Flow Shop Scheduling Problems Under Uncertainty Based on Fuzzy Cut-Set*

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Abstract. Production scheduling is an important part in the factories, and there are various uncertainties in the production scheduling of industrial processes. A scheduling mathematical model for flow shop problems with uncertain processing time has been established based on fuzzy programming theory. And in this paper, the fuzzy model can be translated into two mathematical models about the characteristic of scheduling problems. Furthermore, a fuzzy immune scheduling algorithm combined with the feature of the Immune Algorithm is proposed, which prevents the possibility of stagnation in the iteration process and achieves fast convergence for global optimization. The effectiveness and efficiency of the fuzzy scheduling model and the proposed algorithm are demonstrated by simulation results.

1 Introduction

Production scheduling plays an important role in practice industries, which deals with the allocation of limited resources for tasks. There exists a broad literature on the subject, which presents various algorithms for various types of problems. But, in practice, there can be uncertainty in a number of factors such as processing times and costs. For this reason, it is important to be able to take the imprecision into account in the modeling of the problems itself, so that the algorithms are run on the basis of the information about the problem we really possess.

The prevalent approach to the treatment of these uncertainties is through the use of probabilistic models that describe the uncertain parameters in terms of probability distributions [1]. Li Mingqie et al [2] applied stochastic variables to describing the uncertainties of scheduling problems, in which the mathematical model is changed to the Stochastic Programming problems. However, the evaluation and optimization of these models are computationally expensive. Furthermore, the use of probabilistic models is realistic only when these descriptions of the uncertain parameters are available. When such data is not available, we don't have enough information for inferring or deriving the probabilistic models [3].

Among several theories developed to account for uncertainty, fuzzy set theory is more and more frequently used, because of its simplicity and similarity to human

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reasoning [4]. When fuzzy numbers are most often understood as imprecise or approximate concepts, they may also convey preferences and therefore, in some sense, they may represent flexibilities. So, in this work, we draw upon concepts from fuzzy set theory to describe the imprecision and uncertainties in the durations of batch processing tasks. McCahon and Lee [5] were the first to illustrate the application of fuzzy set theory as a means of analyzing performance characteristics for a flow shop system.

Computational methods based on analogy of biological system have been paid much attention in recent years, such as the Immune Algorithm, which imitates the defending process of an immune system against its invaders in a biological body. In this paper, a new fuzzy scheduling algorithm is proposed based on the Immune Algorithm to solve flow shop problems. The study is organized as follows. In Section 2, the definition and mathematical model of the problems that we intend to treat are given. The proposed fuzzy method is introduced and described in Section 3. In Section 4, the computation procedures and the simulation results are discussed. Conclusions from this work are drawn in Section 5.

2 Problems Statement

One kind of scheduling problem that frequently occurs in real world application environments is the flow shop problem. In a flow shop, different products can be made up in diverse equipments, and all products follow essentially the same processing steps.

2.1 Problem Definition

Flow shop problems can be described as: there are N products, which need to be processed, and the number of processing units is M . The processing time of products i in unit j is $\widetilde{T}_{ij}$, which includes the transfer time, the set-up time and the clean-up time, etc. Because it is mutative and uncertain, it is represented by the fuzzy number.

Every product has the same processing sequence in all units, $\widetilde{S}_{ij}$ and $\widetilde{C}_{ij}$ respectively represents the starting time and the finishing time of product i in unit j . As for the uncertainty of the processing time, the starting time and the finishing time are also uncertain. Accordingly, $\widetilde{S}_{ie}$ and $\widetilde{T}_{ie}$ mean the starting time and the finishing time of the last operation of product i . In the paper, the scheduling criterion is makespan, that is, the last job should be completed as soon as possible.

