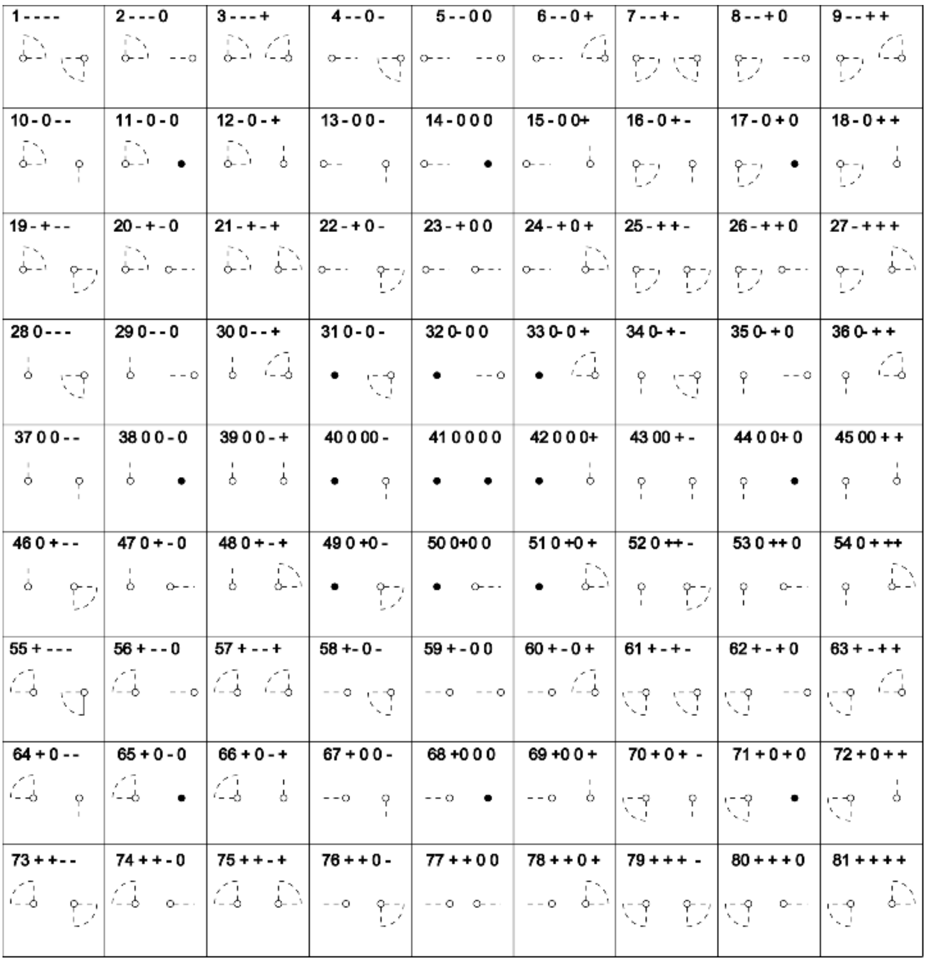

Fig. 3. QTC_c-relation icons

We can represent a trajectory by a label consisting of four characters, each one giving a value respectively for the four conditions above. There are 81 (=3^4) QTC_C-relations. Despite the fact that qualitative reasoning typically deals with a small number of relations, we do believe that the 81 relations form a good foundation for mimicking human reasoning, since they are only based on two constraints resulting in, what we prefer to call, the towards/away-from dichotomy (for the distance constraint) and left/right dichotomy (for the side constraint). Note that in fact QTC_C is a combination of two Double-Cross relations, and that, in [45], Zimmermann and Freksa argue that it has been studied in experimental psychology that humans tend to use rectangular reference systems. As a result, there will be a clear difference in the movements represented by the 81 QTC relations. Each QTC_C-relation can be represented by a so-called relation icon (Fig. 3). The left dot represents the position of k and the right dot the position of l. A dot is filled if the object can be stationary, and open if the object cannot be stationary. Important is that the disk quarters are, topologically spoken, open. I.e., the movement of k in relation $(- - - 0)_C$ in Fig. 3 can be from k to every point on the curved part of the quarter part excluding the horizontal and the vertical line segment. On the other hand, the movement of l in this relation can only be from l straight to k, which is along the dashed line drawn from the open right dot.

