Coopetitive Game, Equilibrium and Their Applications*

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Abstract. Coopetition has become the current trend of economic activities. Coopetitive game is introduced through the comparison of the characteristics of noncooperative game and cooperative game. Furthermore, the coopetitive game is solved adopting one kind of Minimax theorem. Finally, the Cournot coopetition model is presented as an example, and the equilibrium is compared with Nash equilibrium.

1 Introduction

The current business environment, advances in information and communication technologies, and the resultant development of network and virtual organizations have led firms to cooperate and compete simultaneously. The term "co-opetition", coined by management professors Barry Nalebuff (Yale University) and Adam Brandenburger (Harvard University), refers to that phenomenon [1]. In the same year, Maria Bengtsson and Sören Kock entitled coopetition the phenomena including both cooperation and competition, and studied the cooperation and competition in business networks [2, 3]. In fact, cooperation and competition have been studied widely. According to the relationship of the aims in cooperation and competition theory, Deutsch divided the benefit body into three parts: cooperation, competition and independence. [4, 5] Claudia Loebbecke, Paul C.Van Fenema and Philip Powell paid much attention to the knowledge transfer under coopetition and presented the theory of interorganizational knowledge sharing during coopetition. [6, 7] Kjell Hausken studied cooperation and between-group competition and found that competition between groups in defection games might give rise to cooperation though the considerable cost of cooperation might be needed. [8] To Marc's theory, benefit body takes other's actions as positive exterior conditions in cooperation and in competition the other's actions are taken as negative exterior conditions [9].

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In this paper, "coopetition" is defined as the phenomenon that differs from competition or cooperation, and stresses two faces of one relationship, cooperation and competition, in the same situation, in which competitors can strengthen their competitive advantages by cooperation. The "Coopetitive" is the adjective form of the coopetition. In the second section, coopetitive game is introduced and the comparison between noncooperative and cooperative game is studied. The coopetitive equilibrium is given by one kind of Minimax theorem in the third section. Economic examples and the comparisons between noncooperative and competitive game are made in the forth section. Conclusions can be made that the coopetitive game has a prodigious advantages both in modeling and algorithm.

2 Coopetitive Game and Coopetitive Equilibrium

2.1 Noncooperative Game and Cooperative Game and Their Comparison

Game theory can be classified into three types according to the interaction of the players: noncooperative game, cooperative game and coopetitive game. In noncooperative situation, players are self-concerned and each player makes decision by himself based on the strategy preferences. Each player maximizes his payoff against the others'. The equilibria can be obtained at the intersections of players' reaction functions and nearly all of them cannot obtain the satisfactory profits.

In cooperative situation, coalition without any conflict is supposed to construct through contract or nuisance suits commitment, etc. The coalition will maximizes its revenue and allocate it based on certain rules. Unfortunately, the coalitions is usually destroyed because of players' self-concerned actions or some details that are ignored in the cooperative process.

Coopetitive game is presented in this paper to avoid these conflicts. The self-concerned players can form coalition in competitive situation. At the same time, the coopetitive equilibria have advantages over those of noncooperative, and they are stable.

2.2 Coopetitive Game

Definition 1. *A Coopetition game* $< N, (A_i), (uc_i) >$ *includes:*

- *The set of players* $1, 2, \cdots, I$.
- *The pure strategy space* A_i *for each player* i.
- *The payoff coefficient functions* $uc_i(a)$ *for each player* i.

The payoff coefficient function is the standardization of the payoff function u_i, which gives player $i's$ Von Neumann-Morgenstern utility $u_i(a)$ for each profiles $a = (a_1, \cdots, a_I)$. The standardization of payoff function u_i is the ratio between the payoff that a player can get at a certain strategy profile and the highest payoff that he can gain. Therefore, the payoff coefficient function $u_i(a)$ denotes the satisfaction degree that player i can obtain under profile a.

