Intelligent Digital Control for Nonlinear Systems with Multirate Sampling

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Abstract. This paper studies an intelligent digital control for nonlinear systems with multirate sampling. It is worth noting that the multirate control design is addressed for a given nonlinear system represented by Takagi–Sugeno (T–S) fuzzy models. The main features of the proposed method are that it is provided that the sufficient conditions for stabilization of the discrete-time T–S fuzzy system derived by the fast discretization method in the sense of Lyapunov stability criterion, which is can be formulated in the linear matrix inequalities (LMIs).

1 Introduction

Drawing upon recent progress in the Takagi–Sugeno (T–S) fuzzy-model-based digital control, it is observed that a number of important works have used a singlerate controller [1, 2, 4, 3, 9] to meet the stability requirements. The digital control problem was conducted as a stabilizing the discretized model of continuous-time T–S fuzzy plant in [1, 2, 3] and a stabilizing the jumped fuzzy system in [9]. However, their discretized model has the approximation error, which is directly proportional to the sampling time. One gets better exact discretized model if one can A/D and D/A conversions faster. But faster A/D and D/A conversions mean higher cost in implementation. In addition, the digital control system is hybrid system involving continuous-time and discrete-time, but their discussion in [1, 2, 3] only contained the stability of the digital control system in the discrete-time domain. A multirate control approach [10, 13, 11, 12] can be an alternative. Interestingly, advantages of applying faster A/D and D/A conversions are obtained by using A/D and D/A at different rates. Furthermore, in [14], the stability of the closed-loop digital system is well guaranteed for sufficiently fast sampling rate if the closed-loop discrete-time nonlinear system.

Motivated by the above observations, we develop an intelligent multirate control for a class of nonlinear systems under the high speed D/A converter. The main contribution of this paper is that we derive some sufficient conditions in terms of the linear matrix inequalities (LMIs), such that the equilibrium point is a globally asymptotically stable equilibrium point of the discrete-time fuzzy model derived by the fast discretization in the sense of Lyapunov stability criterion.

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2 Problem Statement

In the following, let T and T' be the sampling period and the control update period, respectively. For convenience, we take $T' = \frac{T}{N}$ for a positive integer N, where N is an input multiplicity. Then, $t = kT + lT'$ for $k \in \mathbb{Z}_{\geqslant 0}$ and $l \in \mathbb{Z}_{[0,N-1]}$, where the indexes k and l indicate sampling and control update instants, respectively.

Consider a nonlinear digital control system described by

$$\dot{x}(t) = f(x(t), u_d(t)) \tag{1}$$

for $t \in [kT + lT', kT + lT' + T')$, $(k,l) \in \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{[0,N-1]}$, where $x(t) \in \mathbb{R}^n$ is the state vector, and $u_d(t) = u_d(kT, lT') \in \mathbb{R}^m$ is the multirate digital control input. The control actions are switched with T' and N. Moreover, the digital control signals are fed into the plant with the ideal zero-order hold.

To facilitate the control design, we will develop a simplified model, which can represent the local linear input–output relations of the nonlinear system. This type of models is referred as T–S fuzzy models. The fuzzy dynamical model corresponding to (1) is described by the following IF–THEN rules [1, 2, 4, 3, 5, 6, 7, 8]:

$$\begin{aligned} R_i : \text{IF } z_1(t) \text{ is about } \mathit{\Gamma}_{i1} \text{ and } \cdots \text{ and } z_p(t) \text{ is about } \mathit{\Gamma}_{ip}, \\ \text{THEN } \dot{x}(t) = A_i x(t) + B_i u_d(t) \end{aligned} \tag{2}$$

where $R_i, i \in \mathcal{I}_q = \{1, 2, \ldots, q\}$, is the ith fuzzy rule, $z_h(t), h \in \mathcal{I}_p = \{1, 2, \ldots, p\}$, is the hth premise variable, and $\mathit{\Gamma}_{ih}, (i,h) \in \mathcal{I}_q \times \mathcal{I}_p$, is the fuzzy set. Then, given a pair $(x(t), u_d(t))$, using the center-average defuzzification, product inference, and singleton fuzzifier, the overall dynamics of (2) has the form

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u_d(t) \tag{3}$$

where $A(\theta(t)) = \sum_{i=1}^{q} \theta_i(z(t))A_i$, $B(\theta(t)) = \sum_{i=1}^{q} \theta_i(z(t))B_i$, $\theta_i(z(t)) = w_i(z)/\sum_{i=1}^{q} w_i(z)$, $w_i(z) = \prod_{h=1}^{p} \mathit{\Gamma}_{ih}(z_h(t))$, and $\mathit{\Gamma}_{ih}(z_h(t))$ is the grade of membership of $z_h(t)$ in $\mathit{\Gamma}_{ih}$.

