A Study on Relationship Between Fuzzy Rough Approximation Operators and Fuzzy Topological Spaces

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Abstract. It is proved that a pair of dual fuzzy rough approximation operators can induce a topological space if and only if the fuzzy relation is reflexive and transitive. The sufficient and necessary condition that a fuzzy interior (closure) operator derived from a fuzzy topological space can associate with a fuzzy reflexive and transitive relation such that the induced fuzzy lower (upper) approximation operator is the fuzzy interior (closure) operator is also examined.

1 Introduction

The theory of rough set, proposed by Pawlak [1], is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. The notion of an approximation space consisting of a universe of discourse and an indiscernible relation imposed on it is one of the fundamental concept of rough set theory. Based on the approximation space, the primitive notion of lower and upper approximation operators can be induced. From both theoretic and practical needs, many authors have generalized the concept of approximation operators by using nonequivalent binary relations [2-6], neighborhood systems [7, 8], or by using axiomatic approaches [9-11]. More general frameworks have been obtained under fuzzy environment which involve the rough approximations of fuzzy sets (rough fuzzy sets), the fuzzy approximations of sets (fuzzy rough sets) [12-23]. Extensive research has also been carried out to compare the theory of rough sets with other theories of uncertainty, such as modal logic [18, 24, 25], conditional events [26], and Dempster-Shafer theory of evidence [5, 6, 27, 28]. On the other hand, the relationships between rough sets and topological spaces were studied by many authors [29-32]. The relationships between crisp rough sets and crisp topological spaces were studied in detail. The relationship between fuzzy rough sets and fuzzy topological spaces was investigated by Boixader et al [12], but their studies were restricted to fuzzy T-rough sets defined by fuzzy T-similarity relations which were equivalence crisp relations when they degenerated into crisp ones. In this paper, we focus mainly on the study of the relationship between fuzzy rough approximation operators and fuzzy topological spaces in general case.

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2 Fuzzy Rough Approximation Operators

Let X be a nonempty set called the universe of discourse. The class of all subsets (respectively, fuzzy subsets) of X will be denoted by $\mathcal{P}(X)$ (respectively, by $\mathcal{F}(X)$). For any $A \in \mathcal{F}(X)$, the complement of A is denoted by $\sim A$.

Definition 1. *Let R be a fuzzy binary relation on U, i.e. $R \in \mathcal{F}(U \times U)$. R is referred to as a serial fuzzy relation if $\forall x \in U$, $\bigvee_{y \in U} R(x,y) = 1$; R is referred to as a reflexive fuzzy relation if $R(x,x) = 1$ for all $x \in U$; R is referred to as a transitive fuzzy relation if $R(x,z) \geq \bigvee_{y \in U} (R(x,y) \wedge R(y,z))$ for all $x, z \in U$.*

Definition 2. *If R is a fuzzy relation on U, then the pair (U, R) is referred to as a fuzzy approximation space. Let $A \in \mathcal{F}(U)$, the lower and upper approximations of A, $\underline{R}(A)$ and $\overline{R}(A)$, with respect to the fuzzy approximation space (U, R) are fuzzy sets of U whose membership functions, for each $x \in U$, are defined, respectively, by*

$$\underline{R}(A)(x) = \bigwedge_{y \in U} [(1 - R(x,y)) \vee A(y)],$$
$$\overline{R}(A)(x) = \bigvee_{y \in U} [R(x,y) \wedge A(y)].$$

The pair $(\underline{R}(A), \overline{R}(A))$ is referred to as a fuzzy rough set, and $\underline{R}, \overline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ are referred to as lower and upper fuzzy rough approximation operators, respectively.

