NMF with LogGabor Wavelets for Visualization

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Abstract. Many problems in image representation and classification involve some form of dimensionality reduction. Non-negative matrix factorization (NMF) is a recently proposed unsupervised procedure for learning spatially localized, parts-based subspace representation of objects. Here we present an improvement of the classical NMF by combining with Log-Gabor wavelets to enhance its part-based learning ability. In addition, we compare the new method with principal component analysis (PCA) and locally linear embedding (LLE) proposed recently in Science. Finally, we apply the new method to several real world datasets and achieve good performance in representation and classification.

1 Introduction

Recently, a new approach called non-negative matrix factorization (NMF), is proposed by Lee and Seung [6]. The new one demonstrates how to obtaining a reduced representation of global data in an unsupervised way. Non-negative matrix factorization is different from other methods by adding its non-negative constraints. When applied to image analysis and representation, the obtained NMF basis are localized features that correspond with intuitive notions of the parts of the images. It is supported by psychological and physiological evidence that perception of the whole is based on parts-based representations. And many recent learning strategies focus on the fact that an object can be divided into distinguished parts and only a subset of them are necessary for identification.

In this paper, we combine NMF with Log-Gabor wavelets to improve the performance of learning parts of images of the classical NMF. And then we compare the new method with PCA and LLE. Finally, we apply the new method to several real world datasets to verify its good performance in image representation and classification.

2 NMF vs. PCA and LLE Techniques

2.1 Non-negative Matrix Factorization

Non-negative matrix factorization (NMF), proposed recently by Lee and Sueng, is an outstanding method for obtaining a reduced representation of global data. When

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applied to images analysis, the obtained NMF basis are localized features that correspond with intuitive notions of the parts of images.

The goal of NMF is to find two new matrices W and H to approximate the whole database V as

$$V_{i\mu} \approx (WH)_{i\mu} = \sum_{a=1}^{r} W_{ia} H_{a\mu} \tag{1}$$

The r columns of W are the so called basis images. The update rules for W and H are:

$$\begin{aligned} W_{ia} &\leftarrow W_{ia} \sum_{\mu} \frac{V_{i\mu}}{(WH)_{i\mu}} H_{a\mu} \\ W_{ia} &\leftarrow \frac{W_{ia}}{\sum W_{ja}} \end{aligned} \tag{2}$$

$$H_{a\mu} \leftarrow H_{a\mu} \sum_{i} W_{ia} \frac{V_{i\mu}}{(WH)_{i\mu}} \tag{3}$$

and all elements in W and H are non-negative.

2.2 Performance Comparison of NMF with PCA and LLE

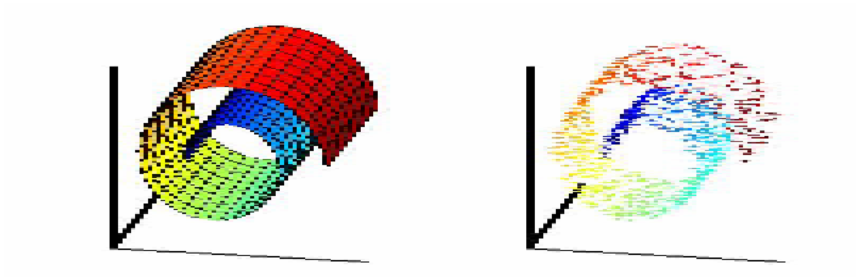
To illustrate the performance of data representation and dimensionality reduction by NMF, PCA and LLE vividly, we applied these methods to a manifold in 3D space. Fig 1 shows the original data and the results by enforcing those three methods. After mapping the manifold to 2D space, the properties of these methods give rise to deep visual impression on us.