Definition 2. *A subset B of the polytope A is called an action strategic extreme set, if $a, b \in A$ and $\lambda a + (1-\lambda)b \in B$ for some $\lambda \in (0,1)$ imply $a, b \in B$.*

For any $a \in A$, $M(a) = \{i' \in I | uc'_i(a) = \min_{i \in I} uc_i(a)\}$ is defined as the index set of a,which means the member of the players who obtain the lowest payoff coefficient function under profile a.

Definition 3. *A point a in A is called a critical strategy if there exists an extreme set Bsuch that for $a \in A, b \in B$and $M(a) \subseteq M(b)$ imply $M(a) = M(b)$. In the other words, a critical point is a point with maximum index set in certain extreme set.*

Definition 4. *Coopetitive equilibrium of game $< N, (A_i), (uc_i) >$ is some critical strategy $a^* \in A$, and $a^* = \arg\max_{a \in A} uc_{i \in M(a)}(a)$.*

The corresponding strategy profiles and the utility profiles under the equilibria are called equilibrium strategy profiles and equilibrium utility profiles respectively.

According to the definition, the coopetitive equilibrium can be obtained in this way: given the strategy profiles, each player finds the conservative (or minimum) payoff coefficients and selects a higher one among them. The coopetitive equilibrium is the one of the strategy profiles that much more players choose with higher satisfaction degrees, and is the counterbalance among players as well.

3 Minimax Theorem and Coopetitive Equilibrium

3.1 Minimax Theorem [10]

Let $\{G_i(x)\}_{i \in I}$ be a family of finitely many continuous concave functions on a polytope X and $I = \{1, \cdots, n\}$. Note that in general, $F(x) = \max_{i \in I} G_i(x)$ on X is not a concave function. However, its behavior is similar to a concave function.

A subset Y of the polytope X is called an extreme set of X if $x, y \in X$ and $\lambda x + (1-\lambda)y \in X$ for some λ in the interval $(0,1)$ imply $x, y \in X$. For example, every vertex is an extreme set and the set X, itself, is also an extreme set. For any $x \in X$, the index set of x is defined as $M(x) = \{i' \in I | G'_i(x) = \max_{i \in I} G_i(x)\}$. A point x in X is called a critical point if there exists an extreme set Ysuch that $x \in Y$and that $y \in Y$ and $M(x) \subseteq M(y)$ imply $M(x) = M(y)$. In the other words, a critical point is a point with maximum $M(x)$ in some extreme set Y.

There is an intuitive interpretation for the critical points. Partition the polytope X into finitely many small regions $X'_i = \{x \in X | G'_i(x) = \max_{i \in I} G_i(x)\}$, every critical point is a "vertex" of some X'_i. Note that X'_i is not necessarily a polytope. If only one small region X'_i is considered, then we cannot say that the minimum value of F(x) on X_i' takes place at "vertices" of X_i'. Thus, the following result is nontrivial.

Theorem 1. *Suppose that $F(x) = \max_{i \in I} G_i(x)$ where I is finite and $G_i(x)$ is a continuous, concave function. Then the minimum value of $F(x)$ for x over a polytope X is achieved at some critical points.*

The following corollaries can be obtained and the proofs are the same as that of theorem 1.

Corollary 1. *Suppose that $f(x) = \min_{i \in I} g_i(x)$ where I is finite and $g_i(x)$ is a continuous, convex function. Then the maximum value of $f(x)$ for x over a polytope X is achieved at some critical points.*

Corollary 2. *The critical strategy of the coopetition game $< N, (A_i), (uc_i) >$ is the critical point of the payoff coefficient function uc_i.*

3.2 Coopetitive Equilibrium

Based on the corollary 1 and corollary 2, the coopetitive equilibrium is one of the critical strategy profiles and the solution is to optimize:

$$\max_{a \in A} uc_{i \in M(a)}(a)$$

In this paper, we apply the following algorithm:

Step 1. Put into the set of players, the pure strategy space A_i and the payoff coefficient functions $uc_i(a)$ for each player i;