3 Main Results

To develop the discretized version of (3), we apply the fast discretization technique [11] to (3). In specific, we first derive a multirate discretized version of (3), and then we apply a discrete-time lifting technique to the multirate discrete-time model. Connecting the fast-sampling operator and the fast-hold operator with $[kT + lT',\ kT + lT' + T')$, $(k,l) \in \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{[0,N-1]}$, to (3) leads the multirate discrete-time plant model.

Assumption 1. *Suppose that $\theta_i(z(t))$ for $t \in [kT + lT', kT + lT' + T')$ is $\theta_i(z(k+l))$. Then, the nonlinear matrices $\sum_{i=1}^{q} \theta_i(z(t))A_i$ and $\sum_{i=1}^{q} \theta_i(z(t))B_i$ of (3) can be approximated as the piecewise constant matrices $A(\theta(k+l))$ and $B(\theta(k+l))$, respectively.*

Proposition 1. *The multirate discrete-time model of* (3) *can be given by*

$$x(k+l+1) \approx G(\theta(k+l))x(k+l) + H(\theta(k+l))u_d(k+l) \tag{4}$$

for $t \in [kT + lT', kT + lT' + T')$, $(k, l) \in \mathbb{Z}_{\geqslant 0} \times \mathbb{Z}_{[0,N-1]}$, *where* $G(\theta(k+l)) = \sum_{i=1}^{q} \theta_i(z(k+l))G_i$, $H(\theta(k+l)) = \sum_{i=1}^{q} \theta_i(z(k+l))H_i$, $G_i = \exp(A_i T')$, *and* $H_i = (G_i - I)A_i^{-1}B_i$.

Proof. The proof is omitted due to lack of space.

Assumption 2. *Suppose that the firing strength* $\theta_i(z(t))$, *for* $t \in [kT, kT + T)$ *is* $\theta_i(z(k))$.

Proposition 2. *Given the system* (4) *for* $l \in \mathbb{Z}_{[0,N-1]}$, *a lifted sampled input*

$$\widetilde{u}_d(k) = \begin{bmatrix} u_d(kT) \\ u_d(kT + T') \\ \vdots \\ u_d(kT + NT' - T') \end{bmatrix} \in \mathbb{R}^{mN} \tag{5}$$

leads a lifted system

$$x(k+1) \approx \widetilde{G}(\theta(k))x(k) + \widetilde{H}(\theta(k))\widetilde{u}(k) \tag{6}$$

for $t \in [kT, kT + T), k \in \mathbb{Z}_{\geqslant 0}$, *where* $\widetilde{G}(\theta(k)) = G^N(\theta(k))$ *and* $\widetilde{H}(\theta(k)) = \left[G^{N-1}(\theta(k))H(\theta(k)) \; G^{N-2}(\theta(k))H(\theta(k)) \; \cdots \; H(\theta(k))\right]$.

Proof. The proof is omitted due to lack of space.

We convert the multirate digital control problem to the solvability of LMIs. For the system (6), we consider the following multirate feedback controller $u(k) = K_l(\theta(k))x(k)$ and have the lifted control input represented as

$$\widetilde{u}(k) = \widetilde{K}(\theta(k))x(k) \tag{7}$$

where $\widetilde{K}(\theta(k)) = \left[K_0^T(\theta(k)) \; K_1^T(\theta(k)) \; \cdots \; K_{N-1}^T(\theta(k))\right]^T$, $K_l(\theta(k)) = K_0(\theta(k)) \times (G(\theta(k)) + H(\theta(k))K_0(\theta(k)))^l$, and $K_0(\theta(k)) = \sum_{i=1}^{q} \theta_i(z(k))K_{0i}$.

The next theorem provides the sufficient conditions for the stabilization in the sense of the Lyapunov asymptotic stability for (6).

Theorem 1. *The given system* (6) *under* (7) *is globally asymptotically stable in the sense of Lyapunov stability criterion if there exist* $Q = Q^T \succ 0$ *and constant matrices* F_i *such that*

$$\begin{bmatrix} -Q & * \\ G_i Q + H_i F_i & -Q \end{bmatrix} \prec 0 \quad i \in [1, q] \tag{8}$$

$$\begin{bmatrix} -Q & * \\ \frac{G_i Q + H_i F_j + G_j Q + H_j F_i}{2} & -Q \end{bmatrix} \prec 0 \quad i < j \in [1, q] \tag{9}$$

where $*$ *denotes the transposed element in symmetric position.*

Proof. The proof is omitted due to lack of space.

4 Closing Remarks

This paper proposed the multirate control design using the LMI approach for the fuzzy system. Some sufficient conditions were derived for stabilization of the discretized model via the fast discretization. Future work will be devoted to the extension to the nonautomonous system.

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