A Fuzzy Multi-criteria Decision Making Model for the Selection of the Distribution Center

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Abstract. The location selection of distribution is one of the most important decision issues for logistic managers. In order to encompass vagueness in decision data, a new fuzzy multiple criteria decision-making method is proposed to solve the distribution center selection problem under fuzzy environment. In the proposed method, the ratings of alternatives and the weights of the criteria are given in terms of linguistic variables which is in turns represented by triangular fuzzy numbers.

1 Introduction

In terms of logistical system design and administration, distribution center is a common problem encountered by logistic managers. During the last decade, seeking reduced transportation cost in the increased economic scale of production has shifted the focus to the selection of distribution center. A distribution center links suppliers (source) and consumers (demand). A distribution center selection problem is homomorphic to a plant location selection problem. Factors such as investment cost, climate condition, labor force quality and quantity, transportation availability may be considered in the selection of the plant location [4,18,19,20,22]. These factors can be classified into objective factors and subjective factors. Many precision-based methods for location selection have been developed. Mathematical programming is usually utilized to determine the optimal location of facilities [1,7,11]. Tompkins and White [22] introduced a method which used the preference theory to assign weights to subjective factors by making all possible pairwise comparisons between factors. Spohrer and Kmak [18] proposed a weight factor analysis method to integrate the quantitative data and qualitative ratings to choose a suitable plant location from numerous alternatives. All the methods stated above are based on the concept of accurate measure and crisp evaluation.

In the selection of a best distribution center, the values for the qualitative criteria are often imprecise. The desired value and importance weight of criteria are usually described in linguistic terms such as "very low", "medium", "high", "fair", and "very high". A distribution center selection problem can modeled as a multiple criteria decision making (MCDA) problem. In traditional MCDM, performance rating and weights are measured in crisp numbers [10,12,21]. Under many circumstances where performance rating and weights can not be given

L. Wang, K. Chen, and Y.S. Ong (Eds.): ICNC 2005, LNCS 3612, pp. 1290–1299, 2005.

precisely, the fuzzy set theory is introduced to model the uncertainty of human judgements and such problems is known as fuzzy multiple criteria decision making (FMCDM). In FMCDM, performance ratings and weights are usually represented by fuzzy numbers. A FMCDM with m alternatives and n criteria can be modeled as follows:

$$D = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} & \dots & \tilde{A}_{1n} \\ \tilde{A}_{21} & \tilde{A}_{22} & \dots & \tilde{A}_{2n} \\ \tilde{A}_{m1} & \tilde{A}_{m2} & \dots & \tilde{A}_{mn} \end{bmatrix}$$

and

$$W = \begin{bmatrix} \tilde{W}_1 & \tilde{W}_2 & \dots & \tilde{W}_n \end{bmatrix}$$

where $\tilde{A}_{ij}$ is the fuzzy number representing the performance of ith alternative under jth criterion and $\tilde{W}_j$ is the fuzzy number representing the weight of jth criterion.

In dealing with fuzzy numbers, aggregation of fuzzy numbers and ranking fuzzy number are some of the important issues in group decision. Methods of aggregation such as OAM can be found in [14]. Many methods for fuzzy ranking have been proposed [2,3,5,6,8,9,13,17,23,24]. They can be classified into two categories. The first category is based on defuzzification. Various methods of defuzzification have been proposed. In the first category, fuzzy numbers are defuzzified into crisp numbers or the so-called utilities in some literatures. The ranking are then done based on these crisp numbers. Though it is easy to compute, the main drawback of this type is that defuzzification tends to loss some information and thus is unable to grasp the sense of uncertainty. The other category is based on fuzzy preference relation. The advantage of this type is that uncertainties of fuzzy numbers are kept during ranking process. However, the fuzzy preference relations proposed thus far are too complex to compute. Yuan [24] has proposed criteria for measuring ranking method. Lee [13] has proposed a new fuzzy ranking method based on fuzzy preference relation satisfying all criteria proposed by Yuan. In [15], we extended the definition of fuzzy preference relation [16] and propose an extended fuzzy preference relation which satisfies additivity and is easy to compute. In this paper, we are going to propose a new method for FMCDM for the selection of distribution center.