3 Composition for QTC_C

A composition table is a central issue in qualitative reasoning. The idea behind a composition table is to compose a finite set of new facts and rules from existing ones, i.e., if two existing relations $R_1(k,l)$ and $R_2(l,m)$ share a common object (l), they can be composed into a new relation set $R_3(k,m)$, depicted by:

$$R_1(k,l) \otimes R_2(l,m) = R_3(k,m)$$

A composition table contains the set of compositions that are possible between all relations in a certain calculus; the left column containing R_1, the top row containing R_2, and the other entries containing $R_3 = R_1 \otimes R_2$.

3.1 Central Concepts

In this section, we focus on the two central concepts laying at the basis of the so-called composition-rule table for QTC_C. This table, which is generated by use of diagrammatic reasoning, is a compacted representation of a composition table. Let us consider both central concepts in detail:

a) Which rotation do we need, such that l of R_2 matches l of R_1?
Generating the composition relation $R_3(k,m)$, means finding out how k moves with respect to m and vice versa, based on $R_1(k,l)$ and $R_2(l,m)$; or in other words: based on the movement of k with respect to l (and vice versa), and the movement of l with respect to m (and vice versa). As said before, the movements are represented via relation icons. In order to be able to compose $R_1(k,l)$ and $R_2(l,m)$ by use of diagrammatic reasoning, we combine the relation icons representing R_1 and R_2. This will be done in two steps. In the first step, the relation icon of $R_2(l,m)$ is translated onto the relation icon of $R_1(k,l)$, in such a way that both origins of the vector

representing the movement of l in $R_1(k,l)$ and the vector representing the movement of l in $R_2(l,m)$ match. In the second step, the relation icon representing R_2 is rotated in such a way that the vector representing the movement of l in R_2 matches the vector representing the movement of l in R_1. Depending on the 6561 (81×81) composition combinations that are possible in QTC_C, one finds eight basic rotation[4] possibilities that can be classified in two groups:

- crisp rotations (0°, 90°, 180°, 270°): only one rotation angle is possible.
- range rotations (0°-90°, 90°-180°, 180°-270°, 270°-360°): there is a range of rotations, over which the second relation has to be rotated.

Notwithstanding that this is not the full spectrum of rotation possibilities, all others can be generated by combining multiples of these eight basic possibilities.

b) How is *k* moving with respect to *l* in R_1, and how is *m* moving with respect to *l* in R_2?

After the matching of the relation icons, the composition of $R_1(k,l)$ and $R_2(l,m)$ can be generated, by studying how k is moving with respect to l in R_1 and how m is moving with respect to l in R_2. This question can be answered by taking the appropriate characters from the QTC_C labels. Based on the definition of QTC_C, one can say:

- the movement of k with respect to l in R_1 can be found in the QTC_C label representing $R_1(k,l)$. The first character of this label represents the movement of k with respect to $RL\perp1$. The third character of this label represents the movement of k with respect to RL_{kl}.
- the movement of m with respect to l in R_2 can be found in the QTC_C label representing $R_2(l,m)$. The second character of this label represents the movement of m with respect to $RL\perp2$. The fourth character of this label represents the movement of m with respect to RL_{ml}.

3.2 Diagrammatic Reasoning

By combining the two central concepts with the diagrammatic reasoning process explained in this section, one can generate all 6561 compositions. Fig. 4-8 contain specific basic rotation possibilities. 'Specific', because only the rotation possibilities each time a new diagrammatic reasoning technique had to be used, are handled. The rotation possibility 'the rotation angle is 270°' is for example not handled since it is analogous with the rotation possibility 'the rotation angle is 90°'. Each possibility consists of a starting situation (a), representing $R_1(k,l)$ and $R_2(l,m)$ after the first part of the matching process, i.e., after translation. Each possibility also contains one (b) or multiple (b_1, b_2, etc.) composition results. Also the double-cross, needed to determine $R_3(k,m)$, is represented. The first and the second character in each zone of the first cross (centre at k) stand for respectively the first and the third character that the QTC_C-relation will get if the velocity vector of k is inside the specific zone. The first and the second character in each zone of the second cross (centre at l) stand for respectively the second and the fourth character that the QTC_C-relation will get if the velocity vector of l is