Step 2. Calculate the payoff coefficients of each player on every vertex;

Step 3. Let $i = 0, k = n - i$. When $k \neq 1$,let payoff coefficient functions be equal of any k players'; If there is no intersection of any k players' payoff coefficient functions, let $i = i + 1$ and repeat step 3; Otherwise, register the strategy profiles x, the utility profiles u, the sets M of players with higher payoff coefficients and their coefficients g_{max} , and the other's payoff coefficients g_{else} at any intersection, and go to step 4.
While $k = 1$, the coopetitive equilibria are the same as Nash equilibria.

Step 4. Maximize the payoff coefficients at the intersections of all the k payoff coefficient functions, and register the corresponding strategy profiles, which are the coopetitive equilibria.

4 Cournot Coopetition Model and Cournot Coopetitive Equilibrium

4.1 Cournot Coopetition Model

In Cournot Model, I oligarchs (firms) produce a homogeneous good. The strategies are quantities. All firms simultaneously choose their respective output lever

x_i from feasible sets $[0, \infty)$, They sell their outputs at the market-cleaning price $p(x)$, where $x = x_1 + x_2 + ... + x_I$. Firm i's cost of production is $C_i(x_i) = c_i x_i$, and firm i's total profit is $u_i(x_1, x_2, ..., x_I) = x_i p(x) - c_i(x_i)$. For linear demand $p(x) = max(0, a - x)$, the maximal profit that firm i can obtain in the monopoly market is $u_i^{\max} = (a - c_i)^2/4$. Firm i's payoff coefficient function under strategy profile x is $uc_i(x) = u_i(x)/u_i^{\max}$.

We call the constant a in the linear demand p potential demand. The cooptitive equilibria depend on the potential demands and the costs of firms.

Definition 5. *The demand-cost difference is defined as the difference between potential demand and the cost, say $(a - c_i)$; the cost-cost difference is defined as the cost difference of any two firms , say $(c_i - c_j)$.*

Lemma 1. *The demand-cost difference and the cost-cost difference determine the satisfaction degrees of the firms at critical points in coopetitive games.*

Proof. Suppose the costs of any two firms are c_i and c_j, $c_i > c_j$, $a - c_i = m(c_i - c_j)$, and then

$$g(x) = \frac{x_i(a - c_i - \sum_{k=1}^{n} x_k)}{(a - c_i)^2/4} = \frac{x_j(m(c_i - c_j) - \sum_{k=1}^{n} x_k)}{m^2(c_i - c_j)^2/4}$$

□

The satisfaction degrees of the firm can be obtained given the demand-cost difference and the cost-cost difference.

Lemma 2. *Firms' satisfaction degrees at coopetitive equilibrium may differ from each other. The greater the demand-cost difference is or the smaller the cost-cost difference is, the closer the utilities of the firms are.*

Proof. Suppose the costs of any three firms are $c_i > c_j > c_k$, $a - c_i = m_1(c_j - c_k)$, $a - c_j = m_2(c_j - c_k)$ and $m_1 < m_2$.

Let $g^j = g^k$ at equilibria, and then

$$\frac{x_j(a - c_j - \sum_{l=1}^{n} x_l)}{m_2^2(c_j - c_k)^2/4} = \frac{x_k(a - c_k - \sum_{l=1}^{n} x_l)}{(m_2 + 1)^2(c_j - c_k)^2/4}$$

$$\frac{u^j(x)}{u^k(x)} = \frac{x_j(a - c_j - \sum_{l=1}^{n} x_l)}{x_k(a - c_k - \sum_{l=1}^{n} x_l)} = \frac{m_2^2}{(m_2 + 1)^2}$$

Let $g^i < g^j$, and then

$$\frac{u^i(x)}{u^j(x)} = \frac{x_i(a - c_i - \sum_{k=1}^{n} x_k)}{x_j(a - c_j - \sum_{k=1}^{n} x_k)} < \frac{(a - c_i)^2}{(a - c_j)^2} = \frac{m_1^2}{m_2^2}$$

□

According to the above lemmas, theorem 2 can be obtained.

