Reciprocal Logic: Logics for Specifying, Verifying, and Reasoning About Reciprocal Relationships

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Abstract. To specify, verify, and reason about various reciprocal relationships in a human society and/or a cyber space, we need a right fundamental logic system to provide us with a criterion of logical validity of reasoning as well as a representation and specification language. This paper proposes a new family of conservative extensions of relevant logic, named "reciprocal logic," for specifying, verifying, and reasoning about reciprocal relationships. The paper shows that various reciprocal logics can be obtained by introducing predicates and related axioms about reciprocal relationships into strong relevant logics and spatial-temporal relevant logics. A case study is focused on trust relationships.

1 Introduction

In human society and/or a cyber space, there are many reciprocal relationships that must concern two parties, such as parent-child relationship, relative relationship, friendship, adjacent relationship, high and low relationship, cooperative relationship, complementary relationship, adverse relationship, dependent relationship, trust relationship, trade relationship, buying and selling relationship, and so on. As a result, many reciprocal relationships appear in various disciplines including Artificial Intelligence, Cryptography, Economics, Game Theory, Information Security Engineering, Knowledge Engineering, Linguistics, Logic, Multi-agent Systems, Philosophy, Psychology, and Software Engineering.

To specify, verify, and reason about various reciprocal relationships in a human society and/or a cyber space, we need a right fundamental logic system to provide us with a criterion of logical validity of reasoning as well as a representation and specification language. The question, "Which is the right logic?" invites the immediate counter-question "Right for what?" Only if we certainly know what we need, we can make a good choice. It is obvious that different applications may require different characteristics of logic.

The present author considers that we should consider the following essential requirements for the fundamental logic. First, as a general logical criterion for the validity of reasoning as well as proving, the logic must be able to underlie relevant reasoning as well as truth-preserving reasoning in the sense of conditional, i.e., for any reasoning based on the logic to be valid, if its premises are true in the sense of conditional, then its conclusion must be relevant to the premises and true in the sense of

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conditional. Second, the logic must be able to underlie ampliative reasoning in the sense that the truth of conclusion of the reasoning should be recognized after the completion of the reasoning process but not be invoked in deciding the truth of premises of the reasoning. From the viewpoint to regard reasoning as the process of drawing new conclusions from given premises, any meaningful reasoning must be ampliative but not circular and/or tautological. Third, the logic must be able to underlie paracomplete reasoning and paraconsistent reasoning. In particular, the so-called principle of Explosion that everything follows from a contradiction cannot be accepted by the logic as a valid principle. In general, our knowledge about various reciprocal relationships may be incomplete or even inconsistent in many ways, i.e., it gives us no evidence for deciding the truth of either a proposition or its negation, or even it directly or indirectly includes some contradictions. Therefore, reasoning with incomplete and/or inconsistent knowledge is the rule rather than the exception in our everyday lives and almost all scientific disciplines. Finally, because reciprocal relationships themselves may change over space and time, the right fundamental logic system must be able to underlie spatial reasoning, or temporal reasoning, or both.

Classical mathematical logic (**CML** for short) cannot satisfy any of the above essential requirements because of the following facts: a reasoning based on **CML** is not necessarily relevant; the classical truth-preserving property of a reasoning based on **CML** is meaningless in the sense of conditional; a reasoning based on **CML** must be circular and/or tautological but not ampliative; reasoning under inconsistency is impossible within the framework of **CML** [1, 2, 4, 5]. The above facts are also true to those classical conservative extensions or non-classical alternatives of **CML** including temporal (classical) logics [3, 11, 12] and spatial (classical) logics [8-10] where the classical account of validity is adopted as the logical validity criterion and the notion of conditional is directly or indirectly represented by the material implication. **CML** does not underlie spatial reasoning and temporal reasoning explicitly.

Traditional relevant (or relevance) logics ware constructed during the 1950s in order to find a mathematically satisfactory way of grasping the elusive notion of relevance of antecedent to consequent in conditionals, and to obtain a notion of implication which is free from the so-called ‘paradoxes’ of material and strict implication [1, 2]. Some major traditional relevant logic systems are ‘system **E** of entailment’, ‘system **R** of relevant implication’, and ‘system **T** of ticket entailment’. A major characteristic of the relevant logics is that they have a primitive intensional connective to represent the notion of (relevant) conditional and their logical theorems include no implicational paradoxes. The underlying principle of the relevant logics is the relevance principle, i.e., for any entailment provable in **E**, **R**, or **T**, its antecedent and consequent must share a propositional variable. Variable-sharing is a formal notion designed to reflect the idea that there be a meaning-connection between the antecedent and consequent of an entailment. It is this relevance principle that excludes those implicational paradoxes from logical axioms or theorems of relevant logics. Also, since the notion of entailment is represented in the relevant logics by a primitive intensional connective but not an extensional truth-function, a reasoning based on the relevant logics is ampliative but not circular and/or tautological. Moreover, because the relevant logics reject the principle of Explosion, they can certainly underlie paraconsistent reasoning. However, traditional relevant logics still include conjunction-implicational paradoxes and disjunction-implicational paradoxes [4, 5]. As a result, they only can guarantee the relevance between the premises of a valid argument and its conclusion and the validity of its conclusion in a sense of weak relevance. Relevant logics do not underlie spatial reasoning and temporal reasoning explicitly.