[4] Just as in trigonometry, we take anti-clockwise angles as being positive.

inside the specific zone. Because of the visualization aspect, only the labels not containing 0 and having no overlap with the velocity vectors are represented. Let us describe the diagrammatic reasoning process of the specific basic rotation possibilities:

Basic rotation possibility: 0° (Fig. 4). If one needs to combine two relations that can be matched without rotation, things are quite straightforward. Let us give an example. Fig. 4a shows R_1 and R_2 after translation: $R_1(k,l) = (- + 0\ 0)_C$ and $R_2(l,m) = (- + 0\ 0)_C$. Fig. 4b contains the composition result $R_3(k,m)$, which could be determined for this rotation possibility without rotation. The labels in the cross centered at k, are – (standing for the first character of $R_3(k,m)$) and 0 (standing for the third character of $R_3(k,m)$). The labels in the cross centered at m, are + (standing for the second character of $R_3(k,m)$) and 0 (standing for the fourth character of $R_3(k,m)$). Thus, $R_3(k,m) = (- + 0\ 0)_C$.

To let de reader become familiar with the technique of diagrammatic reasoning used here, let us give a second example of this rotation possibility:

- Fig. 4a': $R_1(k,l) = (- + - 0)_C$ and $R_2(l,m) = (- 0\ 0 -)_C$.
- Fig. 4b': $R_3(k,m) = (- 0 - -)_C$.

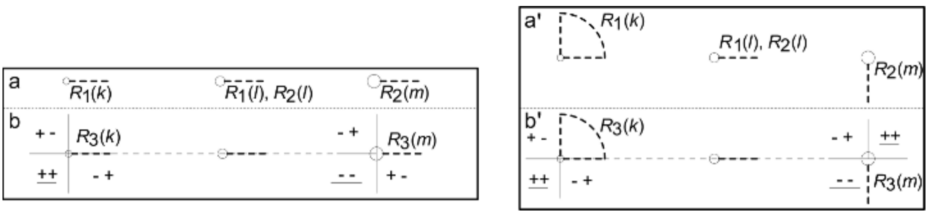

Fig. 4. Composition rules for basic rotation possibility: 0°

There are 9 options for the labels of k in R_1 and there are 9 options for the labels of m in R_2: (– –), (– 0), (– +), (0 –), (0 0), (0 +), (+ –), (+ 0), and (+ +). As a result, there are 9×9=81 compositions for this rotation possibility in QTC_C, as well as for the other rotation possibilities.

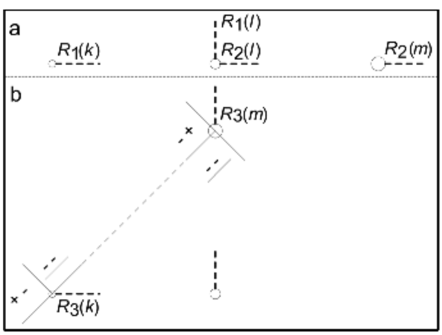

Fig. 5. Composition rules for basic rotation possibility: 90°

Basic rotation possibility: 90° (Fig. 5). In contrast to the previous rotation possibility, a rotation is needed here, which will of course have its consequences for the orientation of the double-cross as can be seen in the example in Fig. 5b. The figure shows R_1, R_2 and R_3, or in other words $R_1 \otimes R_2 = R_3$, or in QTC labels:

$$(-\ 0\ 0\ +)_C \otimes (-\ +\ 0\ 0)_C = (-\ +\ +\ +)_C$$

Basic rotation possibility: 180° (Fig. 6). There is no big difference between this rotation possibility and the former one. Worthwhile mentioning is that when the distance between *l* and *m* varies, different situations may occur. I.e., when the distance between *l* and *m* varies between:

- is smaller than the distance between *k* and *l* (Fig. 6b$_1$);
- is equal to the distance between *k* and *l* (Fig. 6b$_2$);
- is larger than the distance between *k* and *l* (Fig. 6b$_3$).

