Determination of Fabric Viscosity Parameters Using Iterative Minimization

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Abstract. In this paper, we present an experimental work using a MOCAP system and an iterative minimization technique to compute damping parameters and to measure their contribution for the simulation of cloth in free fall movement.

Energy damping is an important phenomenon to consider for the 3D simulation of warp and weft materials, since it has a great influence on the animation realism.

This phenomenon can be generated either by friction between moving cloth and air, or by friction between the warp and the weft threads of the fabric.

The contribution of this paper is to determine viscous parameters of cloth using precise trajectory data of a real cloth.

1 Introduction

A great deal of work on simulating the motion of cloth, and generally of fabric, has already been done [1][2][3] and several cloth simulators have been developed [4][5][6].

The motion of fabric is determined by its resistance to bending, stretching, shearing, by aerodynamic effects such as friction and collisions [7].

Realism of a simulation is usually used as a criterion to evaluate the accuracy of simulation and energy damping plays an important role in this search of realism [8].However, the viscous model parameters used in previously developed cloth simulators have not been estimated experimentally.

Authors mentioned the use of damping models but do not present the method to compute these parameters. [8] has developed an algorithm based on perceptually motivated metric, to estimate cloth damping parameters from video. However, [8] also estimates cloth parameters from video which is a less precise method than using a MOCAP sytem.

2 Fabric and Damping Model

We model fabric (limited to warp/weft textile materials) using the mass-spring system developed by Provot [9] and improved by Baraff & Witkin [4].The springs

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have to be fed with correct parameters to meet the realism that we look for simulation. We use the Kawabata Evaluation System [10] to get the parameters to fed the springs with. The damping model used is the Rayleigh damping model. Its mathematical formula is :

$$[C] = \alpha[M] + \beta[K] \tag{1}$$

where $[C]$ is the damping matrix (n x n), $[M]$ is the mass diagonal matrix (n x n), $[K]$ is the stiffness matrix (n x n), α and β are the damping constants, and n is the total number of masses used to model the fabric.

However, our mechanical model uses 3 different types of springs. So, the stiffness matrix $[K]$ is decomposed as the sum of 3 stiffness matrices modeling bending, shear and tensile :

$$[K] = [K_b] + [K_{sh}] + [K_t]$$

Equation (1) becomes :

$$[C] = \alpha[M] + \beta_b[K_b] + \beta_{sh}[K_{sh}] + \beta_t[K_t] \tag{2}$$

The linearity of Rayleigh's model makes it possible to derive the equation above. The total damping force is :

$$F_{damp} = [C]V \tag{3}$$

where V is the velocity vector of all masses.

3 Experimental Setup

The experiment consists in dropping a piece of fabric in free fall and measuring its trajectory using a motion capture system (MOCAP).(see figure 1) The viscous parameters are then obtained by the adjustment of the simulated trajectory of this fabric computed by our simulator, to the real trajectory. A sample of 50cm by 50cm of a fabric (woven in warp/weft) with reflective round markers stuck on its both sides is thrown in a free fall and the MOCAP system starts recording the successive positions of the markers.

4 Damping Parameters Identification

Given the data collected by the MOCAP, it is possible to compute the speed of each mass i and its acceleration (by finite differences). The fundamental principle of dynamics (F.P.D) is then written for each mass

$$\forall i, m_i A_i = F_i$$

where F_i is the sum of external forces applied on mass i.

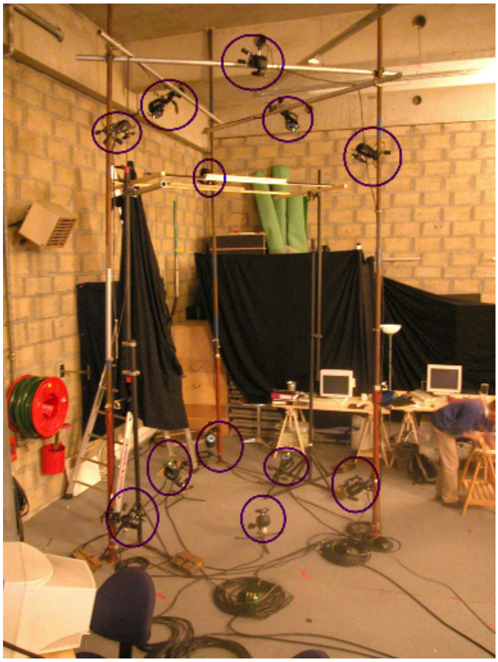

Fig. 1. 12 cameras of the Motion Capture System (MOCAP)