Thus, no existing logic can satisfy all essential requirements for the fundamental logic. This paper proposes a new family of conservative extensions of relevant logic, named "***reciprocal logic***," for specifying, verifying, and reasoning about reciprocal relationships. The paper shows that various reciprocal logics can be obtained by introducing predicates and related axioms about reciprocal relationships into strong relevant logics and spatial-temporal relevant logics. A case study is focused on trust relationships.

2 Strong Relevant Logics and Spatio-temporal Relevant Logics

In order to establish a satisfactory logic calculus of conditional to underlie relevant reasoning, the present author has proposed ***strong relevant*** (or ***relevance***) ***logics*** [4]. The logics require that the premises of an argument represented by a conditional include no unnecessary and needless conjuncts and the conclusion of that argument includes no unnecessary and needless disjuncts. As a modification of traditional relevant logics, strong relevant logics reject all conjunction-implicational paradoxes and disjunction-implicational paradoxes in traditional relevant logics. What underlies the strong relevant logics is the strong relevance principle: If A is a theorem of strong relevant logics, then every propositional variable in A occurs at least once as an antecedent part and at least once as a consequent part. In the framework of strong relevant logics, if a reasoning and/or argument is valid, then both the relevance between its premises and its conclusion and the validity of its conclusion in the sense of conditional can be guaranteed in a certain sense of strong relevance.

The logical connectives, axiom schemata, and inference rules of strong relevant logics are as follows:

Primitive Logical Connectives: { $\Rightarrow$ (entailment), $\neg$ (negation), $\wedge$ (extensional conjunction) }

Defined Logical Connectives: { $\otimes$ (intensional conjunction, $A\otimes B =_{df} \neg(A\Rightarrow\neg B)$), $\oplus$ (intensional disjunction, $A\oplus B =_{df} \neg A\Rightarrow B$), $\Leftrightarrow$ (intensional equivalence, $A\Leftrightarrow B =_{df} (A\Rightarrow B)\otimes(B\Rightarrow A)$), $\vee$ (extensional disjunction, $A\vee B =_{df} \neg(\neg A\wedge\neg B)$), $\rightarrow$ (material implication, $A\rightarrow B =_{df} \neg(A\wedge\neg B)$ or $A\rightarrow B =_{df} \neg A\vee B$), $\leftrightarrow$ (extensional equivalence, $A\leftrightarrow B =_{df} (A\rightarrow B)\wedge(B\rightarrow A)$) }

Axiom Schemata: E1: $A\Rightarrow A$, E2: $(A\Rightarrow B)\Rightarrow((C\Rightarrow A)\Rightarrow(C\Rightarrow B))$, E2′: $(A\Rightarrow B)\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow C))$, E3: $(A\Rightarrow(A\Rightarrow B))\Rightarrow(A\Rightarrow B)$, E3′: $(A\Rightarrow(B\Rightarrow C))\Rightarrow((A\Rightarrow B)\Rightarrow(A\Rightarrow C))$, E3″: $(A\Rightarrow B)\Rightarrow((A\Rightarrow(B\Rightarrow C))\Rightarrow(A\Rightarrow C))$, E4: $(A\Rightarrow((B\Rightarrow C)\Rightarrow D))\Rightarrow((B\Rightarrow C)\Rightarrow(A\Rightarrow D))$, E4′: $(A\Rightarrow B)\Rightarrow(((A\Rightarrow B)\Rightarrow C)\Rightarrow C)$, E4″: $((A\Rightarrow A)\Rightarrow B)\Rightarrow B$, E4‴: $(A\Rightarrow B)\Rightarrow((B\Rightarrow C)\Rightarrow(((A\Rightarrow C)\Rightarrow D)\Rightarrow D))$, E5: $(A\Rightarrow(B\Rightarrow C))\Rightarrow(B\Rightarrow(A\Rightarrow C))$, E5′: $A\Rightarrow((A\Rightarrow B)\Rightarrow B)$, N1: $(A\Rightarrow(\neg A))\Rightarrow(\neg A)$, N2: $(A\Rightarrow(\neg B))\Rightarrow(B\Rightarrow(\neg A))$, N3: $(\neg(\neg A))\Rightarrow A$, C1: $(A\wedge B)\Rightarrow A$, C2: $(A\wedge B)\Rightarrow B$, C3: $((A\Rightarrow B)\wedge(A\Rightarrow C))\Rightarrow(A\Rightarrow(B\wedge C))$, C4: $(\boldsymbol{L}A\wedge\boldsymbol{L}B)\Rightarrow\boldsymbol{L}(A\wedge B)$, where $\boldsymbol{L}A =_{df} (A\Rightarrow A)\Rightarrow A$, D1: $A\Rightarrow(A\vee B)$, D2: $B\Rightarrow(A\vee B)$,