Lemma 3 ([20]). *The lower and upper fuzzy rough approximation operators, $\underline{R}$ and $\overline{R}$, satisfy the properties: $\forall A, B \in \mathcal{F}(U)$, $\forall A_j \in \mathcal{F}(U)(\forall j \in J)$, $\forall \alpha \in I = [0,1]$,*

$$\begin{array}{ll}
\text{(FL1) } \underline{R}(A) = \sim \overline{R}(\sim A), & \text{(FU1) } \overline{R}(A) = \sim \underline{R}(\sim A); \\
\text{(FL2) } \underline{R}(A \cup \widehat{\alpha}) = \underline{R}(A) \cup \widehat{\alpha}, & \text{(FU2) } \overline{R}(A \cap \widehat{\alpha}) = \overline{R}(A) \cap \widehat{\alpha}; \\
\text{(FL3) } \underline{R}(\bigcap_{j \in J} A_j) = \bigcap_{j \in J} \underline{R}(A_j), & \text{(FU3) } \overline{R}(\bigcup_{j \in J} A_j) = \bigcup_{j \in J} \overline{R}(A_j),
\end{array}$$

where $\widehat{\alpha}$ is the constant fuzzy set, i.e. $\widehat{\alpha}(x) = a, \forall x \in U$.

Properties (FL1) and (FU1) show that the fuzzy rough approximation operators $\underline{R}$ and $\overline{R}$ are dual each other. Properties with the same number may be regarded as dual properties. It can be checked that

$$\begin{array}{ll}
\text{(FL4) } A \subseteq B \Longrightarrow \underline{R}(A) \subseteq \underline{R}(B), & \text{(FU4) } A \subseteq B \Longrightarrow \overline{R}(A) \subseteq \overline{R}(B); \\
\text{(FL5) } \underline{R}(\bigcup_{j \in J} A_j) \supseteq \bigcup_{j \in J} \underline{R}(A_j), & \text{(FU5) } \overline{R}(\bigcap_{j \in J} A_j) \subseteq \bigcap_{j \in J} \overline{R}(A_j).
\end{array}$$

Properties (FL2) and (FU2) imply the following properties (FL2)′ and (FU2)′:

$$\text{(FL2)}' \quad \underline{R}(U) = U, \qquad \text{(FU2)}' \quad \overline{R}(\emptyset) = \emptyset.$$

Definition 4. *Let $L, H : \mathcal{F}(U) \to \mathcal{F}(U)$ be two operators. They are referred to as dual operators if for all $A \in \mathcal{F}(U)$:*

$$\text{(Fl1)} \quad L(A) = \sim H(\sim A), \qquad \text{(Fu1)} \quad H(A) = \sim L(\sim A).$$

Rough set approximation operators can also be characterized by axioms. In the axiomatic approach, rough sets are axiomatized by abstract operators. For the case of fuzzy rough sets, the primitive notion is a system $(\mathcal{F}(U), \cap, \cup, \sim, L, H)$, where $L, H : \mathcal{F}(U) \to \mathcal{F}(U)$ are operators from $\mathcal{F}(U)$ to $\mathcal{F}(U)$.

Lemma 5 ([20]). *Let $L, H : \mathcal{F}(U) \to \mathcal{F}(U)$ be two operators. Then there exists a fuzzy relation R on U such that for all $A \in \mathcal{F}(U)$*

$$L(A) = \underline{R}(A), \quad \text{and} \quad H(A) = \overline{R}(A)$$

if and only if (iff) L and H satisfy the following axioms: $\forall A, B \in \mathcal{F}(U)$, $\forall A_j \in \mathcal{F}(U) (\forall j \in J)$, and $\forall \alpha \in I$

$$\begin{array}{ll} \text{(Fl1)}\ L(A) = \sim H(\sim A), & \text{(Fu1)}\ H(A) = \sim L(\sim A); \\ \text{(Fl2)}\ L(A \cup \widehat{\alpha}) = L(A) \cup \widehat{\alpha}, & \text{(Fu2)}\ H(A \cap \widehat{\alpha}) = H(A) \cap \widehat{\alpha}; \\ \text{(Fl3)}\ L(\bigcap_{j \in J} A_j) = \bigcap_{j \in J} L(A_j), & \text{(Fu3)}\ H(\bigcup_{j \in J} A_j) = \bigcup_{j \in J} H(A_j). \end{array}$$