In Fig. 6b$_2$, the position of *l* after the matching process is equal to the position of *m* after the matching process. Since QTC$_C$ only studies objects having the RCC-relation DC, the relation in Fig. 6b$_2$ can be ignored. Note also the big difference in orientation of the double cross between Fig. 6b$_1$ and Fig. 6b$_3$.

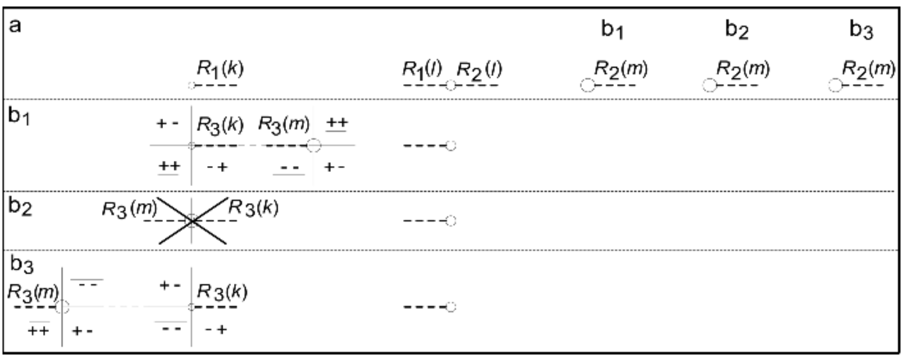

Fig. 6. Composition rules for basic rotation possibility: 180°

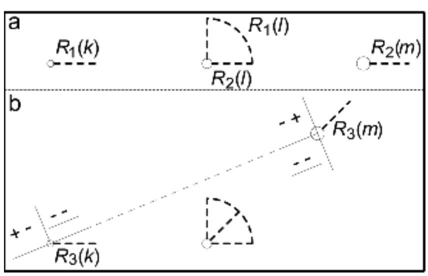

Fig. 7. Composition rules for basic rotation possibility: 0°-90°

Basic rotation possibility: 0°-90° (Fig. 7). Although all angles between 0° and 90° may be used as rotation angle, there is only one result.

Basic rotation possibility: 90°-180° (Fig. 8). There are five different options.[5]