The result (c) in Fig 1, discovered by LLE, demonstrates its neighbor relationship preserving property. Just imagine that using a scissors to cut the manifold into small squares that represent

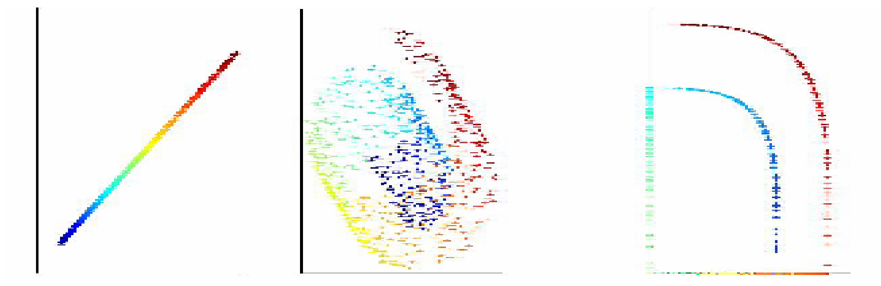
Locally linear patches of the nonlinear scroll-shape surface, and then put these squares onto a flat tabletop while preserving the angular relationship between neighboring squares. But if the data points in the original space are sparse enough, LLE leads to bad performance.

As shown in Fig 1, PCA demonstrates the maximum projection of the original data in lower dimensional space. It is an optimal representation of the original space. In other words, PCA is the optimal method for dimensionality reduction in the sense of mean-square error.

While the result (e) in Fig 1, discovered by NMF, is a compromise between PCA and LLE to some sense. It preserves the neighbor relationship and also gives a good representation of the original data in some way.



(a) a manifold in 3D space (b) sampled from (a)



(c) discovered by LLE (d) discovered by PCA (e) discovered by NMF

Fig. 1. Mapping a manifold in 3D space to 2D space by LLE, PCA and NMF respectively. The results are shown in (c)(d)(e).

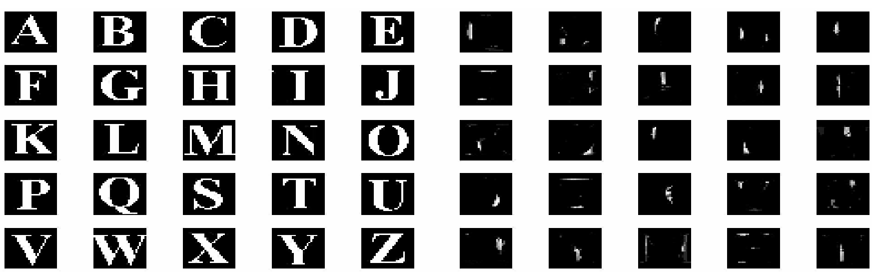


Fig. 2. Some English letters (left) and its basis images discovered by NMF with $r = 25$

After comparison with other methods, NMF is applied to real world datasets such as characters and human ears to demonstrate its parts-based learning ability. The results are shown in Fig 2.

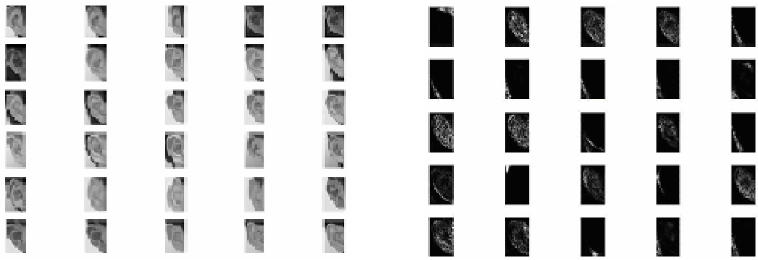


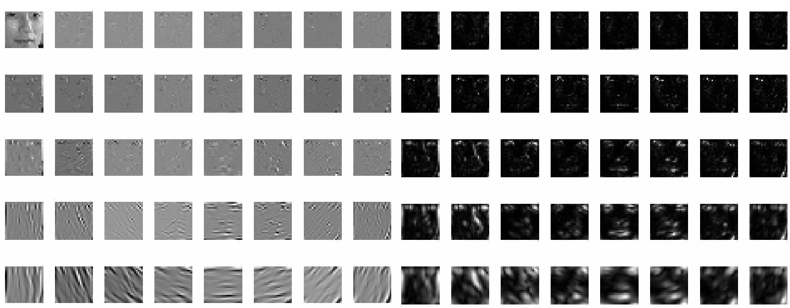
Fig. 3. Images of 30 human ears (left) and its basis images discovered by NMF with $r = 25$

3 LogGabor Wavelets for Image Representation

Here we choose Log-Gabor wavelets because they have no DC response and a better response to high frequency details [10]. The transfer function of Log-Gabor in frequency domain is

$$g(\omega) = e^{\frac{-(\log(\omega/\omega_0))^2}{2(\log(\beta/\omega_0))^2}} \tag{4}$$

where ω is frequency, and ω_0 is the tuning frequency of the filter. β controls the spread of the filter. Fig 4 shows the result of Log-Gabor filter convolving with a face image at five scale and 8 spread. The first block image of (a) in Fig 4 is the original image.