2.2 Mathematical Model

The definition and assumptions can be represented by the following model:

$$min \quad \left\{ \widetilde{Z} = max \quad \left(\widetilde{S}_{ie} + \widetilde{T}_{ie}\right) \right\}$$

$$s.t. \qquad \widetilde{S}_{ij} \geq \widetilde{S}_{i(j-1)} + \widetilde{T}_{i(j-1)} \tag{1}$$

$$\tilde{S}_{ij} \geq \tilde{S}_{(i-1)j} + \tilde{T}_{(i-1)j} \qquad i \in N, \ \ j \in M \tag{2}$$

$$\tilde{S}_{ij} \geq 0 \tag{3}$$

Eq. (1) is the sequence constraint of products, which represents that product i can not start in unit j until its completion in the previous unit.

Eq. (2) is the resource constraint, which means product i can not be processed until completing the product $i-1$ by unit j.

Eq. (3) represents each product may be started at time zero, i.e. the starting time of each one must more than or equal to zero.

2.3 Description of the Solution

In the section, we discuss the flow shop scheduling with fuzzy processing time. Let us assume that the processing time of each job in each unit is given as a triangular fuzzy number. A triangular fuzzy number $\tilde{A}$ is denoted by its three parameters as follows.

$$\tilde{A} = (\quad A_L, \ A_C, \ A_U \quad)$$

Where, A_L is the lower limit, A_C means the center value, and A_U is the upper limit.

There are many approaches to solving the fuzzy mathematical model based on fuzzy theory. We use the concept of fuzzy α -cuts solution in this paper.

α is also called the level of probability. Since the α -cut of a fuzzy number is a closed and convex subset, it can be written a closed interval with $A_\alpha = \left[a_\alpha^L, a_\alpha^R \right]$. As for $\alpha \in [0,1]$, the higher the value of α, the smaller the scope of the feasible solution, and vice versa.

Then, the initial scheduling model can be transformed into the following two programming problems based on the concept of the fuzzy α -level theory.

The optimal programming model of α -level:

$$min \quad \left\{ Z_\alpha^L = max \quad \left(S_{ie\alpha}^{\ L} + T_{ie\alpha}^{\ L} \right) \right\} \tag{4}$$

$$s.t. \qquad S_{ij\alpha}^{\ L} \geq S_{i(j-1)\alpha}^{\ L} + T_{i(j-1)\alpha}^{\ L}$$

$$S_{ij\alpha}^{\ L} \geq S_{(i-1)j\alpha}^{\ L} + T_{(i-1)j\alpha}^{\ L} \qquad i \in N, \quad j \in M$$

$$S_{ij\alpha}^{\ L} \geq 0$$

The worst programming model of α -level:

$$min \quad \{ \; Z_\alpha^R = max \quad \left(S_{ie_\alpha}^{\;R} + T_{ie_\alpha}^{\;R} \right) \} \tag{5}$$

$$s.t. \qquad S_{ij_\alpha}^{\;R} \geq S_{i(j-1)_\alpha}^{\;R} + T_{i(j-1)_\alpha}^{\;R}$$

$$S_{ij_\alpha}^{\;R} \geq S_{(i-1)j_\alpha}^{\;R} + T_{(i-1)j_\alpha}^{\;R} \qquad i \in N, \quad j \in M$$

$$S_{ij_\alpha}^{\;R} \geq 0$$

By solving the two programming problems, we can get the interval $\left[(Z^*)_\alpha^L \, , \; (Z^*)_\alpha^R \right]$ about the α -level set of the optimal objective Z^* for the initial fuzzy problem. This can help managers gain the variation range of the optimal objective function under a certain possible extent.