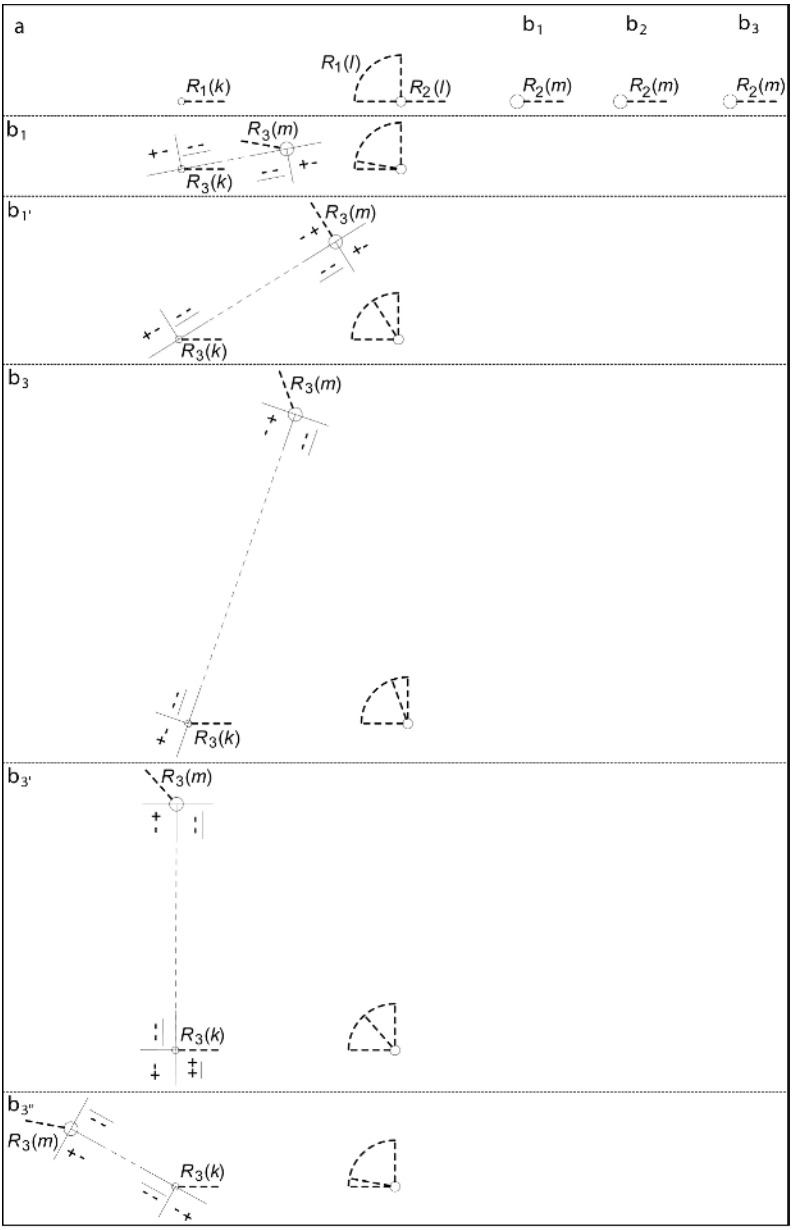

Fig. 8. Composition rules for basic rotation possibility: 90°-180°

[5] Due to space limitations, we do not go in detail on the diagrammatic reasoning process that determines the landmark values. A description of this process can be found in [16].

3.3 Composition-Rule Table

As described in the previous section, all 6561 compositions of QTC_C can be read from the diagrams, generated by diagrammatic reasoning. 1296 of the 6561 relations are invalid, due to the impossibility of inference between a moving point and a stationary point is impossible. 964 of the remaining 5265 relations are strong 964 and 4301 are weak. It is highly preferable to construct a compact table in which the compositions can be found by a simple table look-up. Such a compact table has been called the composition-rule table and will be worked out in this section. Although this so-called composition-rule table is not a traditional composition table containing all entries, this table does give all information contained in a composition table. In addition, it forms a basis of how to implement the composition rules in a practical information system.[6]

Table 1. Composition-rule table for QTC_C[7]

			X	0°		90°		180°				270°		0°-90°		90°-180°										180°-270°										270°-360°	
R_1C_1	R_1C_3			C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3	C_1	C_3
0	0	0	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
–	0	R	X	–	0	–	+	–	0	+	0	–	–	–	+	–	+	–	+	–	+	0	+	+	+	+	–	0	–	–	–	–	–	–	–	–	–
–	–	TR	X	–	–	–	U	–	–	+	+	U	–	–	U	–	U	–	U	–	U	–	+	U	+	+	U	+	–	U	–	U	–	U	–	U	–
0	–	T	X	0	–	–	–	0	–	0	+	+	–	–	–	–	–	–	–	–	–	–	0	–	+	+	+	+	0	+	–	+	–	+	–	+	–
+	–	TL	X	+	–	U	–	+	–	–	+	+	U	U	–	U	–	U	–	U	–	–	–	–	U	U	+	+	+	+	U	+	U	+	U	+	U
+	0	L	X	+	0	+	–	+	0	–	0	+	+	+	–	+	–	+	–	+	–	0	–	–	–	–	+	0	+	+	+	+	+	+	+	+	+
+	+	BL	X	+	+	+	U	+	+	–	–	U	+	+	U	+	U	+	U	+	U	+	–	U	–	–	U	–	+	U	+	U	+	U	+	U	+
0	+	B	X	0	+	+	+	0	+	0	–	–	+	+	+	+	+	+	+	+	+	+	0	+	–	–	–	–	0	–	+	–	+	–	+	–	+
–	+	BR	X	–	+	U	+	–	+	+	–	–	U	U	+	U	+	U	+	U	+	+	+	+	U	U	–	–	–	–	U	–	U	–	U	–	U
R_2C_2	R_2C_4			C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4	C_2	C_4
0	0	0	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
+	0	R	X	+	0	+	+	–	0	+	0	+	–	+	+	–	+	0	+	+	+	+	+	+	+	+	–	+	–	+	–	0	–	–	–	+	–
+	+	TR	X	+	+	U	+	–	–	+	+	+	U	U	+	–	U	–	+	U	+	U	+	U	+	+	U	+	U	+	U	+	–	U	–	+	U
0	+	T	X	0	+	–	+	0	–	0	+	+	+	–	+	–	–	–	0	–	+	–	+	–	+	+	+	+	+	+	+	+	0	+	–	+	+
–	+	TL	X	–	+	–	U	+	–	–	+	U	+	–	U	U	–	–	–	–	U	–	U	–	U	U	+	U	+	U	+	+	+	+	U	U	+
–	0	L	X	–	0	–	–	+	0	–	0	–	+	–	–	+	–	0	–	–	–	–	–	–	–	–	+	–	+	–	+	0	+	+	+	–	+
–	–	BL	X	–	–	U	–	+	+	–	–	–	U	U	–	+	U	+	–	U	–	U	–	U	–	–	U	–	U	–	U	–	+	U	+	–	U
0	–	B	X	0	–	+	–	0	+	0	–	–	–	+	–	+	+	+	0	+	–	+	–	+	–	–	–	–	–	–	–	–	0	–	+	–	–
+	–	BR	X	+	–	+	U	–	+	+	–	U	–	+	U	U	+	+	+	+	U	+	U	+	U	U	–	U	–	U	–	–	–	–	U	U	–