D3: $((A\Rightarrow C)\wedge(B\Rightarrow C))\Rightarrow((A\vee B)\Rightarrow C)$, DCD: $(A\wedge(B\vee C))\Rightarrow((A\wedge B)\vee C)$, C5: $(A\wedge A)\Rightarrow A$, C6: $(A\wedge B)\Rightarrow(B\wedge A)$, C7: $((A\Rightarrow B)\wedge(B\Rightarrow C))\Rightarrow(A\Rightarrow C)$, C8: $(A\wedge(A\Rightarrow B))\Rightarrow B$, C9: $\neg(A\wedge\neg A)$, C10: $A\Rightarrow(B\Rightarrow(A\wedge B))$, IQ1: $\forall x(A\Rightarrow B)\Rightarrow(\forall xA\Rightarrow\forall xB)$, IQ2: $(\forall xA\wedge\forall xB)\Rightarrow\forall x(A\wedge B)$, IQ3: $\forall xA\Rightarrow A[t/x]$ (if x may appear free in A and t is free for x in A, i.e., free variables of t do not occur bound in A), IQ4: $\forall x(A\Rightarrow B)\Rightarrow(A\Rightarrow\forall xB)$ (if x does not occur free in A), IQ5: $\forall x_1 \ldots \forall x_n(((A\Rightarrow A)\Rightarrow B)\Rightarrow B)$ $(n\geq 0)$

Inference Rules: $\Rightarrow$E: from A and $A\Rightarrow B$ to infer B (Modus Ponens), $\wedge$I: from A and B to infer $A\wedge B$ (Adjunction), $\forall$I: if A is an axiom, so is $\forall xA$ (Generalization of axioms)

Various relevant logic systems are defined as follows, where we use 'A | B' to denote any choice of one from two axiom schemata A and B: $\mathbf{T}_{\Rightarrow} =_{\text{df}}$ {E1, E2, E2′, E3 | E3″} + $\Rightarrow$E, $\mathbf{E}_{\Rightarrow} =_{\text{df}}$ {E1, E2 | E2′, E3 | E3′, E4 | E4′} + $\Rightarrow$E, $\mathbf{E}_{\Rightarrow} =_{\text{df}}$ {E2′, E3, E4″} + $\Rightarrow$E, $\mathbf{E}_{\Rightarrow} =_{\text{df}}$ {E1, E3, E4‴} + $\Rightarrow$E, $\mathbf{R}_{\Rightarrow} =_{\text{df}}$ {E1, E2 | E2′, E3 | E3′, E5 | E5′} + $\Rightarrow$E, $\mathbf{T}_{\Rightarrow,\neg} =_{\text{df}} \mathbf{T}_{\Rightarrow}$ + {N1, N2, N3}, $\mathbf{E}_{\Rightarrow,\neg} =_{\text{df}} \mathbf{E}_{\Rightarrow}$ + {N1, N2, N3}, $\mathbf{R}_{\Rightarrow,\neg} =_{\text{df}} \mathbf{R}_{\Rightarrow}$ + {N2, N3}, $\mathbf{T} =_{\text{df}} \mathbf{T}_{\Rightarrow,\neg}$ + {C1~C3, D1~D3, DCD} + $\wedge$I, $\mathbf{E} =_{\text{df}} \mathbf{E}_{\Rightarrow,\neg}$ + {C1~C4, D1~D3, DCD} + $\wedge$I, $\mathbf{R} =_{\text{df}} \mathbf{R}_{\Rightarrow,\neg}$ + {C1~C3, D1~D3, DCD} + $\wedge$I, **Tc** $=_{\text{df}} \mathbf{T}_{\Rightarrow,\neg}$ + {C3, C5~C10}, **Ec** $=_{\text{df}} \mathbf{E}_{\Rightarrow,\neg}$ + {C3~C10}, **Rc** $=_{\text{df}} \mathbf{R}_{\Rightarrow,\neg}$ + {C3, C5~C10}, **TQ** $=_{\text{df}}$ **T** + {IQ1~IQ5} + $\forall$I, **EQ** $=_{\text{df}}$ **E** + {IQ1~IQ5} + $\forall$I, **RQ** $=_{\text{df}}$ **R** + {IQ1~IQ5} + $\forall$I, **TcQ** $=_{\text{df}}$ **Tc** + {IQ1~IQ5} + $\forall$I, **EcQ** $=_{\text{df}}$ **Ec** + {IQ1~IQ5} + $\forall$I, **RcQ** $=_{\text{df}}$ **Rc** + {IQ1~IQ5} + $\forall$I. Here, $\mathbf{T}_{\Rightarrow}$, $\mathbf{E}_{\Rightarrow}$, and $\mathbf{R}_{\Rightarrow}$ are the purely implicational fragments of **T**, **E**, and **R**, respectively, and the relationship between $\mathbf{E}_{\Rightarrow}$ and $\mathbf{R}_{\Rightarrow}$ is known as $\mathbf{R}_{\Rightarrow} = \mathbf{E}_{\Rightarrow}$ + A$\Rightarrow$***L***A; $\mathbf{T}_{\Rightarrow,\neg}$, $\mathbf{E}_{\Rightarrow,\neg}$, and $\mathbf{R}_{\Rightarrow,\neg}$ are the implication-negation fragments of **T**, **E**, and **R**, respectively; **Tc**, **Ec**, **Rc, TcQ**, **EcQ**, and **RcQ** are strong relevant logics.

