A New Vector Median Filter Based on Fuzzy Metrics

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Abstract. Vector median filtering is a well known technique for reducing noise in color images. These filters are defined on the basis of a suitable distance or similarity measure, being the most common used the Euclidean and City-Block distances. In this paper, a Fuzzy Metric, in the sense of George and Veeramani (1994), is defined and applied to color image filtering by means of a new Vector Median Filter. It is shown that the standard Vector Median Filter is outperformed when using this Fuzzy Metric instead of the Euclidean and City-Block distances.

1 Introduction

Images are acquired by photoelectronic or photochemical methods. The sensing devices and the transmission process tend to degrade the quality of the digital images by introducing noise, geometric deformation and/or blur due to motion or camera misfocus [8,27]. The presence of noise in an image may be a drawback in any subsequent processing to be done over the noisy image such as edge detection, image segmentation or pattern recognition. As a consequence, filtering the image to reduce the noise without degrading its quality, preserving edges, corners and other image details, is a major step in any computer vision application [28].

One of the most important families of nonlinear filters is based on the ordering of vectors in a predefined sliding window [27,28]. The output of these filters is defined as the lowest ranked vector according to a specific ordering criterion using a particular *distance measure*. Probably, the most well-known vector filter is the *vector median filter* (VMF) [3] which uses the L_1 (City-Block) or L_2 (Euclidean) norm to order vectors according to their relative magnitude differences. The direction of the image vectors can also be used as an ordering criterion to remove vectors with atypical direction, which means atypical chromaticity. The *basic vector directional filter* (BVDF) [33] parallelizes the VMF operation employing the angle between color vectors as a distance criterion. The BVDF uses only information about directions, so, it is not able to remove achromatic noisy pixels from the image. The *Directional Distance Filter* (DDF) [16] overcomes the difficulties of the BVDF by using both magnitude and direction in the distance criterion.

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However, those vector filters are designed to perform a fixed amount of smoothing and they are not able to adapt to local image statistics. Within this aim, many different filters have been recently introduced in the literature [1,2,17,19,20,21,22,23,24,25,31,32].

In the color image processing field both magnitude and chromatic relations play a major role [6]. These relationships are usually represented using a distance or similarity measure. Many different distance and similarity measures have been introduced in the literature [28,6,7,35,36,29]. Some of them are based on fuzzy theory [6,7,35,36,29] and have been recently applied with many different purposes in image processing, such as, image retrieval [9], image comparison [34], object recognition [11], or region extraction [10].

In this paper, a fuzzy metric in the terms of George and Veeramani [12] is defined and applied to color image filtering by adapting the well-known VMF. The paper is organized as follows. The fuzzy metric is defined in section 2. In Section 3, the proposed filtering is explained. In section 4, some experimental results are shown. Finally, conclusions are presented in section 5.

2 An Appropriate Fuzzy Metric

One of the most important problems in Fuzzy Topology is to obtain an appropriate concept of fuzzy metric. This problem has been investigated by many authors from different points of view. In particular, George and Veeramani [12] have introduced and studied the following notion of fuzzy metric which constitutes a slight modification of the one due to Kramosil and Michalek [18].

According to [12] a fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a (nonempty) set, $*$ is a continuous t-norm and M is a fuzzy set of $X \times X \times]0, +\infty[$ satisfying the following conditions for all $x, y, z \in X,\ s, t > 0$:

(FM1) $M(x, y, t) > 0$
(FM2) $M(x, y, t) = 1$ if and only if $x = y$
(FM3) $M(x, y, t) = M(y, x, t)$
(FM4) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$
(FM5) $M(x, y, \cdot) :]0, +\infty[\longrightarrow [0, 1]$ is continuous.

$M(x, y, t)$ represents the degree of nearness of x and y with respect to t. If $M(x, y, *)$ is a fuzzy metric space we will say that $(M, *)$ is a fuzzy metric on X. In the following, by a fuzzy metric we mean a fuzzy metric in the George and Veeramani's sense.

The authors proved in [12] that every fuzzy metric $(M, *)$ on X generates a Hausdorff topology on X. Actually, this topology is metrizable as it was proved in [13,14], and so the above definition can be considered an appropriate concept of fuzzy metric space.