Definition 6. *Let $L, H : \mathcal{F}(U) \to \mathcal{F}(U)$ be a pair of dual operators. If L satisfies axioms (Fl2) and (Fl3) or equivalently H satisfies axioms (Fu2) and (Fu3), then the system $(\mathcal{F}(U), \cap, \cup, \sim, L, H)$ is referred to as a fuzzy rough set algebra, and L and H are referred to as lower and upper fuzzy approximation operators respectively.*

Lemma 7 ([20]). *Assume that $L, H : \mathcal{F}(U) \to \mathcal{F}(U)$ is a pair of dual fuzzy approximation operators, i.e., L satisfies axioms* (Fl1), (Fl2) *and* (Fl3)*, and H satisfies* (Fu1), (Fu2) *and* (Fu3)*. Then there exists a serial fuzzy relation R on U such that $L(A) = \underline{R}(A)$ and $H(A) = \overline{R}(A)$ for all $A \in \mathcal{F}(U)$ iff L and H satisfy axioms:*

$$\begin{array}{ll} \text{(Fl0)} & L(\widehat{\alpha}) = \widehat{\alpha}, \quad \forall \alpha \in I; \\ \text{(Fu0)} & H(\widehat{\alpha}) = \widehat{\alpha}, \quad \forall \alpha \in I. \end{array}$$

Lemma 8 ([20]). *Assume that $L, H : \mathcal{F}(U) \to \mathcal{F}(U)$ is a pair of dual fuzzy approximation operators. Then there exists a reflexive fuzzy relation R on U such that $L(A) = \underline{R}(A)$ and $H(A) = \overline{R}(A)$ for all $A \in \mathcal{F}(U)$ iff L and H satisfy axioms:*

$$\begin{array}{ll} \text{(Fl6)} & L(A) \subseteq A, \quad \forall A \in \mathcal{F}(U); \\ \text{(Fu6)} & A \subseteq H(A), \quad \forall A \in \mathcal{F}(U). \end{array}$$

Lemma 9 ([20]). *Assume that $L, H : \mathcal{F}(U) \to \mathcal{F}(U)$ is a pair of dual fuzzy approximation operators. Then there exists a transitive fuzzy relation R on U such that $L(A) = \underline{R}(A)$ and $H(A) = \overline{R}(A)$ for all $A \in \mathcal{F}(U)$ iff L and H satisfy axioms:*

$$\begin{array}{ll} \text{(Fl7)} & L(A) \subseteq L(L(A)), \quad \forall A \in \mathcal{F}(U); \\ \text{(Fu7)} & H(H(A)) \subseteq H(A), \forall A \in \mathcal{F}(U). \end{array}$$

3 Fuzzy Topological Spaces and Fuzzy Rough Approximation Operators

Definition 10 ([33]). *A subset τ of $\mathcal{F}(U)$ is referred to as a fuzzy topology on U iff it satisfies*

(1) *If $\mathcal{A} \subseteq \tau$, then $\bigcup_{A \in \mathcal{A}} A \in \tau$,*
(2) *If $A, B \in \tau$, then $A \cap B \in \tau$,*
(3) *If $\widehat{\alpha} \in \mathcal{F}(U)$ is a constant fuzzy set, then $\widehat{\alpha} \in \tau$.*

Definition 11 ([33]). *A map $\Psi : \mathcal{F}(U) \to \mathcal{F}(U)$ is referred to as a fuzzy interior operator iff for all $A, B \in \mathcal{F}(U)$ it satisfies:*

(1) $\Psi(A) \subseteq A$,
(2) $\Psi(A \cap B) = \Psi(A) \cap \Psi(B)$,
(3) $\Psi^2(A) = \Psi(A)$,
(4) $\Psi(\widehat{\alpha}) = \widehat{\alpha}, \quad \forall \alpha \in I$.