However, both traditional relevant logics and strong relevant logics do not underlie spatial reasoning and temporal reasoning explicitly. In order to specify, verify, and reason about spatio-temporal knowledge, we have proposed ***spatio-temporal relevant logics*** [7], which are obtained by introducing region connection predicates and axiom schemata of RCC [8-10], point position predicates and axiom schemata, and point adjacency predicates and axiom schemata into ***temporal relevant logics*** [6]. Below we present a modification of spatio-temporal relevant logics in [7].

Let $\{r_1, r_2, r_3, \ldots\}$ be a countably infinite set of individual variables, called ***region variables***. Atomic formulas of the form $\boldsymbol{C}(r_1, r_2)$ are read as 'region r_1 connects with region r_2.' Let $\{p_1, p_2, p_3, \ldots\}$ be a countably infinite set of individual variables, called ***point variables***. Atomic formulas of the form $\boldsymbol{I}(p_1, r_1)$ are read as 'point p_1 is included in region r_1.' Atomic formulas of the form $\boldsymbol{Id}(p_1, p_2)$ are read as 'point p_1 is identical with p_2.' Atomic formulas of the form $\boldsymbol{Arc}(p_1, p_2)$ are read as 'points p_1, p_2 are adjacent such that there is an arc from point p_1 to point p_2, or more simply, points p_1 is adjacent to point p_2.' Note that an arc has a direction. Atomic formulas of the form $\boldsymbol{Path}(p_1, p_2)$ are read as 'there is a directed path from point p_1 to point p_2.' Here, $\boldsymbol{C}(r_1, r_2)$, $\boldsymbol{I}(p_1, r_1)$, $\boldsymbol{Id}(p_1, p_2)$, $\boldsymbol{Arc}(p_1, p_2)$, and $\boldsymbol{Path}(p_1, p_2)$ are primitive binary predicates to represent relationships between geometric objects. Note that here we use a many-sorted language.

Temporal Operators: { ***G*** (future-tense always or henceforth operator, ***G****A* means 'it will always be the case in the future from now that *A*'), ***H*** (past-tense always operator, ***H****A* means 'it has always been the case in the past up to now that *A*'), ***F*** (future-tense sometime or eventually operator, ***F****A* means 'it will be the case at least once in the future from now that *A*'), ***P*** (past-tense sometime operator, ***P****A* means 'it has been the case at least once in the past up to now that *A*') } Note that these temporal operators are not independent and can be defined as follows: $\boldsymbol{G}A =_{df} \neg\boldsymbol{F}\neg A$, $\boldsymbol{H}A =_{df} \neg\boldsymbol{P}\neg A$, $\boldsymbol{F}A =_{df} \neg\boldsymbol{G}\neg A$, $\boldsymbol{P}A =_{df} \neg\boldsymbol{H}\neg A$.

Primitive Binary Predicate: { ***C*** (connection, $\boldsymbol{C}(r_1, r_2)$ means 'r_1 connects with r_2'), ***I*** (inclusion, $\boldsymbol{I}(p_1, r_1)$ means 'p_1 is included in r_1'), ***Id*** (the same point, $\boldsymbol{Id}(p_1, p_2)$ means 'point p_1 is identical with p_2'), ***Arc*** (arc, $\boldsymbol{Arc}(p_1, p_2)$ means 'p_1 is adjacent to p_2'), ***Path*** (path, $\boldsymbol{Path}(p_1, p_2)$ means 'there is a directed path from p_1 to p_2') }

Defined Binary Predicates: $\boldsymbol{DC}(r_1, r_2) =_{df} \neg\boldsymbol{C}(r_1, r_2)$ ($\boldsymbol{DC}(r_1, r_2)$ means 'r_1 is disconnected from r_2'), $\boldsymbol{Pa}(r_1, r_2) =_{df} \forall r_3(\boldsymbol{C}(r_3, r_1)\Rightarrow\boldsymbol{C}(r_3, r_2))$ ($\boldsymbol{Pa}(r_1, r_2)$ means 'r_1 is a part of r_2'), $\boldsymbol{PrPa}(r_1, r_2) =_{df} \boldsymbol{Pa}(r_1, r_2)\wedge(\neg\boldsymbol{Pa}(r_2, r_1))$ ($\boldsymbol{PrPa}(r_1, r_2)$ means 'r_1 is a proper part of r_2'), $\boldsymbol{EQ}(r_1, r_2) =_{df} \boldsymbol{Pa}(r_1, r_2)\wedge\boldsymbol{Pa}(r_2, r_1)$ ($\boldsymbol{EQ}(r_1, r_2)$ means 'r_1 is identical with r_2'), $\boldsymbol{O}(r_1, r_2) =_{df} \exists r_3(\boldsymbol{Pa}(r_3, r_1)\wedge\boldsymbol{Pa}(r_3, r_2))$ ($\boldsymbol{O}(r_1, r_2)$ means 'r_1 overlaps r_2'), $\boldsymbol{DR}(r_1, r_2) =_{df} \neg\boldsymbol{O}(r_1, r_2)$ ($\boldsymbol{DR}(r_1, r_2)$ means 'r_1 is discrete from r_2'), $\boldsymbol{PaO}(r_1, r_2) =_{df} \boldsymbol{O}(r_1, r_2)\wedge(\neg\boldsymbol{Pa}(r_1, r_2))\wedge(\neg\boldsymbol{Pa}(r_2, r_1))$ ($\boldsymbol{PaO}(r_1, r_2)$ means 'r_1 partially overlaps r_2'), $\boldsymbol{EC}(r_1, r_2) =_{df} \boldsymbol{C}(r_1, r_2)\wedge(\neg\boldsymbol{O}(r_1, r_2))$ ($\boldsymbol{EC}(r_1, r_2)$ means 'r_1 is externally connected to r_2'), $\boldsymbol{TPrPa}(r_1, r_2) =_{df} \boldsymbol{PrPa}(r_1, r_2)\wedge\exists r_3(\boldsymbol{EC}(r_3, r_1)\wedge\boldsymbol{EC}(r_3, r_2))$ ($\boldsymbol{TPrPa}(r_1, r_2)$ means 'r_1 is a tangential proper part of r_2'), $\boldsymbol{NTPrPa}(r_1, r_2) =_{df} \boldsymbol{PrPa}(r_1, r_2)\wedge(\neg\exists r_3(\boldsymbol{EC}(r_3, r_1)\wedge\boldsymbol{EC}(r_3, r_2)))$ ($\boldsymbol{NTPrPa}(r_1, r_2)$ means 'r_1 is a nontangential proper part of r_2').