4.1 Global Minimization

We use global minimization in order to compute the best damping parameters that fit our data.

$$\text{Let } F_{error} = [M]A - [M]g - F_{springs} - F_{damp} \tag{4}$$

where $F_{springs}$ is the springs total force on masses. Damping parameters are obtained by minimizing the norm of F_{error}.

$$\Phi(\alpha, \beta_b, \beta_s, \beta_t) = F_{error}^T.F_{error} \tag{5}$$

$\Phi(\alpha, \beta_b, \beta_s, \beta_t)$ is a definite positive quadratic form, so we can find its minimum by computing its partial derivatives and making them equal to zero. We obtain a linear system.

$$M \begin{pmatrix} \alpha \\ \beta_b \\ \beta_s \\ \beta_t \end{pmatrix} = b \tag{6}$$

We could compute the conditioning number of matrix M to evaluate the solution stability using :

$$\kappa(M) = \| M \| . \| M^{-1} \| \tag{7}$$

$\kappa(M)$ can also be computed as the ratio between the greatest eigenvalue and the smallest one, since M is symmetric definite positive.

The diagonal terms of M are "proportional" to the square of the damping forces corresponding to air viscosity, bending, shear and tension which have sequentially values with an order of magnitude greater than the previous one. So, M is a largely diagonal dominant matrix and its determinant can be approximated by the product of its diagonal elements. So :

$$\kappa(M) \geq \frac{\frac{trace(M)}{4}}{\sqrt[4]{det(M)}} \gg 1 \tag{8}$$

We notice that the conditioning is very large, so the system is ill-conditioned and the solution will not be stable.Thus, we propose to compute damping parameters using iterative minimization.

4.2 Iterative Minimization

The aspect of M suggests us to estimate α first, then β_b, β_s and finally β_t as the corresponding damping forces increase in this order.

Identification of the Parameter of Viscous Damping with the Air. In order to make this identification, all springs are omitted. In fact, the viscous damping force between fabric and air is applied only on masses (springs have a null weight).

Writing the F.P.D for each mass i, we obtain the following equation :

$$m_i A_i = m_i g + F_{damp}^{air}$$

where g is the gravity and F_{damp}^{air} the viscous damping force of the air.

$$F_{damp}^{air} = \alpha_i m_i V_i$$

Let $F_{error}^{air} = m_i A_i - m_i g$. We have to compute the α that minimizes

$$\Phi(\alpha) = \| F_{error}^{air} - F_{damp}^{air} \|^2$$

$$\alpha_i = \frac{(F_{error}^{air} . V_i)}{m_i \| V_i \|^2} \tag{9}$$

So, for each mass and for each frame, we obtain an α_i. As the textile material is homogeneous, all α_i are equal and do not depend on the speed. So, we compute α_f for each frame as the mean of the α_i of this frame and then, we compute α as the mean of the α_f in the *viscous* part of the movement.

In fact, let's analyze the example shown in figure 2. The part of the movement between the frames 0 and 40 corresponds to the beginning of the free fall movement. The speed of the fabric is still very low and the movement of the fabric is still polluted by the launch (very noisy data).