Definition 12 ([33]). *A map $\Phi : \mathcal{F}(U) \to \mathcal{F}(U)$ is referred to as a fuzzy closure operator iff for all $A, B \in \mathcal{F}(U)$ it satisfies:*

(1) $A \subseteq \Phi(A)$,
(2) $\Phi(A \cup B) = \Phi(A) \cup \Phi(B)$,
(3) $\Phi^2(A) = \Phi(A)$,
(4) $\Phi(\widehat{\alpha}) = \widehat{\alpha}, \quad \forall \alpha \in I$.

The elements of a fuzzy topology τ are referred to as open fuzzy sets, and it is easy to show that a fuzzy interior operator Ψ defines a fuzzy topology $\tau_{\Psi} = \{A \in \mathcal{F}(U) : \Psi(A) = A\}$. So, the open fuzzy sets are the fixed points of Ψ.

Theorem 13. *Assume that R is a fuzzy relation on U. Then the operator $\Phi = \overline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ is a fuzzy closure operator iff R is a reflexive and transitive fuzzy relation.*

Proof. " $\Rightarrow$ " Assume that $\Phi = \overline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ is a fuzzy closure operator. Since $A \subseteq \Phi(A), \forall A \in \mathcal{F}(U)$, by Lemma 8 we know that R is a reflexive fuzzy relation. Since $\Phi^2(A) = \Phi(A)$, i.e. $\overline{R}(\overline{R}(A)) = \overline{R}(A), \forall A \in \mathcal{F}(U)$, by using the reflexivity of R we must have

$$\overline{R}(\overline{R}(A)) \subseteq \overline{R}(A), \quad \forall A \in \mathcal{F}(U).$$

Hence by Lemma 9 we conclude that R is a transitive fuzzy relation.

" $\Leftarrow$ " If R is a reflexive and transitive fuzzy relation, by Lemma 8 we have

$$A \subseteq \Phi(A), \forall A \in \mathcal{F}(U).$$

It should be noted that the reflexivity of R implies that R is serial, it follows from Lemma 7 that

$$\Phi(\widehat{\alpha}) = \widehat{\alpha}, \quad \forall \alpha \in I.$$

Since R is transitive, by Lemma 9 we have that

$$\Phi^2(A) \subseteq \Phi(A), \quad \forall A \in \mathcal{F}(U).$$

On the other hand, by using the reflexivity it is easy to see that

$$\Phi^2(A) \supseteq \Phi(A), \quad \forall A \in \mathcal{F}(U).$$

Hence

$$\Phi^2(A) = \Phi(A), \quad \forall A \in \mathcal{F}(U).$$

From Lemma 3 we have that

$$\Phi(A \cup B) = \Phi(A) \cup \Phi(B).$$

Thus we have proved that $\Phi = \overline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ is a fuzzy closure operator.

Theorem 14. *Assume that R is a fuzzy relation on U. Then the operator $\Psi = \underline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ is a fuzzy interior operator iff R is a reflexive and transitive fuzzy relation.*

Proof. It is similar to the proof of Theorem 13.

Theorem 15. *Assume that R is a reflexive and transitive fuzzy relation on U. Then there exists a fuzzy topology τ_R on U such that $\Psi = \underline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ and $\Phi = \overline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ are the fuzzy interior and closure operators respectively.*

Proof. Suppose that R is a reflexive and transitive fuzzy relation on U. Then by Theorem 13 and Theorem 14 we conclude that $\Psi = \underline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ and $\Phi = \overline{R} : \mathcal{F}(U) \to \mathcal{F}(U)$ are the fuzzy interior and closure operators respectively. Define

$$\tau_R = \{A \in \mathcal{F}(U) : \Psi(A) = \underline{R}(A) = A\}.$$

It can easily be checked that τ_R is a fuzzy topology on U. Evidently, Ψ and Φ are respectively the interior and closure operators induced by τ_R.