Axiom Schemata: T1: $\boldsymbol{G}(A\Rightarrow B)\Rightarrow(\boldsymbol{G}A\Rightarrow\boldsymbol{G}B)$, T2: $\boldsymbol{H}(A\Rightarrow B)\Rightarrow(\boldsymbol{H}A\Rightarrow\boldsymbol{H}B)$, T3: $A\Rightarrow\boldsymbol{G}(\boldsymbol{P}A)$, T4: $A\Rightarrow\boldsymbol{H}(\boldsymbol{F}A)$, T5: $\boldsymbol{G}A\Rightarrow\boldsymbol{G}(\boldsymbol{G}A)$, T6: $(\boldsymbol{F}A\wedge\boldsymbol{F}B)\Rightarrow\boldsymbol{F}(A\wedge\boldsymbol{F}B)\vee\boldsymbol{F}(A\wedge B)\vee\boldsymbol{F}(\boldsymbol{F}A\wedge B)$, T7: $(\boldsymbol{P}A\wedge\boldsymbol{P}B)\Rightarrow\boldsymbol{P}(A\wedge\boldsymbol{P}B)\vee\boldsymbol{P}(A\wedge B)\vee\boldsymbol{P}(\boldsymbol{P}A\wedge B)$, T8: $\boldsymbol{G}A\Rightarrow\boldsymbol{F}A$, T9: $\boldsymbol{H}A\Rightarrow \boldsymbol{P}A$, T10: $\boldsymbol{F}A\Rightarrow\boldsymbol{F}(\boldsymbol{F}A)$, T11: $(A\wedge\boldsymbol{H}A)\Rightarrow\boldsymbol{F}(\boldsymbol{H}A)$, T12: $(A\wedge\boldsymbol{G}A)\Rightarrow\boldsymbol{P}(\boldsymbol{G}A)$, RCC1: $\forall r_1\forall r_2(\boldsymbol{C}(r_1, r_2)\Rightarrow\boldsymbol{C}(r_2, r_1))$, RCC2: $\forall r_1(\boldsymbol{C}(r_1, r_1))$, PRCC1: $\forall p_1\forall r_1\forall r_2((\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{DC}(r_1, r_2))\Rightarrow\neg\boldsymbol{I}(p_1, r_2))$, PRCC2: $\forall p_1\forall r_1\forall r_2((\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{Pa}(r_1, r_2))\Rightarrow\boldsymbol{I}(p_1, r_2))$, PRCC3: $\forall r_1\forall r_2(\boldsymbol{O}(r_1, r_2)\Rightarrow\exists p_1(\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{I}(p_1, r_2)))$, PRCC4: $\forall r_1\forall r_2(\boldsymbol{PaO}(r_1, r_2)\Rightarrow\exists p_1(\boldsymbol{I}(p_1, r_1)\wedge \boldsymbol{I}(p_1, r_2))\wedge\exists p_2(\boldsymbol{I}(p_2, r_1)\wedge\neg\boldsymbol{I}(p_2, r_2))\wedge\exists p_3(\neg\boldsymbol{I}(p_3, r_1)\wedge\boldsymbol{I}(p_3, r_2)))$, PRCC5: $\forall r_1\forall r_2(\boldsymbol{EC}(r_1, r_2)\Rightarrow\exists p_1(\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{I}(p_1, r_2)\wedge\forall p_2(\neg\boldsymbol{Id}(p_2, p_1)\Rightarrow\neg\boldsymbol{I}(p_2, r_1)\wedge\neg\boldsymbol{I}(p_2, r_2))))$, PRCC6: $\forall p_1\forall r_1\forall r_2((\boldsymbol{I}(p_1, r_1)\wedge\boldsymbol{TPrPa}(r_1, r_2))\Rightarrow\boldsymbol{I}(p_1, r_2))$, PRCC7: $\forall p_1\forall r_1\forall r_2((\boldsymbol{I}(p_1, r_1)\wedge \boldsymbol{NTPrPa}(r_1, r_2))\Rightarrow\boldsymbol{I}(p_1, r_2))$, APC1: $\forall p_1\forall p_2(\boldsymbol{Arc}(p_1, p_2)\Rightarrow\boldsymbol{Path}(p_1, p_2))$, APC2: $\forall p_1\forall p_2\forall p_3((\boldsymbol{Path}(p_1, p_2)\wedge\boldsymbol{Path}(p_2, p_3))\Rightarrow\boldsymbol{Path}(p_1, p_3))$.