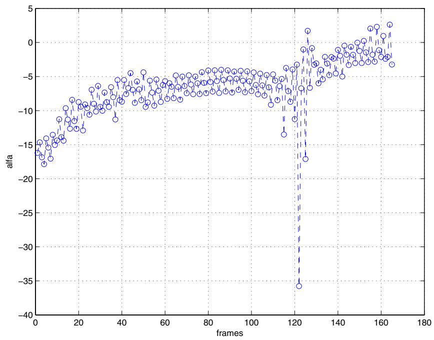

Fig. 2. Viscous damping parameter of the air per frame

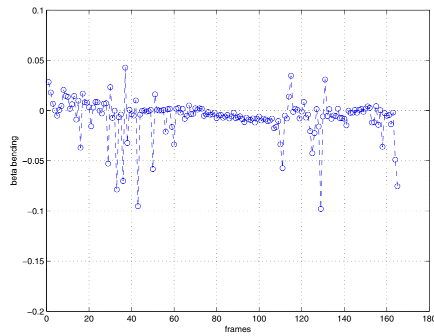

Fig. 3. Viscous damping parameter of bending springs

Beyond frame 100, the fabric has a chaotic turbulent movement and the interaction type between the air and the fabric can no longer be modeled using the Rayleigh model.

So, we compute α as the mean of the α_f in the *viscous* part of the movement, ie between the frames 40 and 100.

Indeed, in this part of the movement, the fabric has already acquired a minimum speed that enables us to measure more reliably a force of viscous friction with the air (since this force is proportional to the speed of the masses).

In addition, we observe that the movement of the fabric on the video is slowed down in this part without having turbulent or chaotic movements, and we know that α_f does not depend on the frame, so we have to restrict the computation to the horizontal part of figure 2.

Identification of the Parameter of Viscous Damping of Bending Springs. After computing the viscous friction parameter α between the fabric and the air, we include the bending springs in the simulation. Hence, the model evolves and allows to take into account forces between two adjacent facets.

A bending spring connects 2 adjacent facets (4 masses) and models the reaction of fabric to bending. Bending forces are very weak compared to tension forces or shearing forces, so errors induced by bending springs are much smaller as well. That is why we have added these springs first to the model (and omit shearing and tensile springs).

We write the F.P.D for each mass i:

$$m_i A_i = m_i g + \alpha m_i V_i + F_i^b + F_{damp}^b(i)$$

where $F_{damp}^b(i)$ is the viscous damping force of the bending springs.

$$F_{damp}^b(i) = \beta_i^b (K_b V)(i) \tag{10}$$

where $K_b = \frac{dF^b}{dP}$, F^b is the vector of forces produced on masses by the bending springs and P is the position vector of all masses.

Let $F^b(P_i)$ be the vector of forces produced on mass i by the bending springs. $F^b = \sum_i F^b(P_i)$ and $F^b(P_i) = \sum_r F^b_r(P_i)$ where $F^b_r(P_i)$ is the vector of forces produced on mass i by the bending springs r connected to mass i.

$$F^b_r(P_i) = \mathcal{M}_r^{Kaw} \frac{d\theta}{dP_i}$$

where $\mathcal{M}_r^{Kaw}$ is the torque intensity produced by the spring, given by *Kawabata* and θ is the angle between the two facets of the bending spring r. So, K_b is a 3n by 3n matrix whose (i,j) bloc (3 by 3) is:

$$\frac{dF^b(P_i)}{dP_j} = \sum_r \frac{dF^b_r(P_i)}{dP_j} \tag{11}$$

where

$$\frac{dF^b_r(P_i)}{dP_j} = \mathcal{M}_r^{Kaw} \frac{\partial^2\theta}{\partial P_i \partial P_j} + \frac{\partial \mathcal{M}_r^{Kaw}}{\partial \theta} \left(\frac{d\theta}{dP_i}\right) \left(\frac{d\theta}{dP_j}\right)^T \tag{12}$$

Equation (10) shows that the computation of the damping force of a bending spring on a mass i uses properties of other masses (its neighbors). Thus, we will directly search for a β^b for each frame.

Let $F^b_{error} = MA - Mg - \alpha MV - \beta^b K_b V - F^b$ where F^b is the bending force vector.