According to the fuzzy theory, the fuzzy addition and maximum operations have the resolvability. Thus, the details of solution can be described as follows:

If $i = 1, \; j = 1$

$$\begin{aligned} S_{ij_\alpha}^{\;L} &= 0, \qquad C_{ij_\alpha}^{\;L} = S_{ij_\alpha}^{\;L} + T_{ij_\alpha}^{\;L} = T_{ij_\alpha}^{\;L} \\ S_{ij_\alpha}^{\;R} &= 0, \qquad C_{ij_\alpha}^{\;R} = S_{ij_\alpha}^{\;R} + T_{ij_\alpha}^{\;R} = T_{ij_\alpha}^{\;R} \end{aligned} \tag{6}$$

If $i = 1, \quad j > 1$

$$\begin{aligned} S_{ij_\alpha}^{\;L} &= C_{i(j-1)_\alpha}^{\;L}, \qquad C_{ij_\alpha}^{\;L} = S_{ij_\alpha}^{\;L} + T_{ij_\alpha}^{\;L} = C_{i(j-1)_\alpha}^{\;L} + T_{ij_\alpha}^{\;L} \\ S_{ij_\alpha}^{\;R} &= C_{i(j-1)_\alpha}^{\;R}, \qquad C_{ij_\alpha}^{\;R} = S_{ij_\alpha}^{\;R} + T_{ij_\alpha}^{\;R} = C_{i(j-1)_\alpha}^{\;R} + T_{ij_\alpha}^{\;R} \end{aligned} \tag{7}$$

If $i > 1, \quad j = 1$

$$\begin{aligned} S_{ij_\alpha}^{\;L} &= C_{(i-1)j_\alpha}^{\;L}, \qquad C_{ij_\alpha}^{\;L} = S_{ij_\alpha}^{\;L} + T_{ij_\alpha}^{\;L} = C_{(i-1)j_\alpha}^{\;L} + T_{ij_\alpha}^{\;L} \\ S_{ij_\alpha}^{\;R} &= C_{(i-1)j_\alpha}^{\;R}, \qquad C_{ij_\alpha}^{\;R} = S_{ij_\alpha}^{\;R} + T_{ij_\alpha}^{\;R} = C_{(i-1)j_\alpha}^{\;R} + T_{ij_\alpha}^{\;R} \end{aligned} \tag{8}$$

If $i > 1, \quad j > 1$

$$\begin{aligned} S_{ij_\alpha}^{\;L} &= max\left(C_{(i-1)j_\alpha}^{\;L}, C_{i(j-1)_\alpha}^{\;L} \right), \qquad C_{ij_\alpha}^{\;L} = S_{ij_\alpha}^{\;L} + T_{ij_\alpha}^{\;L} \\ S_{ij_\alpha}^{\;R} &= max\left(C_{(i-1)j_\alpha}^{\;R}, C_{i(j-1)_\alpha}^{\;R} \right), \qquad C_{ij_\alpha}^{\;R} = S_{ij_\alpha}^{\;R} + T_{ij_\alpha}^{\;R} \end{aligned} \tag{9}$$

The function objective at the α -level:

$$\begin{aligned} min \quad \left\{ \quad Z_\alpha^L = max \quad \left(S_{ie_\alpha}^{\;L} + T_{ie_\alpha}^{\;L} \right) \quad \right\} \\ min \quad \left\{ \quad Z_\alpha^R = max \quad \left(S_{ie_\alpha}^{\;R} + T_{ie_\alpha}^{\;R} \right) \quad \right\} \end{aligned} \tag{10}$$

Then, the scope of the optimal objective under α -level can be represented by the interval $\left[Z_\alpha^L , Z_\alpha^R \right]$. The following is the optimization strategy of the fuzzy scheduling algorithm.

3 The Fuzzy Scheduling Algorithm Based on the IA

The Immune Algorithm is inspired by the characteristic of the natural immune system. One of its most outstanding features is a self-organizing memory which is dynamically maintained and which allows items of information to be forgotten. And it promotes diversification. It does not attempt to focus on local optima; Furthermore, the algorithm operation on the memory cell will achieve very fast convergence during the search process. So, these features make it widely used in many fields, such as Intelligent Control, Pattern Recognition and Optimization Design [6-7].

Because the scheduling problem is a sequence operation, the objective function has much to do with not only the value of the optimal solution but also the position in the coding strings. Character coding technique is adapted for the representations of those antibodies in the Immune Algorithm. That is, according to the characteristic of scheduling problems, each character represents a processing job. The order of the character in the strings is the processing sequence of jobs.

