# If CiE Did not Exist, It Would be Necessary to Invent It ⋆

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As it happens, "Computability in Europe" *was* invented, just over two years ago, and in a short time has grown beyond all expectations. But even though the surprise of finding together so many researchers into different aspects of computability has not worn off, **CiE** does represent a strand of scientific endevour going back to the earliest times. Even before Euclid of Alexandria devised his algorithm for finding the greatest common divisor of two integers, human survival depended on the identification of *algorithmic content* in the everyday world. What distinguished Euclid, and successors like Newton, Leibniz, Frege, Peano, Babbage, Russell, Hilbert, Gödel and Turing, is the reaching for control over that content through theory and abstraction. Perhaps Albert Einstein had something like this in mind in 1950 when he wrote (p.54 of *Out of My Later Years*, Philosophical Library, New York):

> "When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them."

What is peculiarly contemporary about **CiE** is the scrutiny it brings to bear on the *quest* for algorithmic content, something that was not possible before Turing and his fellow 1930s pioneers in the area.

Through the work of computability theorists, the search for algorithmic content goes beyond the ad hoc, and develops into an activity guided by an expanded *consciousness* of what we are doing. We can now explain why certain basic problems are harder than others. We can use our knowledge of logical structure and language to devise more efficient computer programs. We can relate the structures of computability theory to real-world situations, and find models which aid prediction, or make problems in making predictions mathematically explicit. And, the hope is, we can get enough insight into how physical systems 'compute' to ease us past the computational barriers our theory has brought to our notice. The questions surrounding 'New Computational Paradigms' are indeed fundamental ones. Answers, as so often in the past, will depend on the sort of mix of the practical and the theoretical that Alan Turing, if he were still with us, would have recognised, and found fascinating.

⋆ With apologies to Voltaire ...

S.B. Cooper, B. Löwe, and L. Torenvliet (Eds.): CiE 2005, LNCS 3526, pp. 1–7, 2005.

In current terminology, the scientific approach of **CiE** is interdisciplinary, approaching real-world problems from different perspectives and using diverse techniques. **CiE** seeks to bridge the theoretical divide between mathematics and computer science, and between computability theory and science, which are traceable back almost to the birth of computability theory in the mid-1930s. Of course, the natural scientist of the Enlightenment would have had no problem with the so-called interdisciplinarity of **CiE**. It is only since the sixteenth and seventeenth centuries that scientific specialisms have solidified into exclusive disciplines, regulated by senior figures whose role it is to perseverate the assumptions and conventions of their areas, complete with their own technical priorities.

Even 'new paradigms' constitutes a project started by Turing — the natural scientist par excellence, at least to the computability theorist. On the one hand, the yawning gap between computation and the real-world played a key role in both his scientific and personal lives, just as it now dominates the work of **CiE**. This was a gap he was ever, both practically and conceptually, seeking to bridge, and many of his ideas anticipate current research. On the other hand, this was a preoccupation which took him — and now promises to take us — beyond the safe confines of what Thomas Kuhn calls 'normal science'. Here is how Kuhn describes normal science in his influential book *The Structure of Scientific Revolutions* (pp.162–164 of the Third Edition, The University of Chicago Press, 1996):

> "Normally, the members of a mature scientific community work from a single paradigm or from a closely related set. . . . once the reception of a common paradigm has freed the scientific community from the need constantly to re-examine its first principles, the members of that community can concentrate exclusively upon the subtlest and most esoteric of the phenomena that concern it. Inevitably, that does increase both the effectiveness and the efficiency with which the group as a whole solves new problems."

What has become problematic in many areas of science and the humanities is dealing with globally determined phenomena. Everywhere, we see nonlinear development, breakdown in inductive and predictive structures, computer simulation replacing mathematical solutions, and the puzzle of emergence of new relations in the midst of turbulence. We also see the ad hoc development of particular solutions to everyday problems which seem to challenge existing conceptual frameworks. And we have quite basic obstacles to raising the capabilities of present-day computers to the needs of the working scientist. All this is reflected in the wide variety of theoretical directions to be found within **CiE**. The aim is to bring a new understanding to existing developments, and to establish the sort of consciousness of computational issues upon which exciting new practical innovations can be based.

However, the reader coming to this volume for the weird and wonderful from today's scientific fringe will be disappointed. There are indeed contributions which acknowledge the extent to which the Turing machine paradigm is already shifting — see, for instance, Dina Goldin and Peter Wegner on *The Church-Turing Thesis: Breaking the Myth* — but this tends to be work which has gone through a long period of gestation, and received a measure of acceptance and respect within the computer science community. Here is how one of our reviewers of Goldin and Wegner's article described the situation:

> "Fifty years ago the Turing Thesis was OK, now it is still OK provided we know to what computational scenario it should be related. If we are changing the scenario (as the practice of computing prompts us), we have to update the notion of a Turing machine (or of any other fundamental model of computation) as well - that's all."