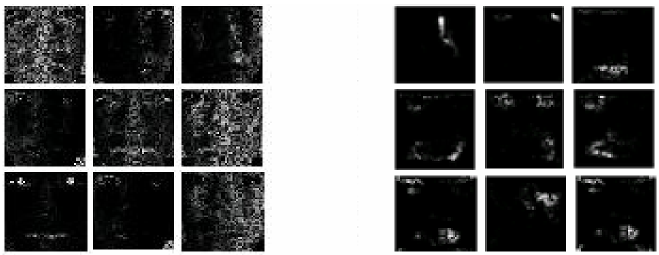


(a) (b)

Fig. 4. Log-Gabor representation of a face image. (a) real part of the representation and (b) the magnitude of the representation.

4 NMF with Log-Gabor Wavelets for Representation

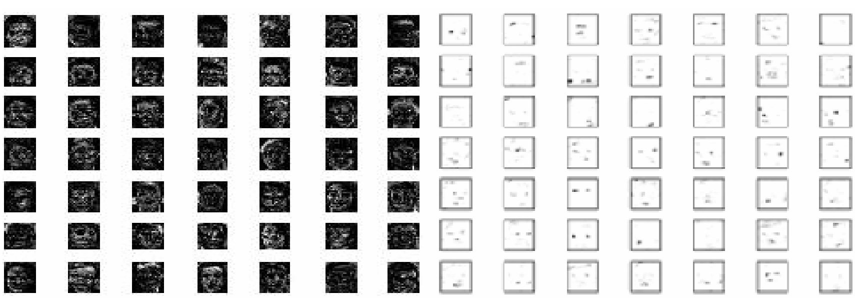
As mentioned in section 1, NMF takes a longer time to give a desirable result. And for images contained complicated structure, such as face images, the basis images discovered by NMF are not wholly part-based perception. Fig 5 shows the basis images learned by NMF without and with Log-Gabor wavelets. Here, the face image is the same as in Fig 4. When combined with Log-Gabor wavelets, NMF yields powerful performance in learning parts of the images. It is attributed to the non-negative constraints of NMF on the one hand, and the preprocessing the images by Log-Gabor wavelets on the other hand.



(a) (b)

Fig. 5. Basis images learned by NMF (a) without Log-Gabor (b) with Log-Gabor $r=9$

(a) (c)



(b) (d)

Fig. 6. Comparison of NMF without and with Log-Gabor, PCA eigenfaces. (a) parts of ORL database (b) basis images learned by NMF only (c) PCA eigenfaces (d) basis images learned by NMF with log-Gabor wavelets.

Next, NMF is applied to ORL face database combined with Log-Gabor wavelets. In addition, PCA is also applied to the face database to have a comparison with NMF. Fig 6 shows the results.

The experiments related to NMF (b) (d) in Fig 6 choose $r = 49$. Higher pixel values are in darker color in (d) in order to make it clearer. This is different from the other three. The basis images of learned by NMF only are as holistic as the PCA basis (eigenfaces) for the training set (a) in Fig 6. It is noticed that the result demonstrated in [6] does not appear so probably because the faces used for producing that result are well aligned and processed. The new method, NMF combined with Log-Gabor wavelets, learns basis components which not only lead to non-subtractive representations, but also yields truly localized features and parts-based representations. Also, the features formed in basis components discovered by the new method become more localized as the r increases.

5 Conclusions

We have introduced a new method, original NMF with Log-Gabor wavelets, for image representation and visualization. The new method improves the classical NMF in terms of part-based learning ability largely because of a sparse and informative representation given by Log-Gabor wavelets. It gives a meaningful perceptional representation in image analysis and a high recognition performance in image classification. When compared with other methods such as linear PCA and nonlinear LLE, the new method shows robustness to variations in illumination, occlusion and facial expression.

Our next goal is to further improve the performance of NMF such as accelerating the convergence time, learning the basis r by machine and learning more informative local features.

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