A fuzzy metric $(M, *)$ on X is said to be stationary if M does not depend on t, i.e. for each $x, y \in X$ the function $M_{x,y}(t) = M(x, y, t)$ is constant [15].

A subset A of X is said to be F-bounded [12] if there exist $t > 0$ and $s \in]0, 1[$ such that $M(x, y, t) > s$ for all $x, y \in A$.

Example 4.4 of [30] suggests the next proposition.

Proposition 1. *Let X be the closed real interval $[a,b]$ and let $K > |a| > 0$. Consider for each $n = 1, 2, \cdots$ the function $M_n : X^n \times X^n \times]0, +\infty[\longrightarrow]0, 1]$ given by*

$$M_n(x, y, t) = \prod_{i=1}^{n} \frac{min\{x_i, y_i\} + K}{max\{x_i, y_i\} + K} \tag{1}$$

where $x = (x_1, \cdots, x_n), y = (y_1, \cdots, y_n)$, and $t > 0$. Then, $(M_n, \cdot)$ is a stationary F-bounded fuzzy metric on X^n, where the t-norm $\cdot$ is the usual product in $[0,1]$.

Proof. Axioms (FM1)-(FM3) and (FM5) are obviously fulfilled. We show, by induction, the triangular inequality (FM4).

An easy computation shows that M_1 verifies (FM4). Now, suppose it is true for M_{n-1}. Then, for each $x = (x_1, \ldots, x_n), y = (y_1, \ldots, y_n), z = (z_1, \ldots, z_n)$ and for each $t, s > 0$ we have

$$\begin{aligned} M_n(x,z,t+s) &= \prod_{i=1}^{n} \frac{min\{x_i,z_i\}+K}{max\{x_i,z_i\}+K} = \prod_{i=1}^{n-1} \frac{min\{x_i,z_i\}+K}{max\{x_i,z_i\}+K} \cdot \frac{min\{x_n,z_n\}+K}{max\{x_n,z_n\}+K} \geq \\ &\geq \prod_{i=1}^{n-1} \frac{min\{x_i,y_i\}+K}{max\{x_i,y_i\}+K} \cdot \prod_{i=1}^{n-1} \frac{min\{y_i,z_i\}+K}{max\{y_i,z_i\}+K} \cdot \frac{min\{x_n,y_n\}+K}{max\{x_n,y_n\}+K} \cdot \frac{min\{y_n,z_n\}+K}{max\{y_n,z_n\}+K} = \\ &= \prod_{i=1}^{n} \frac{min\{x_i,y_i\}+K}{max\{x_i,y_i\}+K} \cdot \prod_{i=1}^{n} \frac{min\{y_i,z_i\}+K}{max\{y_i,z_i\}+K} = M_n(x,y,t) \cdot M_n(y,z,s), \end{aligned} \tag{2}$$

so M_n is a fuzzy metric on X^n, for $n = 1, 2, \ldots$ and clearly it is stationary.

Finally, X^n is F-bounded, for $n = 1, 2, \ldots$ Indeed, if we write $\mathbf{a} = (\overbrace{a, \ldots, a}^{n})$ and $\mathbf{b} = (\overbrace{b, \ldots, b}^{n})$, then for each $x, y \in X^n$ and $t > 0$ we have

$$M_n(x, y, t) \geq M_n(\mathbf{a}, \mathbf{b}, t) = \left(\frac{a + K}{b + K}\right)^n > 0, \text{ for } n = 1, 2, \ldots \tag{3}$$

□

In next sections we will use the above fuzzy metric and it will be denoted $M_n(x, y)$, since it does not depend on t.

2.1 Computational Analysis

Computationally efficient distances are of interest in the field of order statistic filters [4,5]. For this reason, the use of the L_1 Norm is preferred to the L_2 Norm in many cases [28].

The particular case of the proposed fuzzy metric M_n suitable for 3-channel image processing tasks will be M_3, where $M_3(\mathbf{I}_i, \mathbf{I}_j)$ will denote the fuzzy distance between the pixels $\mathbf{I}_i$ and $\mathbf{I}_j$ in the $\mathbf{I}$ image. For each calculation of M_3: 3 comparisons, 6 additions, 3 divisions and 2 products have to be computed. In the case of L_1 Norm are necessary 3 comparisons (absolute value), 3 subtractions and 2 additions whereas for the L_2 Norm 3 subtractions, 3 powers, 2 additions and 1 square-root have to be done. As can be seen in Table 1, the computational complexity of M_3 is even higher than the L_2 Norm. However, an optimization in the computation of M_3 (Fast M_3) may be applied.