2 Mathematical Preliminaries

Definition 1. *The α-cut of fuzzy set A, A^α, is the crisp set $A^\alpha = \{x \mid \mu_A(x) \geq \alpha\}$. The support of A is the crisp set $Supp(A) = \{x \mid \mu_A(x) > 0\}$. A is normal iff $sup_{x \in U} \mu_A(x) = 1$, where U is the universe set.*

Definition 2. *A fuzzy subset A of real number R is convex iff*

$$\mu_A(\lambda x + (1 - \lambda)y) \geq (\mu_A(x) \wedge \mu_A(y)), \forall x, y \in R, \forall \lambda \in [0, 1],$$

where $\wedge$ denotes the minimum operator.

Definition 3. *A is a fuzzy number iff A is a normal and convex fuzzy subset of R.*

Definition 4. *A triangular fuzzy number A is a fuzzy number with piecewise linear membership function μ_A defined by*

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 0, & otherwise, \end{cases}$$

which can be denoted as a triplet (a_1, a_2, a_3).

Definition 5. *Let A and B be two fuzzy numbers. Let ∘ be a operation on real numbers, such as +, -, *, ∧, ∨, etc. By extension principle, the extended operation ∘ on fuzzy numbers can be defined by*

$$\mu_{A \circ B}(z) = \sup_{x,y:z=x \circ y} \{\mu_A(x) \wedge \mu_B(y)\}. \tag{1}$$

Definition 6. *Let A be a fuzzy number. Then A^L_α and A^U_α are defined as $A^L_\alpha = \inf_{\mu_A(z) \geq \alpha}(z)$ and $A^U_\alpha = \sup_{\mu_A(z) \geq \alpha}(z)$ respectively.*

Definition 7. *A fuzzy preference relation R is a fuzzy subset of $\Re \times \Re$ with membership function $\mu_R(A, B)$ representing the degree of preference of fuzzy number A over fuzzy number B.*

1. *R is reciprocal iff $\mu_R(A, B) = 1 - \mu_R(B, A)$ for all fuzzy numbers A and B.*
2. *R is transitive iff $\mu_R(A, B) \geq \frac{1}{2}$ and $\mu_R(B, C) \geq \frac{1}{2} \Rightarrow \mu_R(A, C) \geq \frac{1}{2}$ for all fuzzy numbers A, B and C.*
3. *R is a fuzzy total ordering iff R is both reciprocal and transitive.*

If fuzzy numbers are compared based on fuzzy preference relations, then A is said to be greater than B iff $\mu_R(A, B) > \frac{1}{2}$.

Definition 8. *An extended fuzzy preference relation R is an extended fuzzy subset of $\Re \times \Re$ with membership function $-\infty \leq \mu_R(A, B) \leq \infty$ representing the degree of preference of fuzzy number A over fuzzy number B.*

1. *R is reciprocal iff $\mu_R(A, B) = -\mu_R(B, A)$ for all fuzzy numbers A and B.*
2. *R is transitive iff $\mu_R(A, B) \geq 0$ and $\mu_R(B, C) \geq 0 \Rightarrow \mu_R(A, C) \geq 0$ for all fuzzy numbers A, B and C.*
3. *R is additive iff $\mu_R(A, C) = \mu_R(A, B) + \mu_R(B, C)$*
4. *R is a total ordering iff R is both reciprocal, transitive and additive.*

If fuzzy numbers are compared based on extended fuzzy preference relations, then A is said to be greater than B iff $\mu_R(A, B) > 0$.

Our extended fuzzy preference relation is defined as follows.