Inference Rules: TG: from A to infer $\boldsymbol{G}A$ and $\boldsymbol{H}A$ (Temporal Generalization)

The minimal or weakest propositional temporal relevant logics are defined as follows: $\mathbf{T_0Tc} =_{df} \mathbf{Tc}$ + {T1~T4} + TG, $\mathbf{T_0Ec} =_{df} \mathbf{Ec}$ + {T1~T4} + TG, $\mathbf{T_0Rc} =_{df} \mathbf{Rc}$ + {T1~T4} + TG. Note that the minimal or weakest temporal classical logic $\mathbf{K_t}$ = all axiom schemata for **CML** + →E + {T1~T4} + TG. Other characteristic axioms such as T5~T12 that correspond to various assumptions about time can be added to $\mathbf{T_0Tc}$, $\mathbf{T_0Ec}$, and $\mathbf{T_0Rc}$ respectively to obtain various propositional temporal relevant logics. Various predicate temporal relevant logics then can be obtained by adding axiom schemata IQ1~IQ5 and inference rule ∀I into the propositional temporal relevant logics. For examples, minimal or weakest predicate temporal relevant logics are as follows: $\mathbf{T_0TcQ} =_{df} \mathbf{T_0Tc}$ + {IQ1~IQ5} + ∀I, $\mathbf{T_0EcQ} =_{df} \mathbf{T_0Ec}$ + {IQ1~IQ5} + ∀I, $\mathbf{T_0RcQ} =_{df} \mathbf{T_0Rc}$ + {IQ1~IQ5} + ∀I.

We can obtain some ***spatial relevant logics*** by adding region connection, point position, and point adjacency axiom schemata into the various predicate strong relevant logics. For examples: **STcQ** $=_{df}$ **TcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}, **SEcQ** $=_{df}$ **EcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}, **SRcQ** $=_{df}$ **RcQ** + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}.

Finally, we can obtain various spatio-temporal relevant logics by adding region connection, point position, and point adjacency axiom schemata into the various predicate temporal relevant logics. For examples: $\mathbf{ST_0TcQ} =_{df} \mathbf{T_0TcQ}$ + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}, $\mathbf{ST_0EcQ} =_{df} \mathbf{T_0EcQ}$ + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}, $\mathbf{ST_0RcQ} =_{df} \mathbf{T_0RcQ}$ + {RCC1, RCC2, PRCC1~PRCC7, APC1, APC2}.

3 Reciprocal Logics

Based on strong relevant logics and spatio-temporal relevant logics, we can construct various ***reciprocal logics*** to underlie specifying, verifying, and reasoning about reciprocal relationships by introducing predicates and related axioms about various reciprocal relationships into strong relevant logics and spatial-temporal relevant logics. Therefore, they are conservative extensions of temporal relevant logics as well as strong relevant logics. As a case study, here we focus our interests on trust relationships. Other reciprocal relationships can be considered and dealt with in the same way.

Various reciprocal relationships may be symmetrical or unsymmetrical, transitive or non-transitive, but in general they are not reflective. Although there are many various definitions on the concept of trust, various trust relationships should have something in common. In general, a trust relationship must concern two parties, say A and B, such that A trusts B to do something, and any trust relationship is not necessarily symmetrical and not necessarily transitive. In many cases, a trust relationship is conditional in the form that A trusts B to do something, if some condition is true. On the other hand, the relationship of trust is just one of many kinds of reciprocal relationships in a human society and/or a cyber space.

Let {pe_1, pe_2, pe_3, …} be a countably infinite set of individual variables, called ***person variables***. Atomic formulas of the form ***TR***(pe_1, pe_2) are read as 'person pe_1 trusts person pe_2.' Let {o_1, o_2, o_3, …} be a countably infinite set of individual variables, called ***organization variables***. Atomic formulas of the form ***TRpo***(pe_1, o_1) are read as 'person pe_1 trusts organization o_1.' Atomic formulas of the form ***TRop***(o_1, pe_1) are read as 'organization o_1 trusts person pe_1.' Atomic formulas of the form ***TRoo***(o_1, o_2) are read as 'organization o_1 trusts organization o_2.'