The composition-rule table (Table 1) contains a top heading ('X 0° 90°…270° 360°') and two sub-headings ('R_1C_1 R_1C_3 $R_3C_1R_3C_3…R_3C_1R_3C_3$' and '$R_2C_2$ R_2C_4 $R_3C_2R_3C_4…R_3C_2R_3C_4$'). The body of the table consists of two parts, an upper part belonging to the first sub-heading and a lower part belonging to the second sub-heading. The top heading shows which column stands for which rotation that has to be made in order that l in R_2 matches l in R_1 (e.g. X: both relations cannot be matched via a rotation; 90°-180°: the range between 90° and 180° matches both relations). Apart from the X-column, every column contains at least one sub-heading

[6] In a way, the concept of the composition-rule table that is presented here for QTC_C, can be compared with the concept of the condensed composition table for the Interval Calculus (the Interval Calculus is presented in [1], its compaction is presented in [2]). For QTC_C, there is a compression from 6561 to 306 entries. For the Interval Calculus, there is a compression from 169 to 7 entries. Both compression results are below 5% (4.7% for QTC_C and 4.1% for the Interval Calculus).

[7] Due to space limitations, R_3 is removed for all sub-headings in the table.

('R_3C_1 R_3C_3' for the upper part and 'R_3C_2 R_3C_4' for the lower part). In the columns 'R_3C_1' and 'R_3C_3', respectively the first and the third character of the composition label can be found. If all qualitative variables are possible for one character, then the character is represented by 'U' standing for the universal set of qualitative values; if multiple combinations of characters are possible for a specific rotation possibility, then multiple columns need to be presented. In the columns 'R_3C_2' and 'R_3C_4', respectively the second and the fourth character of the composition label can be found. The rows of the upper part differentiate between the movements of *k* with respect to *l* (the first and third character in R_1). The rows of the lower part differentiate between the movements of *m* with respect to *l* (the second and fourth character in R_2). The abbreviations next to the characters stand for these movements: T (Top), TL (Top-Left), L (Left), BL (Bottom-Left), B (Bottom), BR (Bottom-Right), R (Right), and TR (Top-Right). Let us explain the composition-rule table by use of the examples for the first 2 rotation possibilities, given in section 3.2:

Basic rotation possibility: 0°. In the examples with 'rotation angle: 0', no rotation has to be made in order that *l* in R_2 matches *l* in R_1. Thus, we select the column labeled '0°'. In the first example, $R_1C_1 = –$, and R_1C_3 is 0. In other words, the movement of *k* with respect to *l* is R, which can be found in the upper part of the table. We thus need to select the row labeled '– 0 R'. The intersection between column '0°' and row '– 0 R' gives: $R_3C_1 = –$ and $R_3C_3 = 0$. Still in the first example, $R_2C_2 = +$ and $R_2C_4 = 0$. In other words, the movement of *m* with respect to *l* is R, which can be found in the lower part of the table. We thus need to select the row labeled with '+ 0 R'. The intersection between column '0°' and row '+ 0 R' gives: $R_3C_2 = +$ and $R_3C_4 = 0$. Combining the solutions from the upper part and the lower part of the table, results in $R_3(k,m) = (– + 0\ 0)_C$. Let us write down the second example in a shorter way:
Rotation constraint:

R_2 needs no rotation to match R_1. Thus, select column '0°'.