Given a fixed parameter K in (1), numerator and denominator of each division in (1) are in a bounded set $[K, 255 + K]$ when processing RGB images. All the possible divisions can be precalculated in a square matrix C where

$$C(i,j) = \frac{min\{i,j\} + K}{max\{i,j\} + K} \quad i,j \in [0,255] \tag{4}$$

Using the pre-computation matrix, the calculation of Fast M_3 for two pixels $\mathbf{I}_i = (\mathbf{I}_i(1), \mathbf{I}_i(2), \mathbf{I}_i(3)), \mathbf{I}_j = (\mathbf{I}_j(1), \mathbf{I}_j(2), \mathbf{I}_j(3))$ is reduced to

$$M_3(\mathbf{I}_i, \mathbf{I}_j) = \prod_{l=1}^{3} C(\mathbf{I}_i(l), \mathbf{I}_j(l)) \tag{5}$$

By means of this optimization, 3 accesses to matrix and 2 products are enough to make the computation.

The time measured for the construction of the matrix C is about 0.8 seconds in a Pentium IV 2.4GHz. Although it supposes an initial cost, the gain is approx. $8\mu s$ (see Table 1) in each computation, so, the initial cost is compensated when 10^5 computations have to be done (which is roughly the computation involved in the filtering of a $50 \cdot 50$ pixels image[1]).

Table 1. Computational comparison between the classical metrics L_1 and L_2 and the proposed fuzzy metric M_3 measured in a Pentium IV 2.4GHz

Metric	1 Computation (μs)	Computations per second
L_1 Norm	28.37	3.524 10^4
L_2 Norm	30.10	3.322 10^4
M_3	34.68	2.883 10^4
Fast M_3	26.98	3.706 10^4

The results presented in Table 1 show that the M_3 Fuzzy Metric is computationally cheaper than the classical L_1 and L_2 Norms when the optimization of the pre-computation matrix is applied.

3 Image Filtering

3.1 Classical Vector Median Filter [3,28]

Let $\mathbf{I}$ represents a multichannel image and let W be a window of finite size n (filter length). The noisy image vectors in the filtering window W are denoted as $\mathbf{I}_j, j = 0, 1, ..., n-1$. The *distance* between two vectors $\mathbf{I}_i, \mathbf{I}_j$ is denoted as $\rho(\mathbf{I}_i, \mathbf{I}_j)$. For each vector in the filtering window, a global, accumulated distance to all other vectors in the window has to be calculated. The scalar quantity $R_i = \sum_{j=0}^{n-1} \rho(\mathbf{I}_i, \mathbf{I}_j)$, is the distance associated to the vector $\mathbf{I}_i$. The ordering of the R_i's: $R_{(0)} \leq R_{(1)} \leq ... \leq R_{(n-1)}$, implies the same ordering of the vectors $\mathbf{I}_i$'s: $\mathbf{I}_{(0)} \leq \mathbf{I}_{(1)} \leq ... \leq \mathbf{I}_{(n-1)}$. Given this order, the output of the filter is $\mathbf{I}_{(0)}$.

[1] For all the filters studied in this article has been used a 8-neighborhood 3×3 size window W.

3.2 Proposed Vector Median Filter

The proposed filter will parallelize the operation of the classical VMF with just one modification. The ordering criterion usually used as defined above has to be inverted due to the axiom (FM2) of the Fuzzy Metric (1), and then the vector median must now be defined as the vector in the sliding window that maximizes the *accumulated* fuzzy distance, as follows.

Being the fuzzy distance between two pixels $\mathbf{I}_i, \mathbf{I}_j$ of the image $\mathbf{I}$ in the n length sliding window W denoted as $M_3(\mathbf{I}_i, \mathbf{I}_j)$, the scalar quantity $M^i = \sum_{j=0, j\neq i}^{n-1} M_3(\mathbf{I}_i, \mathbf{I}_j)$, is the accumulated fuzzy distance associated to the vector $\mathbf{I}_i$. According to VMF, the ordering of the M^i's is now defined as: $M^{(0)} \geq M^{(1)} \geq ... \geq M^{(n-1)}$, therefore, the ordering of the vectors $\mathbf{I}_i$ is: $\mathbf{I}_{(0)} \geq \mathbf{I}_{(1)} \geq ... \geq \mathbf{I}_{(n-1)}$. Given this order, the output of the filter $\mathbf{I}_{out}$ is defined as $\mathbf{I}_{(0)}$.