Definition 9. *For any fuzzy number A, B, extended fuzzy preference relation $F(A,B)$ is defined by the membership function*

$$\mu_F(A,B) = \int_0^1 ((A-B)^L_\alpha + (A-B)^U_\alpha)d\alpha \tag{2}$$

Lemma 1. *F is reciprocal, i.e.,*

$$\mu_F(B,A) = -\mu_F(A,B). \tag{3}$$

Proof: Since $(A-B)^L_\alpha + (A-B)^U_\alpha = -((B-A)^L_\alpha + (B-A)^U_\alpha)$, we have $\mu_F(B,A) = -\mu_F(A,B)$. □

Lemma 2. *F is additive, i.e.,*

$$\mu_F(A,B) + \mu_F(B,C) = \mu_F(A,C) \tag{4}$$

Proof:

$$\begin{aligned}
&\mu_F(A,B) + \mu_F(B,C) \\
&= \int_0^1 ((A-B)^L_\alpha + (A-B)^U_\alpha)d\alpha + \int_0^1 ((B-C)^L_\alpha + (B-C)^U_\alpha)d\alpha \\
&= \int_0^1 A^L_\alpha - B^U_\alpha + A^U_\alpha - B^L_\alpha + B^L_\alpha - C^U_\alpha + B^U_\alpha - C^L_\alpha d\alpha \\
&= \int_0^1 ((A-C)^L_\alpha + (A-C)^U_\alpha)d\alpha.
\end{aligned} \tag{5}$$

□

Lemma 3. *F is transitive, i.e.,*

$$\mu_F(A,B) \geq 0 \text{ and } \mu_F(B,C) \geq 0 \Rightarrow \mu_F(A,C) \geq 0. \tag{6}$$

Proof: By lemma 2, we have $\mu_F(A,C) = \mu_F(A,B) + \mu_F$ (B,C). Since $\mu_F(A,B), \mu_F(B,C) \geq 0$, we have $\mu_F(A,c) \geq 0$. □

Lemma 4. *Let $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. $\mu_F(A,B) \geq 0$ iff*

$$a_1 + 2a_2 + a_3 - b_1 - 2b_2 - b_3 \geq 0 \tag{7}$$

Proof: $\mu_F(A,B) \geq 0$ iff

$$\mu_F(A,B) = \int_0^1 (A-B)^L_\alpha + (A-B)^U_\alpha d\alpha = \frac{a_1 + 2a_2 + a_3 - b_1 - 2b_2 - b_3}{2} \geq 0. \tag{8}$$

□

Definition 10. *Let $\geq$ be a binary relation on fuzzy numbers defined by*

$$A \geq B \text{ iff } \mu_F(A,B) \geq 0. \tag{9}$$

Theorem 1. *$\geq$ is a total ordering relation.*

3 The Fuzzy Decision Making Method

To facilitate our method, define the preference function of one fuzzy number $\tilde{A}_{ij}$ over another number $\tilde{A}_{kj}$ as follows:

$$P(\tilde{A}_{ij}, \tilde{A}_{kj}) = \begin{cases} \mu_F(\tilde{A}_{ij}, \tilde{A}_{kj}) \text{ if } \mu_F(\tilde{A}_{ij}, \tilde{A}_{kj}) \geq 0 \\ 0 \qquad \text{otherwise} \end{cases}$$

Let J be the set of benefit criteria and J' be the set of cost criteria where

$$J = \{1 \leq j \leq n \text{ and } j \text{ belongs to benefit criteria}\}$$

$$J' = \{1 \leq j \leq n \text{ and } j \text{ belongs to cost criteria}\},$$

and

$$J \cup J' = \{1, \ldots, n\}.$$

The strength matrix $S = (S_{ij})$ is given by letting

$$S_{ij} = \begin{cases} \sum_{k \neq i} P(\tilde{A}_{ij}, \tilde{A}_{kj}) & \text{if } j \in J \\ \sum_{k \neq i} P(\tilde{A}_{kj}, \tilde{A}_{ij}) & \text{if } j \in J'. \end{cases} \tag{10}$$

Similarly, the weakness matrix $I = (I_{ij})$ is given by letting

$$I_{ij} = \begin{cases} \sum_{k \neq i} P(\tilde{A}_{kj}, \tilde{A}_{ij}) & \text{if } j \in J \\ \sum_{k \neq i} P(\tilde{A}_{ij}, \tilde{A}_{kj}) & \text{if } j \in J'. \end{cases} \tag{11}$$

The fuzzy weighted strength matrix $\tilde{S} = (\tilde{S}_i)$ can be obtained by

$$\tilde{S}_i = \sum_j S_{ij} \tilde{W}_j \tag{12}$$

and the fuzzy weighted weakness matrix $\tilde{I} = (\tilde{I}_i)$ can be obtained by

$$\tilde{I}_i = \sum_j I_{ij} \tilde{W}_j, \tag{13}$$

where $1 \leq i \leq m$. Now we are ready to present our method for FMCDM.