The computation steps are discussed below.

Step1: Initial antibody population formulation.

In the initial step, the antibodies are generated randomly in the feasible space. A population pool comprises these antibodies. And a group of genes form an antibody. Each antibody represents a possible solution to a schedule of flow shop.

Step 2: Affinity calculation.

Two affinity calculation forms are calculated. One is the affinity ax_V between the antigen and antibody v, the other is the affinity ay_{vw} between antibody v and w, and the expected propagation proportion of each antibody E_v is also computed. These will provide a useful reference in the following evaluation process.

Step 3: Evaluation and selection.

Based on the results of the computation procedure mentioned earlier, the antibody that has high affinities with the antigen is added to the new memory cell. As most selected antibodies exhibit higher affinities with the antigen, the averaged affinity of the new population pool will be higher than that of the original pool. Therefore, a new antibody chosen from the pool comes with a higher affinity with the antigen. The average affinity of a new antibody pool chosen from this new memory cell is higher than that of the old antibody pool.

Step 4: Boost or restriction of antibody generations.

According to the expected propagation proportion, the antibodies in the population pool will be sequenced. The antibody with lower expectation will be restricted. It can be inferred that the antibody with a high affinity and low density will be the most possible candidate. This indeed is a critical role that controls excessive production as for the boost of antibodies with higher affinities and the restriction of antibodies with the higher density.

Step 5: Crossover and mutation

Based on the new antibodies selected from the memory cell, the crossover and mutation of the new antibodies are performed. Crossover is a random process of recombination of strings. With the probability of crossover, a partial exchange of characters between two strings is performed. With the crossover operation, the proposed algorithm is able to acquire more information with the generated individuals. The search space is thus extended and more complete. Mutation is the occasional random alteration of the bits in the string. The mutation operator helps reproduce some individuals that may be vital to the performance.

Step 6: Decisions.

All the antibodies in each generation must be evaluated. The antibodies with higher affinity are tracked to the memory cell for each generation. If the termination criterion is satisfied or no further improvement in relative affinity can be obtained, the optimal search will end. And genes of the antibody can be decoded to be solutions of the scheduling problem. Otherwise, the procedure must turns to step 2.

4 Computational Experience

Consider the following example where ten products have to be processed in five units. The fuzzy processing time data for the example can be found in Table 1. The fuzzy processing times are specified by three parameters, which represent the lower bound, the most likely value and the upper bound on the processing time.

Table 1. The fuzzy processing times of products

	Machine 1	Machine 2	Machine 3	Machine 4	Machine 5
Job 1	(23 25 31)	(11 15 21)	(10 12 14)	(34 40 46)	(6 10 12)
Job 2	(6 7 11)	(37 41 47)	(21 22 24)	(28 36 40)	(6 8 10)
Job 3	(38 41 45)	(137 155 167)	(27 33 37)	(111 121 141)	(145 160 188)
Job 4	(64 74 90)	(8 12 16)	(16 24 30)	(40 48 58)	(66 78 86)
Job 5	(6 7 9)	(69 95 107)	(51 72 84)	(51 52 56)	(148 153 179)
Job 6	(10 12 16)	(8 14 16)	(26 62 74)	(26 32 38)	(140 162 190)
Job 7	(9 11 17)	(5 7 12)	(20 31 35)	(20 26 30)	(26 32 38)
Job 8	(25 31 39)	(35 39 43)	(135 141 175)	(4 6 10)	(15 19 23)
Job 9	(24 32 34)	(84 92 98)	(10 12 14)	(8 14 18)	(84 102 122)
Job 10	(19 27 31)	(109 114 128)	(17 21 23)	(78 90 102)	(44 52 66)

In the algorithm, the population size is 40, the size of memory cell is 20 and the number of iteration is 150. The scheduling algorithm has been executed many times. Figure.1 and 2 are the evolution curves of the algorithm when α is 0.3.