This is, in general, the straightforward adaptation of the VMF when using a similarity measure instead of a distance measure [28].

4 Experimental Results

In this section, the classical gaussian model for the thermal noise and the impulsive noise model for the transmission noise, as defined in [28,32], has been used to add noise to the well-known images Lenna ($256 \cdot 256$), Peppers ($512 \cdot 512$) and Baboon ($512 \cdot 512$). The performance of the filter has been evaluated by using the common measures MSE, SNR and NCD as defined in [32].

Three different types of noise, according to the models in [28,32], have been considered in this section:

- Type A = low contaminated impulsive noise $p = 7\%, p_1 = p_2 = p_3 = 0.3$
- Type B = high contaminated impulsive noise $p = 30\%, p_1 = p_2 = p_3 = 0.3$
- Type C = mixed gaussian impulsive noise $\sigma = 10, p = 15\%, p_1 = p_2 = p_3 = 0.3$

4.1 Adjusting the K Parameter

The K parameter included in the definition of the Fuzzy Metric M_3 (1) has an important influence on the filter performance. The metric is non-uniform in the sense that the measure given by M_3 for two different pairs of consecutive numbers (or vectors) may not be the same. However, this feature may be very interesting since it is known that the human perception of color is also non-uniform [26]. Clearly, increasing the value of K reduces this non-uniformity. This effect is shown in Fig. 1 where the content of the matrix C (4) for different values of K is presented.

After performing several tests, the results seem to show that a suitable value for the K parameter for a variety of noise types is $K = 2^{10}$. The dependence of the performance on the value of K is shown in Fig. 2. The use of a proper value for K may lead to an improvement of the filter performance up to 60%. In Fig. 2 the performance (MSE) of the filter dependent on K is shown for the filtering of the Lenna image contaminated with type B noise. For other performance measures as SNR and NCD the behavior is similar to MSE. The performance is low for lower values of K. Increasing K leads to

