Fuzzy Adaptive Objects (Logic of Monitors)

Germano Resconi¹, Javier Alonso², and Raul Izquierdo²

¹ Catholic University, via Trieste 17, Brescia, Italy resconi@numerica.it
² Object Oriented Technology Laboratory, Department of Computer Science, University of Oviedo, Calvo Sotero s/n 33005 Oviedo, Spain {javier, raul}@a4devis.com

Abstract. The active semantic in Adaptive Object-Model (AOM) is only one example of the more complex active semantics at different orders. When we violate the integrity of the system, uncertainty grows up in the system. When monitors are conflicting worlds in the modal logic, we can study uncertainty with the logic of the monitors that is comparable with the logic in the fuzzy set theory. Fuzzy values (integration degree) of a concatenation of interactive objects can be computed by fuzzy AND, OR and NOT operators presented in this paper.

1 Introduction

An Adaptive Object-Model (AOM) is a system that represents classes, attributes and relationships as metadata. Users change the metadata (object model) to reflect changes in the model. These changes modify the system's behaviour.

The relation among objects or interactive object generates constrains that we control by the monitors and propagators. To enforce constrains requires that the related objects were updated with information describing the trigger when the propagator is instantiated Active semantics uses monitors for a local object. For a far object to actively restore the integrity of the interactive object we use the propagator that propagates through objects the message of the monitors. The active semantics in AOM is only one example of the more complex active semantics at different orders. When we violate the integrity, the monitors give the information how and where the integrity is violated. When we assume that any monitor is a world in the modal logic, the monitors are a set of conflicting worlds. With the logic of the monitors we can study how uncertainty can be computed by the AND, OR and NOT elementary logic operators. The logic of the monitors can be compared with the logic of the fuzzy sets. We create logic expressions with the monitors and we compute the degree of integrity in complex logic situations.

2 Entity and Relationship

Following the Specialization Principle we design Entity –Relationship structure as in the following figure and table.



Fig. 1. Graphic image of a particular coherent case of roles and relations

Table 1. Coherent table of role and relations

Roles	Relation ₁	Relation ₂	Relation ₃	Relation ₄
father	daughter			
daughter	father			
•••				
daughter				mother

Any entity (E1, E2, E3 & E4) has two roles that must be coherent with the entity itself.

E1 = "John", E2 = "Mary", E3 = "Charles" and E4 = "Anna"

So the Entity – Relation can be modelled in this way

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M = \langle ROLE, RELATION, ENTITY, F, G \rangle (1)
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Where ROLE is the set of roles, RELATION is the set of relations, ENTITY is the set of entities and

$$F: ROLE \times RELATION \to ROLE$$
(2)

Is the transition rule for which given the "relation" we can obtain a role from another role. The function G

$$G: ROLE \times RELATION \rightarrow ENTITY$$
 (3)

Is the reply rule for which we can associate any role and relation with an entity.

Because any entity is not a simple element but is an abstract element with different instances, we can describe the internal structure of the entity by the rule

$$H: INSTANCE \times ATTRIBUTE \rightarrow VALUE$$
(4)

Where instances are samples of the Entity class.

3 Active Semantics

Relationships capture the semantics of the interactive objects and become the means by which active semantics is used to express behavioural composition. Adaptive Object Modelling (AOM) is a model based on instances rather than classes. When we define the object type class, any instance of the class is an instance of the object. With the introduction of the object type, we can have active objects and passive objects. In passive objects we have only the reaction of the object to a message. Inside the object type we have the methods, states, monitor and participant structure.



Fig. 2. Internal structure of two interactive objects with monitor and participants in the relationship of the mutual control of the methods

4 Monitors as Worlds in the Meta-theory of Uncertainty

We know that in modal logic the world is the entity by which we can know if a proposition or assertion is TRUE of FALSE. When the enforcement rule (assertion) is violated the assertion is FALSE and the integrity of interactive objects is not valid.

In the Adaptive Object Modelling the implied action of an assertion is to reject any action which would lead to violation of the constraint. But in general we assume that the implied action cannot always eliminate the violation of the constraint. In this case we break the integrity and the system has uncertainty condition. When the assertions are given as TRUE in all the monitors the associated objects are completely integrate and the constrain condition is valid. When all the monitors give the value FALSE then we have a complete violation of the integrity. But, when for a part of the monitors the assertions are false we have a partial integrity of the objects. In conclusion we assume that:

- We associate to a monitor a possible world
- The relation among the monitors is a relation among the worlds
- We associate to any assertion in a monitor (world) a logic value TRUE or FALSE

With the monitors (worlds) we define the Kripke model i

$$M = \langle W, R, V \rangle \tag{5}$$

where W is a non-empty set of possible worlds (monitors), R is any type of relation among the worlds. $R \subseteq W \times W$ is an accessibility relation on W, and V is the function that assigns a logic value TRUE or FALSE to any monitor.

V: Propositions
$$\times$$
 W \rightarrow {T, F} (6)

A proposition p is necessarily true when in all the accessible worlds (monitors) p is true. The proposition p is necessarily false when there is at least one accessible world (monitor) where p is false. The proposition p is possibly true when there is at least one accessible world (monitor) where p is true.