Theorem 2. *More players gain higher satisfaction degrees at coopetitive equilibrium. Moreover, the firms with lower cost can obtain a higher satisfaction degree, vice versa.*

4.2 Cournot Coopetitive Equilibrium

4.2.1 Extreme Sets

According to the definition of extreme set and the feasible sets $0 \leq x_i \leq (a-c_i)$, all the n-dimension vectors $(x_1, x_2, \cdots, x_n)$, $x_i \in [a_i, b_i]$, $0 \leq a_i \leq x_i \leq b_i \leq (a-c_i)$ are extreme sets.

4.2.2 Critical Point

According to corollary 1, the critical points can be obtained at the vertices or at the interior critical points. From the feasible set $0 \leq x_i \leq (a-c_i)$ of firm i, the production of each firm is either zero or $a - c_i$, and there will be no profits for any firm and even internecine $(uc_i(x) \leq 0)$.

The interior critical points are the points whose index sets are not embraced by the other critical points and they can be obtained at the intersections of the payoff coefficient functions. The algorithm presented in section 3.2 is adopted to obtain all of the intersections approximately by simulation because of the difficulties in expressing the formula. For instance, for $a = 1.0, c_1 = 0.4, c_2 = 0.5, c_3 = 0.5, c_4 = 0.6$, we can divide the simulation span into 100 equal intervals, and set the iteration accuracy at 0.001. The iteration accuracy and the division step of the feasible sets can be changed for the different problems. There are many interior critical points whose index sets are $\{1,2,3\}, \{1,2,4\}, \{1,3,4\}$ and $\{2,3,4\}$.

4.2.3 The Coopetition Equilibrium

Comparing the utility coefficients at vertices and the interior critical points, the coopetitive equilibrium can be obtained. For the case of $a = 1.0, c_1 = 0.4, c_2 = 0.5, c_3 = 0.5, c_4 = 0.6$, the firms must choose the strategy profiles to maximize their satisfaction degrees. The maximum satisfaction degrees obtained at interior critical points, and the corresponding strategy profiles x, the utility profiles u, the sets M of players with higher satisfaction and their coefficients g_{max}, and the other's payoff coefficients g_{else} at any intersection are listed in Table 1. Which coalition can come into being among these coalitions? All of the firms will choose to participate the coalition in which they can obtain the highest satisfaction degrees. Coalition $\{1,2,3\}$ can give three members satisfaction degree 0.328 which is much higher than those of the other coalitions, therefore this coalition will come into being.

Table 1. Case 1: $a = 1.0, c_1 = 0.4, c_2 = 0.5, c_3 = 0.5, c_4 = 0.6$

M	x_1	x_2	x_3	x_4	$g_{\max}$	g_{else}	u_1	u_2	u_3	u_4
{1,2,3}*	0.09	0.09	0.09	0.002	0.328	0.064	0.030	0.021	0.021	0.0003
{1,2,4}	0.057	0.053	0.035	0.050	0.257	0.171	0.023	0.016	0.011	0.010
{1,3,4}	0.057	0.035	0.053	0.050	0.257	0.171	0.023	0.011	0.016	0.010
{2,3,4}	0.003	0.075	0.075	0.076	0.325	0.012	0.001	0.016	0.020	0.013

Several other examples are given in Table 2, 3 and 4 respectively. The strategies marked with asterisk denote the equilibrium strategy in the tables.

Remark 1. The above simulation results verify the theorem 2.