Character constraints:

Movement of *k* in R_1: TR. Thus, select row '– – TR' in upper part.
Movement of *m* in R_2: B. Thus, select row '0 – B' in lower part.
Thus, $R_3(k,m) = (– 0 – –)_C$.

Basic rotation possibility: 90°. In this example $(– 0\ 0 +)_C \otimes (– + 0\ 0)_C$, the composition-rule table shows:
Rotation constraint:

R_2 has to be rotated over 90° to match R_1. Thus, select '90°' column.

Character constraints:

Movement of *k* in R_1: R. Thus, select row '– 0 R' in upper part.
Movement of *m* in R_2: R. Thus, select row '+ 0 R' in lower part.
Thus, R_3 contains $(– + + +)_C$.

At first sight, this possibility is far more complex since there are five different options. Because every option stands for a specific rotation angle or rotation range, the first option of the upper part needs to be combined with the first option of the lower part, the second option of the upper part with the second option of the lower part, etc. Thus, we may not take the cross-product. Besides this, the same methodology as before can be used.

4 The Example of 'Puzzling the Past'

Scientists have researched many evolutions of phenomena, often dealing with moving objects. Frequently, a jigsaw puzzle has to be constructed without knowing on beforehand which part will enter the picture and at what time. Therefore, such reconstructions are very complicated to represent, implement, analyze, visualize, etc. This section shows that QTC_C is well suited for this kind of interesting research, since this qualitative calculus can handle incomplete knowledge and composition rules have been constructed for QTC_C[8]. We consider the example of geomorphologic research performed by several teams.

4.1 Initial Research

Suppose two scientific teams are doing research on a site, independent of each other. Both have been asked to describe the movement of object k with respect to object n (R_3). The problem for both teams is that they cannot find data, in order to determine directly the movement of k with respect to n. Therefore, both teams need other data to infer an (incomplete) answer. The following data has been found:

Team 1: $R_1(k,l) = (- + + -)_C$, $R_2\ (l,n) = (- - - +)_C$.
Team 2: $R_1(k,m) = (- + - +)_C$, $R_2(m,n) = (- + - +)_C$.

In order to determine $R_3(k,n)$, the composition-rule table (Table 1) can be used.

Team 1. In order to compose $R_1(k,l) = (- + + -)_C$ and $R_2(l,n) = (- - - +)_C$, the second relation needs a rotation between 180° and 360°. Thus, there are three basic rotation possibilities: 180°-270°, 270°, and 270°-360°.

180°-270°. Select column '180°-270°'. $R_1C_1 = -$, and $R_1C_3 = +$. In other words, the movement of k with respect to l is BR, which can be found in the upper part of the table. Therefore, select the row labeled '– + BR'. The intersection between column '180°-270°' and row '– + BR' gives the qualitative values for the first and the third character of the composition label, having five options:

option 1: $R_3C_1 = \mathrm{U}$ and $R_3C_3 = -$,
option 2: $R_3C_1 = -$ and $R_3C_3 = -$,
option 3: $R_3C_1 = -$ and $R_3C_3 = \mathrm{U}$,
option 4: $R_3C_1 = -$ and $R_3C_3 = \mathrm{U}$,
option 5: $R_3C_1 = -$ and $R_3C_3 = \mathrm{U}$.

$R_2C_2 = -$ and R_2C_4 =s +. In other words, the movement of n with respect to l is TL, which can be found in the lower part of the table. Therefore, select the row labeled '– + TL'. The intersection between column '180°-270°' and row '– + TL' gives the qualitative values for the second and the fourth character of the composition label:

option 1: $R_3C_2 = \mathrm{U}$ and $R_3C_4 = +$,
option 2: $R_3C_2 = \mathrm{U}$ and $R_3C_4 = +$,
option 3: $R_3C_2 = \mathrm{U}$ and $R_3C_4 = +$,

[8] Note that analogous reasoning processes have been worked out for the temporal domain [2, 46].

option 4: R_3C_2 = + and R_3C_4 = +,
option 5: R_3C_2 = + and R_3C_4 = U.