Of course, that may be 'all'. But extracting useful models from new computational situations, or — to approach things more theoretically — to develop new abstractions and make innovative real-world connections, is the essence of the challenge. So we also see here papers dealing with more overtly mathematical extensions of the standard Turing model of computation, such as infinite time Turing machines — see Joel Hamkins on *Infinitary computability with infinite time Turing machines*, and the paper of Philip Welch — and, at the other extreme, a number of contributions dealing with natural computation. Membrane computing is represented by Gheorghe Păun (a seminal figure in this area), Marian Gheorghe et al, and Shankara Narayanan Krishna, and we have Paola Bonizzoni, Felice, and Mauri on DNA computing, and Natalio Krasnogor (with Gheorghe again, and others) on computational modelling of the important microbiological phenomenon of 'quorum sensing'. The resurgent topic of analog computers is touched on by Jérôme Durand-Lose, Giuseppe Trautteur (on *Beyond the Super-Turing Snare: Analog Computation and Virtual Digitality*) and Jeffery Zucker with John Tucker, and neural networks, another area with a long history, is represented by Angelo Cangelosi, and Krzysztof Michalak with Halina Kwasnicka. And, of course, quantum computation is a key topic at **CiE 2005** (where from Harry Buhrman we get just a taster from his talks). New paradigms of computation come in all shapes and sizes, and the growth of quantum computing reminds us that even quite modest improvements (theoretically speaking) in computing efficiency promise big changes in the world we live in.

Although a number of people associated **CiE** are involved with these different areas, most of us do not actually set out to *do* quantum computing, membrane computing, neural networks, evolutionary computation, and so on. The agenda is not so piecemeal. This is not computational tourism, the Readers Digest Condensed Books version, with the grown-ups head for the real thing — WCIT, CINC, CEC, and other myriad specialist meetings. The intervention of **CiE** is aimed at using logical and mathematical methods to reveal underlying structures and unities, to develop general frameworks and conceptual aids — to build the sort of theoretical and practical synergies which gave Turing and von Neumann such a key role in the early days of the first computing revolution. For example, one can recognise within most of the existing proposals for new computational paradigms a high degree of interactivity, as is picked up on in Goldin and Wegner's paper. One aspect of this is the importance now given to connectionist models of computation, anticipated by Turing himself in his discussion of 'unorganised machines' in his 1948 National Physical Laboratory Report *Intelligent machinery*. In 1988 Paul Smolensky observed in his influential *Behavioral and Brain Sciences* paper *On the proper treatment of connectionism* (p.3) that:

> "There is a reasonable chance that connectionist models will lead to the development of new somewhat-general-purpose self-programming, massively parallel analog computers, and a new theory of analog parallel computation: they may possibly even challenge the strong construal of Church's Thesis as the claim that the class of well-defined computations is exhausted by those of Turing machines."

This kind of challenge is being met on a number of levels. On the one hand one has the work on neural networks and logic reported on by Artur Garcez in this volume, while on the other one has models based on computational reducibilities (such as that derived from Turing's oracle machines), which promises a new relevance to the sort of classical computability featuring in the contributions from Finkel, Harris, Kalimullin, Lewis, Angsheng Li, Selivanov, Soskov, Alexandra Soskova, and Terwijn (describing an interesting application of the Medvedev lattice). As our anonymous reviewer pointed out above, "Fifty years ago the Turing Thesis was OK, now it is still OK provided we know to what computational scenario it should be related." The world has not lost its algorithmic content, but there is needed a fundamental process of readjustment to new realities, involving full use of our powers of theoretical deconstruction of computationally complex environments. Relevant here is the paper of Udi Boker and Nachum Dershowitz. As one of our reviewers commented: "The subject of this paper is very central to the topic of the conference: How can we compare the computational power of different computational paradigms?".