Fig.1 and 2 illustrate the evolution curve of the model. In each chart, the above curve is the reciprocal value of the mean objective, which means the reciprocal of the average value of the affinities between the antigen and antibodies of populations in evolution; The middle curve is the mean value of the objective of antibodies in

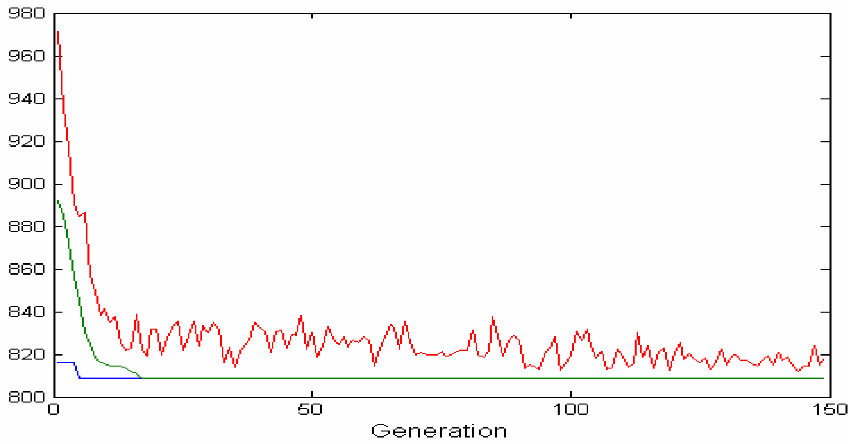

Fig. 1. The evolution curve of the optimal programming model ($\alpha = 0.3$)

Fig. 2. The evolution curve of the worst programming model ($\alpha = 0.3$)

memory cell, which represents the average objective of all individuals in memory cell; and the nether one is the optimal curve, which means the optimal objective of each generation in iteration. Along with the evolution process, the average curve and the optimal curve have not changed intensively, which indicates the convergence of the method.

Let α =0, step size is 0.1, the detail results at the various a-levels are shown in Table 2.

If α increases constantly, the objective tends to converging to the smaller feasible region. That is, the interval of the function objective is getting smaller and smaller, and will converge to the most possible value. As for the same objective, two different

Table 2. The scheduling results at different α -levels

α	The optimal programming model		The worst programming model	
	Job sequence	Functional objective	Job sequence	Functional objective
0	7 6 5 3 1 4 8 9 2 10	775	7 6 9 5 3 2 4 1 8 10	1052
0.1	7 6 5 3 4 8 10 2 1 9	786.2	7 6 4 5 8 9 3 1 10 2	1035.5
0.2	7 6 5 9 1 3 8 2 10 4	797.4	7 6 8 9 2 1 4 5 3 10	1019
0.3	7 6 1 5 2 8 4 9 3 10	808.6	7 6 8 9 4 5 1 3 2 10	1002.5
0.4	7 6 9 5 2 1 4 8 3 10	819.8	7 6 9 1 5 4 10 3 2 8	986
0.5	7 6 9 5 4 3 2 10 8 1	831	7 6 5 3 2 9 10 1 4 8	969.5
0.6	7 6 5 3 9 2 1 4 10 8	842.2	7 6 4 9 5 3 10 2 8 1	953
0.7	7 6 2 9 5 4 1 3 8 10	853.4	7 6 5 9 4 2 3 8 1 10	936.5
0.8	7 4 5 3 2 1 8 10 9 4	864.6	7 6 5 4 3 2 9 1 8 10	920
0.9	7 6 10 1 9 4 5 3 2 8	875.8	7 6 2 9 5 4 1 8 3 10	903.5
1	7 6 5 3 2 4 9 8 1 10	887	7 6 5 1 2 9 4 3 10 8	887