We search for β^b that minimizes $\Phi(\beta) = \| F^b_{error} - F^b_{damp} \|^2$

$$\beta^b = \frac{(F_{error}.K_b V)}{\| K_b V \|^2} \tag{13}$$

Figure 3 shows the β^b value found for each frame. β^b is computed as before, as the mean of β^b in the *viscous* part of the movement, ie between frames 40 and 100. (see figure 3)

Identification of the Parameter of Viscous Damping of Shearing Springs. we use the already determined parameters α and β^b to compute the air and bending springs damping forces. We include the shearing springs in the fabric model. We write the F.P.D for each mass i:

$$m_i A_i = m_i g + \alpha m_i V_i + F^b_i + \beta^b (K_b V)(i) + F^{sh}_i + F_{damp}(i)$$

where $F_{damp}(i)^{sh}$ is the viscous damping force of the shearing springs.

$$F^{sh}_{damp}(i) = \beta^{sh}_i (K_{sh} V)(i) \tag{14}$$

where $K_{sh} = \frac{dF^{sh}}{dP}$, F^{sh} is the vector of forces produced on masses by the shearing springs and P is the position vector of all masses.

Let $F^{sh}(P_i)$ be the vector of forces produced on mass i by the shearing springs. $F^{sh} = \sum_i F^{sh}(P_i)$ and $F^{sh}(P_i) = \sum_r F^{sh}_r(P_i)$ where $F^{sh}_r(P_i)$ is the vector of forces produced on mass i by the shearing springs r connected to mass i.

$$F_r^{sh}(P_i) = F_r^{Kaw} \frac{ds}{dP_i}$$

where F_r^{Kaw} is the spring force intensity given by *Kawabata* and s is the stretch of the spring r. So, K_{sh} is a 3n by 3n matrix whose (i, j) bloc (3 by 3) is:

$$\frac{dF^{sh}(P_i)}{dP_j} = \sum_r \frac{dF_r^{sh}(P_i)}{dP_j} \tag{15}$$

$$\frac{dF_r^{sh}(P_i)}{dP_j} = F_r^{Kaw} \frac{\partial^2 s}{\partial P_i \partial P_j} + \frac{\partial F_r^{Kaw}}{\partial s} \left(\frac{ds}{dP_i}\right) \left(\frac{ds}{dP_j}\right)^T \tag{16}$$

We will use the same approach as that one used for computing β^b. We start by computing a β^{sh} for each frame.

Let $F_{error}^{sh} = F_{error}^b - \beta^{sh} K_{sh} V - F^{sh}$. We search for the β^{sh} that minimizes $\Phi(\beta) = \| F_{error} - F_{damp} \|^2$

$$\beta^{sh} = \frac{(F_{error}^{sh} . K_{sh} V)}{\| K_{sh} V \|^2} \tag{17}$$

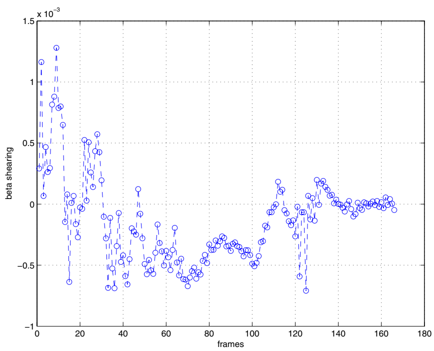

Fig. 4. Viscous damping parameter of shearing springs

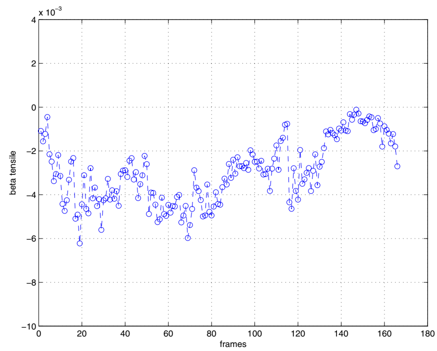

Fig. 5. Viscous damping parameter of tensile springs

Figure 4 shows the β^{sh} value found for each frame. We compute β^{sh} as the mean of β^{sh} in the *viscous* part of the movement, i.e. between frames 40 and 100.

Identification of the Parameter of Viscous Damping of Tensile Springs. We use the same approach as that one used for computing β^{sh}

$$\beta^t = \frac{(F_{error}^t . K_t V)}{\| K_t V \|^2} \tag{18}$$

Figure 5 shows the β^t value found for each frame. As for the shearing part, we compute β^t as the mean of β^t in the *viscous* part of the movement, i.e. between frames 40 and 100.