Resconi, et al. (1992-1996) suggested to adjoin a function

$$\Psi: W \to R \tag{7}$$

where R is the set of real numbers assigned to worlds W in order to obtain the new model

$$S1 = \langle W, R, V, \Psi \rangle \tag{8}$$

That is for every world, there is an associated real number that is assigned to it. With the model S1, we can build the *hierarchical meta-theory* where we can calculate the expression for the membership function in the fuzzy set theory.

Imprecision means that an "entity" (temperature, velocity ...) cannot have a crisp logic evaluation. The meaning of a word in a proposition may usually be evaluated in different ways for different assessments of an entity by different monitors, i.e. worlds. In this case a world is associated to a monitor in the active object.

The violation of the integrity in an interactive object is the principal source of the imprecision in the meaning representation of the interactive object.

When we write for short:

$$\mu_{p_{A}}(\mathbf{x}) = \frac{(set \ of \ monitors \ where \ \mathbf{p}_{A}(\mathbf{x}) \ is \ true)}{|W(\mathbf{x})|} \tag{9}$$

where pA(x) is the integrity attribute for the interactive object x in the set A of interactive objects, the variable $\mu A(x)$ is the membership function in the fuzzy set A of the interactive objects and for a particular interactive object x.

The membership expression is computed as the value of Ψ in S1 stated in (1) above. It is computed by the expression

$$\Psi = \frac{1}{|W|} \tag{10}$$

5 Logic of the Monitors and Fuzzy Set Theory

Because any monitor as a world gives us information where we violate the integrity of the system, we are interested in the development of logic operations by which we can create logic expressions in the logic of the monitors.

5.1 Operation AND

Starting from two propositions, such as ' "John is tall" is true' AND "John is heavy" is true', given that we know the membership functions of $\mu_{p_1}(x)$ for P₁: "John is tall" is true', and $\mu_{p_2}(x)$ for P₂: "John is heavy" is true'. It should be clear that if we know the set of possible worlds W₁={Wi} where p₁ is true and the set of possible worlds W₂={Wj} where p₂ is true, then we can compute

$$\mu_{p_1}(\mathbf{x}) = |\mathbf{W}_1| / |\mathbf{W}| \text{ and } \mu_{p_2}(\mathbf{x}) = |\mathbf{W}_2| / |\mathbf{W}|.$$
(11)

Because $|W_i| = N_i$ is the number of possible worlds where the proposition p_i is true. We generate a new event p such that "p₁ and p₂" is "true" by the expression

$$\mathbf{p} = \mathbf{p}_1 \wedge \mathbf{p}_2 \tag{12}$$

where "and" is interpreted to be equivalent to "^" operation. So we have

$$\mu_{p}(\mathbf{x}) = \mu_{p_{1} \wedge p_{2}}(\mathbf{x}) = \frac{|\mathbf{W}_{1} \cap \mathbf{W}_{2}|}{|\mathbf{W}|}$$
(13)

Remark 1. When we know the value of p_1 and p_2 for every world we evaluate the expression $p = p_1 \land p_2$. Because for p_2 I can choose any type of sentence, we can choose $p_2 = \neg p_1$. When the set of worlds where p_1 is true and the set of worlds where p_2 is true have intersection different from zero, irrational worlds (monitors) can grow up: obtaining worlds where $p = p_1 \land \neg p_1$ is true.

We can easily show that for W_1^{C} , the complement of W_1 we have

$$\mu_{p_1 \land p_2}(x) = \frac{|W_1 \cap W_2|}{|W|} = \frac{|W_2|}{|W|} - \frac{|W_1^C \cap W_2|}{|W|} = \min[\mu_{p_1}(x), \mu_{p_2}(x)] - \frac{|W_1^C \cap W_2|}{|W|}$$
(14)

In this case we have $W1 \cap W2 = W2$ that is the set with the minimum value of cardinality.

When $p_2 = \neg p_1$ we have that all the worlds in W2 are irrational. We can prove that

$$0 \le \mu_{p_1 \land p_2} \le \min(\mu_{p_1}, \mu_{p_2})$$
 (15)

Between a zero irrationality to the maximum of the irrationality in the monitors.

5.2 Operation OR

For the OR combination, following similar steps as in the paragraph above, we have:

$$\max(\mu_{p_1}, \mu_{\neg p_1}) \le \mu_{p_1 \lor \neg p_1} \le 1$$
(16)

In this case, the set where $\neg p1$ is true is included in the set where p1 is true, we break the classical property for which the set of worlds where $\neg p1$ is true is the complement set of the worlds where p1 is true.

5.3 Operation NOT

In the fuzzy calculus we break the classical symmetry for which:

$$\mu_{\neg p} = 1 - \mu_p \tag{17}$$

The set where p is true and the set where $\neg p$ is true are separate sets without any connection one with the other as we have in the classical modal logic.

6 Conclusion

This paper introduces the partial coherence or integrity. When we have defect in knowledge, we violate the integrity of the system of interacting objects. The monitors give us the position in the system where the integrity is violated. Meta-theory of uncertainty by modal logic, where the world is a monitor, can generate a special logic or logic of the monitors by which we can compose the monitors results with the AND, OR and NOT logic operations.

References

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