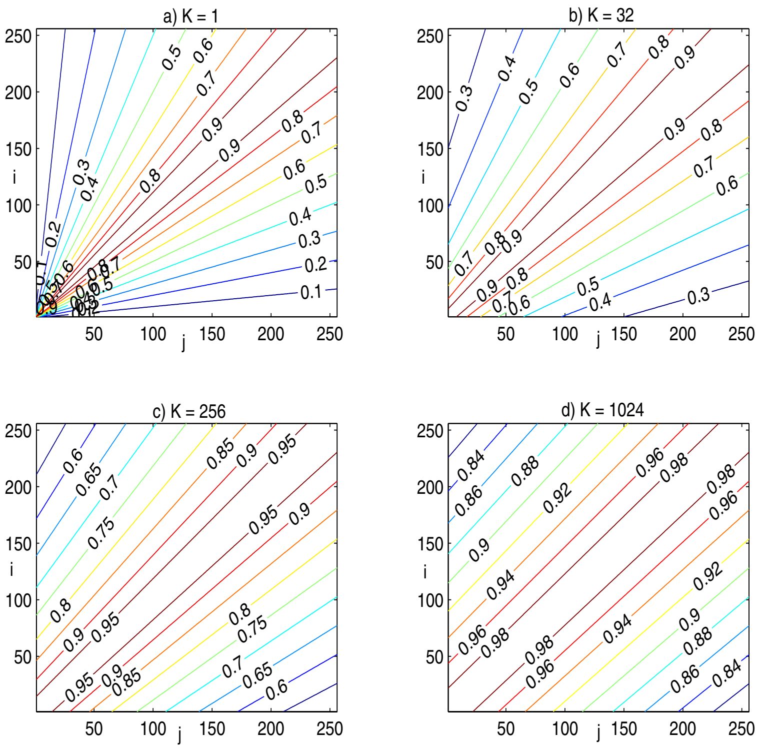

Fig. 1. Content of the pre-computation matrix $C(i, j)$ for several values of K

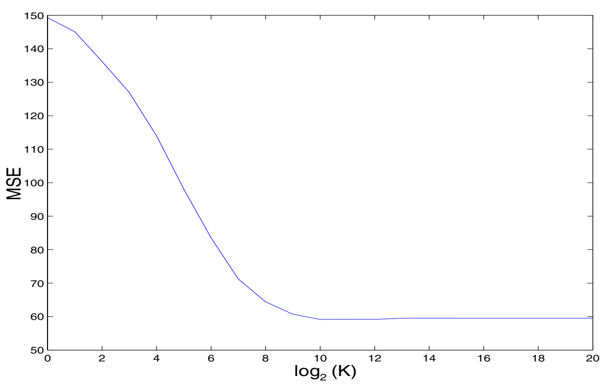

Fig. 2. Performance of the VMF using M_3 in terms of MSE depending on K using the Lenna image contaminated with type B noise

a maximum performance and then it decreases slightly for higher values of K. Finding the optimum K is a problem we are trying to solve since it depends on the particular image and noise. In spite of it, it has been found that in the most of the tested cases the optimum is in the range $[2^9, 2^{15}]$, as the case shown in Fig. 2.

4.2 Comparing Performances

In order to compare the performance of the VMF using the metrics L_1, L_2 and M_3, different images contaminated with different types of noise have been used as commented in section 4.

The results of the performance measured in tems of MSE, SNR and NCD are shown in Tables 2,3 and 4. Fig. 3 presents the peppers image contaminated with type B noise (30% impulsive) and the output of the compared filters, standing out a detail of each image.

Table 2. Comparison of the performance measured in terms of MSE, SNR and NCD using the Lenna image contaminated with different types of noise

Filter	A Noise			B Noise			C Noise		
	MSE	SNR	NCD_{Lab}	MSE	SNR	NCD_{Lab}	MSE	SNR	NCD_{Lab}
None	552.9	15.17	$4.92\ 10^{-2}$	2318.51	9.35	$20.80\ 10^{-2}$	1246.86	12.04	$17.90\ 10^{-2}$
VMF L_1	42.18	26.75	$1.81\ 10^{-2}$	59.63	25.25	$2.19\ 10^{-2}$	91.59	23.38	$6.40\ 10^{-2}$
VMF L_2	45.56	26.41	$1.79\ 10^{-2}$	76.05	24.19	$2.46\ 10^{-2}$	97.01	23.13	$6.35\ 10^{-2}$
VMF M_3	41.81	26.78	$1.80\ 10^{-2}$	59.18	25.28	$2.17\ 10^{-2}$	90.49	23.43	$6.36\ 10^{-2}$

Table 3. Comparison of the performance measured in terms of MSE, SNR and NCD using the Peppers image contaminated with different types of noise

Filter	A Noise			B Noise			C Noise		
	MSE	SNR	NCD_{Lab}	MSE	SNR	NCD_{Lab}	MSE	SNR	NCD_{Lab}
None	566.94	14.42	$4.84\ 10^{-2}$	2493.27	7.99	$21.09\ 10^{-2}$	1324.56	10.73	$19.66\ 10^{-2}$
VMF L_1	18.87	29.19	$4.84\ 10^{-2}$	35.49	26.45	$2.34\ 10^{-2}$	63.10	23.95	$7.53\ 10^{-2}$
VMF L_2	19.30	29.10	$1.88\ 10^{-2}$	40.37	25.89	$2.46\ 10^{-2}$	64.98	23.82	$7.51\ 10^{-2}$
VMF M_3	18.71	29.23	$1.86\ 10^{-2}$	33.35	26.72	$2.29\ 10^{-2}$	62.10	24.02	$7.48\ 10^{-2}$

Table 4. Comparison of the performance measured in terms of MSE, SNR and NCD using the Baboon image contaminated with different types of noise

Filter	A Noise			B Noise			C Noise		
	MSE	SNR	NCD_{Lab}	MSE	SNR	NCD_{Lab}	MSE	SNR	NCD_{Lab}
None	535.33	15.52	$4.83\ 10^{-2}$	2301.44	9.18	$20.76\ 10^{-2}$	1238.64	11.88	$17.37\ 10^{-2}$
VMF L_1	287.66	18.22	$4.07\ 10^{-2}$	326.93	17.66	$4.48\ 10^{-2}$	350.65	17.36	$7.93\ 10^{-2}$
VMF L_2	295.07	18.11	$4.02\ 10^{-2}$	351.71	17.34	$4.61\ 10^{-2}$	359.89	17.24	$7.72\ 10^{-2}$
VMF M_3	287.98	18.21	$4.05\ 10^{-2}$	326.73	17.67	$4.46\ 10^{-2}$	350.27	17.36	$7.88\ 10^{-2}$

The results show that the VMF using the proposed fuzzy metric may give better performance than using the classical metrics.