Primitive Predicate: { ***TR*** (trust, ***TR***(pe_1, pe_2) means 'pe_1 trusts pe_2'), ***B*** (belong to, ***B***(pe_1, o_1) means 'pe_1 belongs to o_1') }

Defined Predicates: ***NTR***(pe_1, pe_2) $=_{df}$ ¬(***TR***(pe_1, pe_2)) (***NTR***(pe_1, pe_2) means 'pe_1 does not trust pe_2'), ***TREO***(pe_1, pe_2) $=_{df}$ ***TR***(pe_1, pe_2)∧***TR***(pe_2, pe_1), (***TREO***(pe_1, pe_2)

means 'pe_1 and pe_2 trust each other'), $\boldsymbol{ITR}(pe_1, pe_2, pe_3) =_{df} \neg(\boldsymbol{TR}(pe_1, pe_2) \wedge \boldsymbol{TR}(pe_1, pe_3))$, ($\boldsymbol{ITR}(pe_1, pe_2, pe_3)$ means 'pe_1 does not trust both pe_2 and pe_3' (incompatibility)), $\boldsymbol{XTR}(pe_1, pe_2, pe_3) =_{df} (\boldsymbol{TR}(pe_1, pe_2) \vee \boldsymbol{TR}(pe_1, pe_3)) \wedge (\boldsymbol{NTR}(pe_1, pe_2) \vee \boldsymbol{NTR}(pe_1, pe_3))$, ($\boldsymbol{XTR}(pe_1, pe_2, pe_3)$ means 'pe_1 trusts either pe_2 or pe_3 but not both' (exclusive disjunction)), $\boldsymbol{JTR}(pe_1, pe_2, pe_3) =_{df} \neg(\boldsymbol{TR}(pe_1, pe_2) \vee \boldsymbol{TR}(pe_1, pe_3))$, ($\boldsymbol{JTR}(pe_1, pe_2, pe_3)$ means 'pe_1 trusts neither pe_2 nor pe_3'(joint denial)), $\boldsymbol{TTR}(pe_1, pe_2, pe_3) =_{df} (\boldsymbol{TR}(pe_1, pe_2) \wedge \boldsymbol{TR}(pe_2, pe_3)) \Rightarrow \boldsymbol{TR}(pe_1, pe_3)$, ($\boldsymbol{TTR}(pe_1, pe_2, pe_3)$ means 'pe_1 trusts pe_3, if pe_1 trusts pe_2 and pe_2 trusts pe_3'), $\boldsymbol{CTR}(pe_1, pe_2, pe_3) =_{df} \boldsymbol{TR}(pe_1, pe_3) \Rightarrow \boldsymbol{TR}(pe_2, pe_3)$, ($\boldsymbol{CTR}(pe_1, pe_2, pe_3)$ means 'pe_2 trusts pe_3, if pe_1 trusts pe_3'), $\boldsymbol{NCTR}(pe_1, pe_2, pe_3) =_{df} \neg \boldsymbol{TR}(pe_1, pe_3) \Rightarrow \boldsymbol{TR}(pe_2, pe_3)$, ($\boldsymbol{NCTR}(pe_1, pe_2, pe_3)$ means 'pe_2 trusts pe_3, if pe_1 does not trust pe_3'), $\boldsymbol{CNTR}(pe_1, pe_2, pe_3) =_{df} \neg \boldsymbol{TR}(pe_1, pe_3) \Rightarrow \neg \boldsymbol{TR}(pe_2, pe_3)$, ($\boldsymbol{CNTR}(pe_1, pe_2, pe_3)$ means 'pe_2 does not trust pe_3, if pe_1 does not trust pe_3'), $\boldsymbol{TRpo}(pe_1, o_1) =_{df} \forall pe_2(\boldsymbol{B}(pe_2, o_1) \wedge \boldsymbol{TR}(pe_1, pe_2))$, ($\boldsymbol{TRpo}(pe_1, o_1)$ means 'pe_1 trusts o_1'), $\boldsymbol{NTRpo}(pe_1, o_1) =_{df} \forall pe_2(\boldsymbol{B}(pe_2, o_1) \wedge \boldsymbol{NTR}(pe_1, pe_2))$, ($\boldsymbol{NTRpo}(pe_1, o_1)$ means 'pe_1 does not trust o_1', note that $\boldsymbol{NTRpo}(pe_1, o_1)$ is not a simple negation of $\boldsymbol{TRpo}(pe_1, o_1)$), $\boldsymbol{TRop}(o_1, pe_1) =_{df} \forall pe_2(\boldsymbol{B}(pe_2, o_1) \wedge \boldsymbol{TR}(pe_2, pe_1))$, ($\boldsymbol{TRop}(o_1, pe_1)$ means 'o_1 trusts pe_1'), $\boldsymbol{NTRop}(o_1, pe_1) =_{df} \forall pe_2(\boldsymbol{B}(pe_2, \mathrm{o}_1) \wedge \boldsymbol{NTR}(pe_2, pe_1))$, ($\boldsymbol{NTRop}(o_1, pe_1)$ means 'o_1 does not trust pe_1', note that $\boldsymbol{NTRop}(o_1, pe_1)$ is not a simple negation of $\boldsymbol{TRop}(o_1, pe_1)$), $\boldsymbol{TRoo}(o_1, o_2) =_{df} \forall pe_1 \forall pe_2(\boldsymbol{B}(pe_1, o_1) \wedge \boldsymbol{B}(pe_2, o_2) \wedge \boldsymbol{TR}(pe_1, pe_2))$, ($\boldsymbol{TRoo}(o_1, o_2)$ means 'o_1 trusts o_2'), $\boldsymbol{NTRoo}(o_1, o_2) =_{df} \forall pe_1 \forall pe_2(\boldsymbol{B}(pe_1, o_1) \wedge \boldsymbol{B}(pe_2, o_2) \wedge \boldsymbol{NTR}(pe_1, pe_2))$, ($\boldsymbol{NTRoo}(o_1, o_2)$ means 'o_1 does not trust o_2', note that $\boldsymbol{NTRoo}(o_1, o_2)$ is not a simple negation of $\boldsymbol{TRoo}(o_1, o_2)$), $\boldsymbol{TRpoEO}(pe_1, o_1) =_{df} \boldsymbol{TRpo}(pe_1, o_1) \wedge \boldsymbol{TRop}(o_1, pe_1)$, ($\boldsymbol{TRpoEO}(pe_1, o_1)$ means 'pe_1 and o_1 trust each other'), $\boldsymbol{TRooEO}(o_1, o_2) =_{df} \boldsymbol{TRoo}(o_1, o_2) \wedge \boldsymbol{TRoo}(o_2, o_1)$, ($\boldsymbol{TRooEO}(o_1, o_2)$ means 'o_1 and o_2 trust each other')