Combining the solutions from the upper part and the lower part of the table, results in:

option 1: $(\mathrm{U\ U} - +)_C$,
option 2: $(- \mathrm{U} - +)_C$,
option 3: $(- \mathrm{U\ U} +)_C$,
option 4: $(- + \mathrm{U} +)_C$,
option 5: $(- + \mathrm{U\ U})_C$.

270°. Dual reasoning as for 'rotation possibility: 180°-270°', results in option 6: $(- \mathrm{U\ U} +)_C$.
270°-360°. Dual reasoning as for 'rotation possibility 180°-270°', results in option 7: $(- \mathrm{U\ U} +)_C$.

The disjunction of all results gives:

$$(\mathrm{U\ U} - +)_C \cup (- \mathrm{U} - +)_C \cup (- \mathrm{U\ U} +)_C \cup (- + \mathrm{U} +)_C \cup (- + \mathrm{U\ U})_C \cup$$
$$(- \mathrm{U\ U} +)_C \cup (- \mathrm{U\ U} +)_C =$$
$$(\mathrm{U\ U} - +)_C \cup (- \mathrm{U\ U} +)_C \cup (- + \mathrm{U\ U})_C.$$

Team 2. In order to compose $R_1(k,m) = (- + - +)_C$ and $R_2(m,n) = (- + - +)_C$, the second relation needs a rotation between -90° and 90°. Thus, there are three basic rotation possibilities: 270°-360°, 0°, and 0°-90°.

270°-360°. Option 1: $(\mathrm{U} + - \mathrm{U})_C$
0°. Option 2: $(- + - +)_C$
0°-90°. Option 3: $(- \mathrm{U\ U} +)_C$

The disjunction of all results gives:

$$(\mathrm{U} + - \mathrm{U})_C \cup (- + - +)_C \cup (- \mathrm{U\ U} +)_C =$$
$$(\mathrm{U} + - \mathrm{U})_C \cup (- \mathrm{U\ U} +)_C.$$

4.2 Follow-Up Research

Both research teams get a different incomplete result. However, the real answer must be a subset of the incomplete answer. Suppose a third and a fourth team want to do further research on this site and can use the results of Team 1 and Team 2. Suppose Team 3 is convinced that Team 1 and Team 2 were correct. This would mean that Team 3 takes the conjunction of both former results as being the new incomplete relation to which the correct answer will certainly belong. Suppose Team 4 doubts the correctness of the data gathered by Team 1 and Team 2, but does not know which team would have had the best results. Team 4 could take the disjunction of both former results as being the set to which the correct result has to belong.

Team 3. $((\mathrm{U\ U} - +)_C \cup (- \mathrm{U\ U} +)_C \cup (- + \mathrm{U\ U})_C) \cap ((\mathrm{U} + - \mathrm{U})_C \cup (- \mathrm{U\ U} +)_C)$

Team 4. $((\mathrm{U\ U} - +)_C \cup (- \mathrm{U\ U} +)_C \cup (- + \mathrm{U\ U})_C) \cup ((\mathrm{U} + - \mathrm{U})_C \cup (- \mathrm{U\ U} +)_C)$

Team 5. Finally, a new team (Team 5) finds a new methodology and can detect directly: $R_3(k,n) = (- + - +)_C$.

Thus, one can see that the incomplete answers of both Team 1 and Team 2, and thus also the answers of Team 3 and Team 4, contained the correct answer.

5 Concluding Remarks

The example of 'puzzling the past', in which a jigsaw puzzle of a configuration of moving points is represented, needs further investigation, since we strongly believe that this example forms a basis for implementation of incomplete spatio-temporal knowledge in information systems. QTC_C and the concept of the composition-rule table can be used in a variety of research domains, such as geomorphology, geology, archaeology, and biology. In complex researches, there is a huge number of anchor points, teams, measurements per team, updates, etc. Such assessments will become complex, but we are convinced that implementation of QTC_C in an information system can lead to interesting results for this widespread but difficult kind of reconstruction processes.

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