Step 1: Identify the criteria for the selection of distribution selection.

Step 2: Aggregate the fuzzy decision matrices and fuzzy weight matrices given by decision makers and normalized the group fuzzy decision matrix. Let $D = (\tilde{A}_{ij})$ be the normalized group fuzzy decision matrix and $W = (\tilde{W}_j)$ be the weight matrix.

Step 3: Calculate the strength matrix by (10).

Step 4: Calculate the weakness matrix by (11).

Step 5: Calculate the fuzzy weighted strength indices by (12).

Step 6: Calculate the fuzzy weighted weakness indices by (13).

Step 7: Derive the strength index S_i from the fuzzy weighted strength and weakness indices by

$$S_i = \sum_{k \neq i} P(\tilde{S}_i, \tilde{S}_k) + \sum_{k \neq i} P(\tilde{I}_k, \tilde{I}_i) \tag{14}$$

Step 8: Derive the weakness index I_I from the fuzzy weighted strength and weakness indices by

$$I_i = \sum_{k \neq i} P(\tilde{S}_k, \tilde{S}_i) + \sum_{k \neq i} P(\tilde{I}_i, \tilde{I}_k) \tag{15}$$

Step 9: Aggregate the strength and weakness indices into total performance indices by

$$t_i = \frac{S_i}{S_i + I_i} \tag{16}$$

Step 10: Rank alternatives by total performance indices t_i for $1 \leq i \leq m$.

4 Numerical Example

Suppose a company desires to select a suitable city for establishing a new distribution center. The evaluation is done by a committee of three decision-makers D_1, D_2, and D_3. After preliminary screening, there are three alternatives A_1, A_2, and A_3 under further evaluation. Assume the linguistic variables employed for weights and ratings are respectively shown in Table 1. The evaluation committee then undergoes the proposed evaluation procedure:

Step 1: Five selection criteria are identified:
(1) investment cost (C_1),
(2) expansion possibility (C_2),
(3) availability of acquirement material (C_3),
(4) human resource (C_4),
(5) closeness to demand market (C_5).

Table 1. Linguistic variables for the importance weights of criteria and the ratings

Importance weights of criteria		Linguistic variables for the ratings	
Very low (VL)	(0,0,0.1)	Very poor(VP)	(0,0,1)
Low (L)	(0,0.1,0.3)	Poor (P)	(0,1,3)
Medium low (ML)	(0.1,0.3,0.5)	Medium poor(ML)	(1,3,5)
Medium (M)	(0.3,0.5,0.7)	Faire (F)	(3,5,7)
Medium high (MH)	(0.5,0.7,0.9)	Medium good (MG)	(5,7,9)
High (H)	(0.7,0.9,1.0)	Good (G)	(7,9,10)
Very high (VH)	(0.9,1.0,1.0)	Very good (VG) (9,10,10)	

Table 2. The importance weights of the criteria

	D_1	D_2	D_3
C_1	H	VH	VH
C_2	H	H	H
C_3	MH	H	MH
C_4	MH	MH	MH
C_5	H	H	H

Table 3. The fuzzy weights of the criteria

	C_1	C_2	C_3	C_4	C_5
Weight	(0.83,0.97,1)	(0.7,0.9,1)	(0.57,0.77,0.93)	(0.5,0.7,0.9)	(0.7.0.9,1)