5 Conclusions

The metric (1) proposed in section 2, which has been proved to be a Fuzzy Metric in the sense of George and Veeramani [12], is a suitable fuzzy metric to be used in multichan-

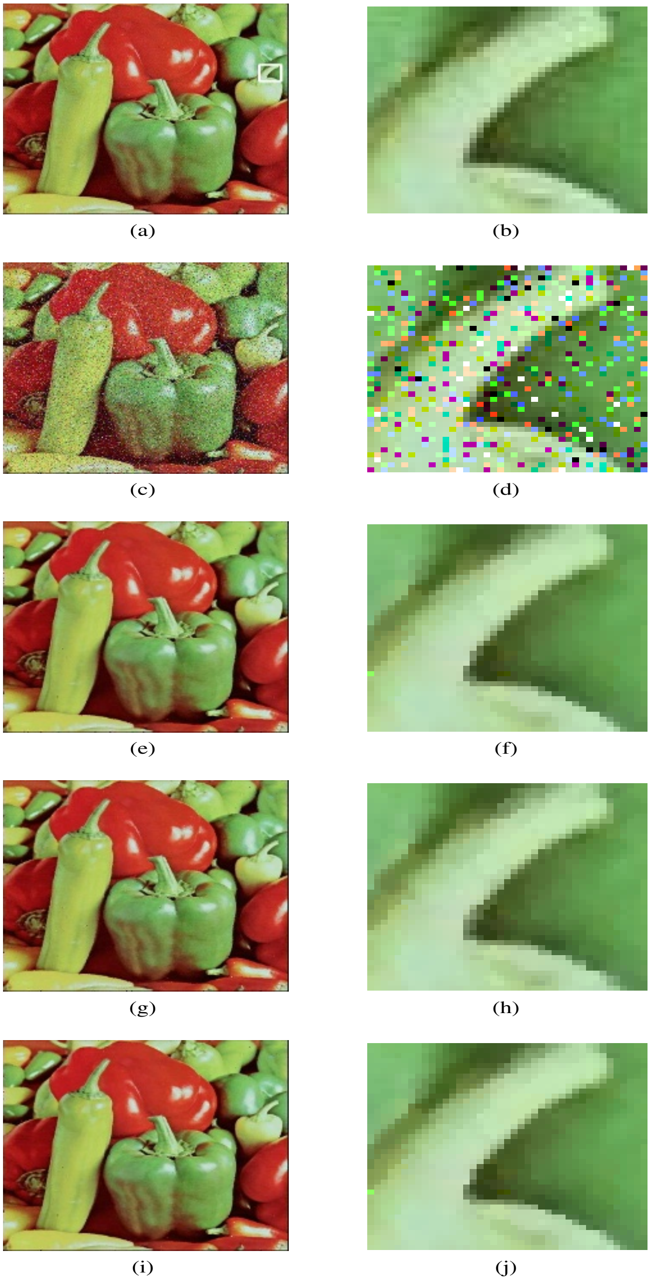

Fig. 3. (a) Original image peppers pointing out the detailed area,(b) detailed area,(c) peppers corrupted with noise type B and (d) detail, (e) result of the VMF using L_1 and (f) detail, (g) result of the VMF using L_2 and (h) detail, (i) result of the proposed filter using M_3 and (j) detail

nel image filtering. The adaptation of the Vector Median Filter (section 3) for the use of the proposed fuzzy metric outperforms the usual VMF's using the classical metrics L_1 and L_2, specially when the impulsive noise present in the image is high, as has been shown in section 4. Moreover, the proposed metric presents a nice computational cost (see section 2.1).

Fuzzy Metrics are a powerful tool which may be successfully applied in image processing tasks since they are able to represent more complex relations than the classical metrics.

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