5 Results

Using the optimized parameters found, we can evaluate the improvement of the simulation. We can compute the error force without taking into account damping $F_{error}^{without-damp}$ at each step of the identification, and compare it with $F_{error}^{with-damp}$. All results are summed up in tables 1 and 2 (fabric 11).

5.1 Air Damping

Results shows that the norm of $F_{error}^{with-damp}$ is smaller than the norm of $F_{error}^{without-damp}$, which validates our work. We notice that damping due to viscous friction with the air allows to decrease the error by about 50% on average for this example.

5.2 Bending Spring Damping

We use the already determined parameter α. We compute the error force taking into account the bending spring damping and compare it with the error force without damping. The difference between the two error forces is very small. So, we can neglect the bending springs viscous damping while simulating the fabric movement in order to decrease the simulation time.

5.3 Shearing Spring Damping

On average the $F_{error}^{with-damp}$ norm is smaller than $F_{error}^{without-damp}$ norm. Shearing spring viscous damping decreases the error by 3%.

5.4 Tensile Spring Damping

Results show that tensile spring viscous damping decreases the error almost for each frame between frames 40 and 100.

The error decrease is most important between frames 50 and 90. On average, tensile spring viscous damping decreases the error by 9%.

5.5 Other Results Summary

The same experiments have been done with other types of fabrics. The results are summed up in tables 1 and 2.

5.6 Simulation

We have used the damping parameters found for our study fabric, and we have simulated a free fall movement using a cloth simulator. Then we have compared the position of the simulated piece of fabric with the real one.

We observe that in the viscous part of the movement, the simulated cloth follows faithfully the trajectory of the real cloth.

Table 1. Damping parameters

	α		β^{bend}		β^{shear}		$\beta^{tensile}$	
	mean	std	mean	std	mean	std	mean	std
fabric 11	-7.0	1.9	-6.8e-3	2.0e-2	-3.1e-4	2.9e-4	-3.9e-6	4.2e-6
fabric 12	-5.9	2.4	-5.2e-3	1.5e-2	-4.0e-4	4.0e-4	-2.7e-6	3.0e-6
fabric 13	-7.2	2.2	-3.1e-4	2.0e-3	-1.8e-4	4.6e-4	-4.0e-6	5.3e-6
fabric 21	-7.2	2.3	-2.0e-4	5.7e-4	-4.3e-4	6.7e-4	-4.6e-7	5.7e-7
fabric 31	-7.4	1.1	-8.4e-4	3.1e-3	-1.1e-3	6.1e-4	-2.2e-8	6.0e-8

Table 2. Error decrease using optimized parameters

	α	β^{bend}	β^{shear}	$\beta^{tensile}$
	error decrease	error decrease	error decrease	error decrease
fabric 11	50%	0.3%	3%	9%
fabric 12	45%	0.3%	1.1%	6%
fabric 13	51%	0.1%	1.4%	3.3%
fabric 21	48%	0%	2.3%	0.5%
fabric 31	72%	0.2%	36%	0.8%

6 Discussion

This paper describes experiments allowing the measurement of damping parameters for cloth simulation, in the case of warp and weft materials. We captured the behavior of pieces of fabric in a free fall movement using a MOCAP system. Then an optimization framework was used in order to compute damping parameters of the fabric. The validation of these measurements was performed by comparing real and simulated fabric free falls.

1. **Air damping:** results obtained for the damping parameter α of the fabric with the air show that we can use an average value equal to -7. When using an optimal α, we can decrease the error (numerical error made by ignoring air damping) by 50%.
   So, air damping has an important influence on the realism of cloth simulation.
2. **Bending damping:** results obtained for bending damping parameter β^{bend} show that bending damping does not decrease the error. So, there is no difference if we add bending damping or not in the simulation. Thus, we will ignore bending damping in future cloth simulation which allows some gain in computing time.
3. **Shear and Tensile damping:** These parameters model an inner phenomenon in the fabric, so they depend on the mechanical properties of the fabric. Fabric 11, 12 and 13 in tables 1 and 2 are three experiments with the same fabric. We notice, that the computed (β^{shear}) and ($\beta^{tensile}$) have almost the same value.

Shear and tensile damping decrease the error force, so they add some realism to coth simulation. However, the amount of error decrease depends on the experiment.

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