Table 4. The ratings of alternatives given by decision makers

Criteria	Alternatives	D_1	D_2	D_3
C_1	A_1	6×10^6	8×10^6	7×10^6
	A_2	3×10^6	4×10^6	5×10^6
	A_3	4×10^6	5×10^6	6×10^6
C_2	A_1	G	VG	F
	A_2	VG	VG	VG
	A_1	MG	G	VG
C_3	A_1	F	G	G
	A_2	G	G	G
	A_1	G	MG	VG
C_4	A_1	VG	G	G
	A_2	G	G	G
	A_1	G	VG	VG
C_5	A_1	F	F	F
	A_2	G	F	G
	A_1	G	G	G

Table 5. The group fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	7×10^6	(6.3,8,9)	(5.7,7.7,9)	(7.7,9.3,10)	(3,5,7)
A_2	4×10^6	(9,10,10)	(7,9,10)	(7,9,10)	(5.7,7.7,9)
A_3	5×10^6	(7,9,10)	(7,9,10)	(8.3,9.7,10)	(7,9,10)

The benefit criteria are C_2, C_3, C_4, and C_5. The cost criterion is C_1. The weights of the criteria are shown in Table 3.

Step 2: The ratings of alternatives given three decision makers are shown in Table 4. The group fuzzy decision matrix is obtained by averaging the ratings of three decision makers and is shown in Table 5. The group fuzzy decision matrix is normalized by dividing ratings with the largest value in the support

Table 6. The normalized group fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(1,1,1)	(0.62,0.8,0.9)	(0.57,0.77,0.9)	(0.77,0.93,1)	(0.3,0.5,0.7)
A_2	(0.57,0.57,0.57)	(0.9,1,1)	(0.7.0.9,1)	(0.7,0.9,1)	(0.57,0.77,0.9)
A_3	(0.71,0.71,0.71)	(0.7,0.87,0.97)	(0.7,0.87,0.97)	(0.83,0.97,1)	(0.7,0.9,1)

Table 7. The strength and weakness matrices

strength						weakness					
	C_1	C_2	C_3	C_4	C_5		C_1	C_2	C_3	C_4	C_5
A_1	0	0	0	0.067	0	A_1	1.43	0.52	0.45	0.067	1.25
A_2	1.14	0.63	0.3	0	0.5	A_2	0	0	0	0.2	0.25
A_3	0.57	0.13	0.2	0.2	1	A_3	0.29	0.25	0.05	0	0

Table 8. The fuzzy weighted strength indices and weakness indices of alternatives

	fuzzy weighted strength index		fuzzy weighted weakness index
A_1	(0.033,0.278,0.339)	A_1	(2.712,3.369,3.674)
A_2	(1.913,2.283,2.462)	A_2	(0.275,0.365,0.43)
A_3	(1.482,1.766,1.947)	A_3	(0.441,0.541,0.582)

Table 9. The strength and weakness indices of alternatives

	strength index		weakness index
A_1	0	A_1	18.376
A_2	11.176	A_2	0
A_3	8.526	A_3	1.325

of the fuzzy numbers in the same criterion. The normalized group fuzzy decision matrix is shown in Table 6.

Step 3: The strength matrix derived by (10) is shown in Table 7.

Step 4: The weakness matrix derived by (11) is shown in Table 7.

Step 5: The fuzzy weighted strength indices of alternatives derived by (12) are shown in Table 8.

Step 6: The fuzzy weighted weakness indices of alternatives derived by (13) are shown in Table 8.

Step 7: The strength indices of alternatives derived by (14) are shown in Table 9.

Step 8: The weakness indices of alternatives derived by (15) are shown in Table 9.

Step 9: The total performance indices aggregated by (16) are $A1 : 0$,$A2 : 1$, and $A3 : 0.866$.

Step 10: The rank of alternatives by total performance indices are $A1 : 3$, $A2 : 1$, and $A3 : 2$. Alternative 2 is the best distribution center.

5 Conclusions

In this paper, we have proposed a new fuzzy multiple criteria decision making (FMCDM) method for the problem of selecting distribution center under fuzzy environment. Our method enables decision makers to assess alternatives with linguistic variables so that vagueness can be encompassed in the assessment of distribution centers. Our method provides the strength index and the weakness index beside the overall performance index so that decision makers can assess distribution centers from different perspectives.