Table 2. Case 2: $a = 1.0, c_1 = 0.4, c_2 = 0.5, c_3 = 0.6, c_4 = 0.6$

M	x_1	x_2	x_3	x_4	$g_{\max}$	u_1	u_2	u_3	u_4
{1,2,3,4}*	0.054	0.05	0.048	0.048	0.24	0.022	0.015	0.010	0.010

Table 3. Case 3: $a = 1.0, c_1 = 0.4, c_2 = 0.4, c_3 = 0.6, c_4 = 0.6$

M	x_1	x_2	x_3	x_4	$g_{\max}$	g_{else}	u_1	u_2	u_3	u_4
{1,2,3}*	0.09	0.09	0.09	0.002	0.328	0.064	0.030	0.030	0.021	0.0003
{1,2,4}	0.087	0.087	0.005	0.106	0.305	0.017	0.027	0.027	0.001	0.012
{1,3,4}	0.078	0.009	0.075	0.078	0.312	0.036	0.028	0.003	0.020	0.013
{2,3,4}	0.009	0.078	0.075	0.078	0.312	0.036	0.0032	0.028	0.020	0.013

Table 4. Case 4: $a = 1.0, c_1 = 0.5, c_2 = 0.5, c_3 = 0.6, c_4 = 0.6$

M	x_1	x_2	x_3	x_4	$g_{\max}$	u_1	u_2	u_3	u_4
{1,2,3,4}*	0.233	0.233	0.086	0.086	0.510	0.032	0.032	0.020	0.020
{1,2,3,4}	0.24	0.24	0.098	0.098	0.676	0.042	0.042	0.027	0.027
{1,2,3,4}	0.25	0.25	0.11	0.11	0.880	0.055	0.055	0.035	0.035

4.3 Comparison Between Coopetitive Equilibrium and Nash Equilibrium

The equilibrium strategies and equilibrium profits under Nash equilibrium in noncooperative game are as follows.

$$x^i_{Nash} = (a + \sum\nolimits_{j=1}^{n} c_j)/(n+1) - c_i \quad i = 1, 2, \cdots, n \tag{1}$$

$$\begin{aligned} u^i_{Nash} = \tfrac{1}{n+1}(a - c_i - \textstyle\sum_{j=1}^{n} x_j)(a + \sum_{j=1}^{n} c_j - (n+1)c_i) \\ i = 1, 2, \cdots, n \end{aligned} \tag{2}$$

Let a=1.0, c_1=0.4, c_2=0.5, c_3=0.5, c_4=0.6, the equilibrium strategy and equilibrium profit profiles are $x_{Nash} = (0.2, 0.1, 0.1, 0)$, $u_{Nash} = (0.04, 0.01, 0.01, 0)$ in noncooperative game. From Table 1, the coopetition equilibrium strategy is $x_{Cooptition}$ = (0.09,0.09,0.09,0.002) and equilibrium profit profile is $u_{Coopetition}$ = (0.0295,0.0205,0.0205,0.0131). By comparison, we can obtain that $\sum\limits_{i=1}^{n} u^i_{Coopetition} > \sum\limits_{i=1}^{n} u^i_{Nash}$, and $\sum\limits_{i=1}^{n} x^i_{Nash} > \sum_{i=1}^{n} x^i_{Coopetition}$.

It is concluded that the coopetitive game has a prodigious advantage both in the modeling and algorithm. The coopetitive equilibrium can be obtained conveniently and convex or concave payoff coefficient functions are the only requirements.

5 Conclusions

In this paper, the advantages and disadvantages of noncooperative and cooperative game are compared and the coopetitive game is presented. The coopetitive equilibrium is defined and the algorithm is given by using one kind of Minimax theorem. This algorithm has great advantages and can solve games with irregular, non-differential concave or convex payoff functions.

The Cournot coopetitive model with linear demand function and asymmetric costs are studied as examples. Conclusions can be made that much more players can obtain higher satisfaction degrees at coopetitive equilibria, which are dependent on the costs and the potential demands. The comparison is made between noncooperative Nash equilibrium and coopetitive equilibrium. The algorithms for solving the coopetitive equilibria with non-linear demand function by minimax theorem need further study.

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