Axiom Schemata: TR1: $\neg(\forall pe_1 \forall pe_2(\boldsymbol{TR}(pe_1, pe_2) \Rightarrow \boldsymbol{TR}(pe_2, pe_1)))$, TR2: $\neg(\forall pe_1 \forall o_1(\boldsymbol{TRpo}(pe_1, o_1) \Rightarrow \boldsymbol{TRop}(o_1, pe_1)))$, TR3: $\neg(\forall pe_1 \forall o_1(\boldsymbol{TRop}(o_1, pe_1) \Rightarrow \boldsymbol{TRpo}(pe_1, o_1)))$, TR4: $\neg(\forall o_1 \forall o_2(\boldsymbol{TRoo}(o_1, o_2) \Rightarrow \boldsymbol{TRoo}(o_2, o_1)))$, TR5: $\neg(\forall pe_1 \forall pe_2 \forall pe_3((\boldsymbol{TR}(pe_1, pe_2) \wedge \boldsymbol{TR}(pe_2, pe_3)) \Rightarrow \boldsymbol{TR}(pe_1, pe_3)))$, TR6: $\neg(\forall pe_1 \forall pe_2 \forall o_1((\boldsymbol{TRpo}(pe_1, o_1) \wedge \boldsymbol{TRop}(o_1, pe_2)) \Rightarrow \boldsymbol{TR}(pe_1, pe_2)))$, TR7: $\neg(\forall pe_1 \forall pe_2 \forall o_1((\boldsymbol{TRop}(o_1, pe_1) \wedge \boldsymbol{TR}(pe_1, pe_2)) \Rightarrow \boldsymbol{TRop}(o_1, pe_2)))$, TR8: $\neg(\forall o_1 \forall o_2 \forall o_3((\boldsymbol{TRoo}(o_1, o_2) \wedge \boldsymbol{TRoo}(o_2, o_3)) \Rightarrow \boldsymbol{TRoo}(o_1, o_3)))$

We can obtain various reciprocal logics for specifying, verifying, and reasoning about trust relationships by adding axiom schemata about trust relationships into the various predicate strong relevant logics, predicate temporal relevant logics, spatial relevant logics, or spatial-temporal relevant logics, respectively. For examples: if we do not take the notions of space and time into account but just consider some "static" trust relationships, then we can use the following logics: **TrTcQ** $=_{df}$ **TcQ** + {TR1~TR8}, **TrEcQ** $=_{df}$ **EcQ** + {TR1~TR8}, **TrRcQ** $=_{df}$ **RcQ** + {TR1~TR8}. When we want to specify, verify, and reason about trust relationships themselves may change over space and time, we should use the following logics: $\mathbf{TrST_0TcQ} =_{df} \mathbf{ST_0TcQ}$ + {TR1~TR8}, $\mathbf{TrST_0EcQ} =_{df} \mathbf{ST_0EcQ}$ + {TR1~TR8}, $\mathbf{TrST_0RcQ} =_{df} \mathbf{ST_0RcQ}$ + {TR1~TR8}. While if minimal or weakest temporal relevant logics are not adequate, then those characteristic axioms such as T5~T12 that correspond to various assumptions about time can be added into $\mathbf{TrST_0TcQ}$, $\mathbf{TrST_0EcQ}$, and $\mathbf{TrST_0RcQ}$ respectively to obtain various stronger logics.

4 Concluding Remarks

We have proposed a new family of conservative extensions of relevant logic, named "reciprocal logic," for specifying, verifying, and reasoning about reciprocal relationships. We showed that various reciprocal logics can be obtained by introducing predicates and related axioms about reciprocal relationships into strong relevant logics and spatial-temporal relevant logics. Our case study is focused on trust relationships.

Because the strong relevance between the antecedent and the consequent of a conditional is intrinsically important to representing conditional reciprocal relationships, it is intrinsically important to construct various reciprocal logics based on strong relevant logics rather than traditional relevant logics as well as classical mathematical logic and its various classical conservative extensions.

On the other hand, the first three essential requirements for the fundamental logic mentioned in Section 1 are also essential to any applied logic for representing and reasoning about knowledge in a domain where there may be some incompleteness or inconsistency. Therefore, strong relevant logics can be considered as the universal basis of various applied logics for knowledge representation and reasoning.

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