Acknowledgement

This research work was partially supported by the National Science Council of the Republic of China under grant No. NSC93-2416-H-019-004-.

References

1. C.H. Aikens, Facility location models for distribution planning, *European J. Poer. Res.* 22 (1985) 263-279.
2. S.M. Baas and H. Kwakernaak, Rating and ranking of multiple-aspect alternatives using fuzzy sets, *Automatica* 13 (1977) 47-58.
3. G. Bortolan and R. Degani, A review of some methods for ranking fuzzy subsets, *Fuzzy Sets and Systems* 15 (1985) 1-19.
4. D.J. Bowersox, D.J. Closs, *Logistical Management - The Integrated Supply Chain Process,* (McGraw-Hill, Singapore, 1996).
5. W. Chang, Ranking of fuzzy utilities with triangular membership functions, *Proc. Internat. Conf. on Policy Analysis and Information Systems* (1981) 263-272.
6. S. Chen, Ranking fuzzy numbers with maximizing set and minimizing set, *Fuzzy Sets and Systems* 17 (1985) 113-129.
7. M.D. Dahlberg, J.H. May, Linear programming for sitting of energy facilities, *J. Energy Eng.* (1908) 5-14.
8. M. Delgado, J.L. Verdegay and M.A. Vila, A procedure for ranking fuzzy numbers using fuzzy relations, *Fuzzy Sets and Systems* 26 (1988) 49-62.
9. D. Dubois and H. Prade, Ranking fuzzy numbers in the setting of possibility theory, *Inform. Sci.* 30 (1983) 183-224.
10. J.S. Dyer, P.C. Fishburn, R.E. Steuer, J. Wallenius, S. Zionts, Multiple criteria decision making, Multiattribute utility theory: The next ten years, Management Sci. 38 (5) (1992) 645-654.
11. J.E. Hodder, M.C. Dincer, A multifactor model for international plant location and financing under uncertainty, *Comput. & Oper. Res.* 13(5) (1986) 601-609.
12. C.L. Hwang, K. Yoon, Multiple Attributes Decision Making Methods and Applications, Springer, Berlin Heidelberg, 1981.

13. H.-S. Lee, A new fuzzy ranking method based on fuzzy preference relation, *2000 IEEE International Conference on Systems, Man And Cybernetics* (2001) 3416-3420.
14. H.-S. Lee, Optimal consensus of fuzzy opinions under group decision making environment, *Fuzzy Sets and Systems* 132 (2002) 303-315.
15. H.-S. Lee, An Extended Fuzzy Preference Relation for Comparison of Fuzzy Numbers, The 6h World Multi-Conference on Systemics, Cybernetics and Informatics, July 14-18, 2002, Orlando, USA, XI 76-79.
16. H.-S. Lee, On fuzzy preference relation in group decision making, *International Journal of Computer Mathematics* 82(2) (2005) 133-140.
17. K. Nakamura, Preference relations on a set of fuzzy utilities as a basis for decison making, *Fuzzy Sets and Systems 20*, 147-162 (1986).
18. G.A. Spohrer, T.R. Kmak, Qualitative analysis used in evaluating alternative plant location scenarios, *Indust. Eng.* (August 1984) 52-56.
19. W.J. Stevenson, *Production/Operations Management,* (Richard D. Irwin Inc., Illinois, 1993).
20. D.R. Sule, *Manufacturing Facility; Location, Planning and Design,* (PWS Publishing, Boston, 1994).
21. J. Teghem, Jr., C. Delhaye, P.L. Kunsch, An interactive decision support system (IDSS) for multicriteria decision aid, Math. Comput. Modeling 12:10/11 (1989) 1311-1320.
22. J.A. Tompkins, J.A. White, *Facilities Planning,* (Wiley, New York, 1984).
23. R.R. Yager, A procedure for ordering fuzzy subsets of the unit interval, *Inform. Sci.* 24 (1981) 143-161.
24. Y. Yuan, Criteria for evaluating fuzzy ranking methods, *Fuzzy Sets and Systems 44*, 139-157 (1991).