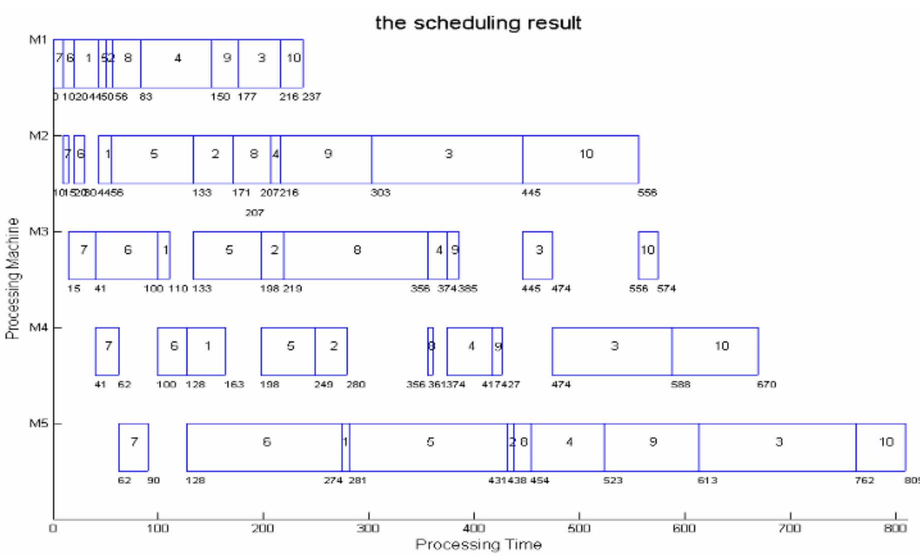

Fig. 3. The Gantt chart of the schedule (the optimal model $\alpha = 0.3$)

job sequences may be obtained. Thus can help managers gain a broader overall view of scheduling and make the proper decision.

When α is 0.3, the Gantt chart of the job sequences of the optimal model and the worst model are respectively shown in Fig.3 and Fig.4. Then, the optimal objective of the example is 808.6, and the worst objective is 1002.5.

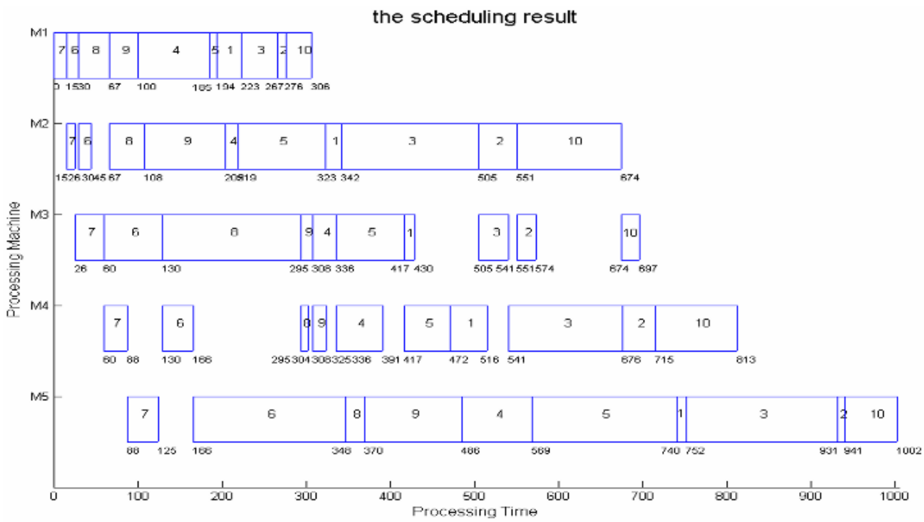

Fig. 4. The Gantt chart of the schedule (the worst model $\alpha = 0.3$)

5 Conclusions

Flow shop scheduling problems under uncertainty are discussed in this paper. As for the uncertainty of processing time, the fuzzy mathematical model is proposed based on the fuzzy cut-set theory. And integrated with the characteristic of the Immune Algorithm, a fuzzy scheduling algorithm is presented to solve the problems. By the results obtained from both simulated and experimental data, it is proven that the feasibility and effectiveness of the scheduling model and the fuzzy scheduling algorithm. And the results can help managers have a broader view of problems and to make the proper decision.

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