Theorem 16. *Let $\Phi : \mathcal{F}(U) \to \mathcal{F}(U)$ be a fuzzy closure operator, then there exists a reflexive and transitive fuzzy relation on U such that $\overline{R}(A) = \Phi(A)$ for all $A \in \mathcal{F}(U)$ iff Φ satisfies the following two conditions*

(1) $\Phi(\bigcup_{j \in J} A_j) = \bigcup_{j \in J} \Phi(A_j), \quad A_j \in \mathcal{F}(U),\ j \in J,$

(2) $\Phi(A \cap \widehat{\alpha}) = \Phi(A) \cap \widehat{\alpha}, \quad \forall A \in \mathcal{F}(U), \forall \alpha \in I.$

Proof. Let $\Phi : \mathcal{F}(U) \to \mathcal{F}(U)$ be a fuzzy closure operator. If their exists a reflexive and transitive fuzzy relation on U such that $\overline{R}(A) = \Phi(A)$ for all $A \in \mathcal{F}(U)$, then by Lemma 3 we know that conditions (1) and (2) hold. Conversely, if Φ satisfies the conditions (1) and (2), then, by Lemmas 5, 8, 9, and Theorem 13, there exists a reflexive and transitive fuzzy relation on U such that $\overline{R}(A) = \Phi(A)$ for all $A \in \mathcal{F}(U)$.

Theorem 17. *Let $\Psi : \mathcal{F}(U) \to \mathcal{F}(U)$ be a fuzzy interior operator, then there exists a reflexive and transitive fuzzy relation on U such that $\underline{R}(A) = \Psi(A)$ for all $A \in \mathcal{F}(U)$ iff Ψ satisfies the following two conditions*

(1) $\Psi(\bigcap_{j\in J} A_j) = \bigcap_{j\in J} \Psi(A_j), \quad A_j \in \mathcal{F}(U),\ j \in J,$
(2) $\Psi(A \cup \widehat{\alpha}) = \Psi(A) \cup \widehat{\alpha}, \quad \forall A \in \mathcal{F}(U), \forall \alpha \in I.$

Proof. Let $\Psi : \mathcal{F}(U) \to \mathcal{F}(U)$ be a fuzzy interior operator. If their exists a reflexive and transitive fuzzy relation on U such that $\underline{R}(A) = \Psi(A)$ for all $A \in \mathcal{F}(U)$, then by Lemma 3 we know that conditions (1) and (2) hold. Conversely, if Ψ satisfies the conditions (1) and (2), then, by Lemmas 5, 8, 9, and Theorem 14, there exists a reflexive and transitive fuzzy relation on U such that $\underline{R}(A) = \Psi(A)$ for all $A \in \mathcal{F}(U)$.

Remark. It should be pointed out that if U is a finite universe of discourse, then properties (FL3) and (FU3) in Lemma 3 can equivalently be replaced by the following properties (FL3)′ and (FU3)′ respectively [21, 22]

(FL3)′ $\underline{R}(A \bigcap B) = \underline{R}(A) \bigcap \underline{R}(B), \forall A, B \in \mathcal{F}(U),$
(FU3)′ $\overline{R}(A \bigcup B) = \overline{R}(A) \bigcup \overline{R}(B), \forall A, B \in \mathcal{F}(U),$

thus the condition (1) in Theorems 16 and 17 can be omitted.

4 Conclusion

We have proved that a pair of dual fuzzy rough approximation operators can induce a topological space if and only if the fuzzy relation is reflexive and transitive. On the other hand, under certain conditions a fuzzy interior (closure) operator derived from a fuzzy topological space can associate with a reflexive and transitive fuzzy relation such that the induced fuzzy lower (upper) approximation operator is the fuzzy interior (closure) operator. In this paper, we only consider the fuzzy rough sets constructed by the triangle normal $T = \min$, we will generalize the research to the $(\mathcal{I}, \mathcal{T})$-fuzzy rough sets. The analysis will facilitate further research in uncertain reasoning under fuzziness.

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