As already suggested, we can think of **CiE** as *computation with consciousness*, in the sense that there is a relatively high level of detachment and abstraction involved. Wonderful things can be achieved without consciousness. Bert Hölldobler and Edward O. Wilson's book on *The Ants* runs to over eight-hundred pages, and ants and similar biological examples have inspired new problem-solving strategies based on 'swarm intelligence'. But the limits to what a real-life ant colony can achieve are more apparent than those of more obviously conscious beings. As the constructors move in and tarmac over our richly structured ant colony, the ants have no hope of expanding their expertise to deal with such eventualities. For us algorithmic content gives rise to new emergent forms, which themselves become victim to our algorithmic appetites, and even the inevitable limits on this inductive process we hope to decode. Maybe it is going *too* far to think of **CiE** as the conscious and interventionist observers of the ant-like activities of our more ad hoc computational colleagues! But any sceptics might remember how Turing himself put even this aspect of the computational process under the mathematical microscope. When Turing says in his 1939 paper:

> "Mathematical reasoning may be regarded ... as the exercise of a combination of ... *intuition* and *ingenuity*. ... In pre-Gödel times it was

> thought by some that all the intuitive judgements of mathematics could be replaced by a finite number of ... rules. The necessity for intuition would then be entirely eliminated. In our discussions, however, we have gone to the opposite extreme and eliminated not intuition but ingenuity, ..."

he is talking about what happens when one persistently transcends a particular context by iterating an overview, such as that of Gödel for first-order Peano arithmetic. We can trace back to this paper the genesis of powerful proof-theoretic methods which have both benefitted from, and fed back into, computability theoretic perspectives. One tends to think of computability as being a language-independent notion, but the need to describe what is going on computationally reasserts the human dimension, and leads us to appropriate proof theoretic hierarchies. This direction is represented here by *Proofs and Computation* Special Sessions contributors Ulrich Berger, Coquand, and Wainer (with Geoff Ostrin), and by proof mining expert Ulrich Kohlenbach.

An important recent development has been the growth of proof complexity since the appearance of Sam Buss' thesis on *Bounded Arithmetic* back in 1985, showing that basic complexity classes could be allied with levels of relatively easily described proof theoretic heirarchies. And if ever there was an area in need of new paradigms, it is computational complexity. There are, of course, deep and intactable problems, basic to how we compute in the future, for which no one seems to be able to even get close to a solution. The appearance of Yuri Matiyasevich and Yiannis Moschovakis (with the intriguing title *Recursion and complexity*) on our list of authors reminds us of similarly basic computational issues arising from traditional logical frameworks. Contributions from Barra and Kristiansen, Ricardo Gavaldá (on computational learning theory), Gibson and Woods, Kristiansen again, Jack Lutz (on *The Dimension of a Point: Computability Meets Fractal Geometry*), Peter Bro Miltersen, Victor Mitrana et al, Pheidas and Vidaux, and Jacobo Torán give just an indication of the great variety of output to be encountered.

Also to be found here are articles and abstracts dealing with *real computation* — another key topic at **CiE 2005** — an implicit acknowledgement of gap between how the working scientist describes the universe in terms of real numbers, and the way in which present-day computers are constrained to work with discrete data. Richard Feynman may have commented, characteristically provocative as ever, in a 1982 article on *Simulating physics with computers* in the *International Journal of Theoretical Physics*:

> "It is really true, somehow, that the physical world is representable in a discretized way, and ... we are going to have to change the laws of physics."

But the practical realities which faced Feynman the scientist in dealing with continuous mathematical models of physical systems have not changed. This area also sees a variety of theoretical approaches to the practical problems involved, some easily located within the familiar framework of what has been known as recursive analysis, and others much more immediately geared to applications. At the more applied end of the spectrum we have Edalat-Khanban-Lieutier, Amin Farjudian, Lieutier, Pattinson, and Ning Zhong. More mathematical approaches, including work on computable and c.e. reals, etc., show up in nice contributions from George Barmpalias, Klaus Meer, Guohua Wu, Zheng Xizhong and Martin Ziegler. Korovina and Kudinov take us in the direction of computability over higher type continuous data, connecting up with more general and set theoretical work, such as that of Peter Koepke, and Alexey Stukachev.

Amongst other important computational notions not yet mentioned are computable models (Hirschfeldt, Morozov), randomness (Reimann), reverse mathematics (Joseph Berger), and other riches too many to itemise in detail.

In the end, the overall impression is one of *normal science at its best* — in its particularities inventive, relevant and soundly based, but a confluence of perspectives exceeding the sum of its parts. This is the essence of most paradigm shifts in history. In retrospect we may recognise a particular 'eureka' moment, but closer inspection often reveals revolutionary new ideas *emerging* out of a number of contributory and seemingly unrelated developments. Only when the picture is focused and comprehensive enough one can one clearly distinguish both its failings and potentialities. As Kuhn says (p.92):

> "...scientific revolutions are inaugurated by a growing sense, ...often restricted to a narrow subdivision of the scientific community, that an existing paradigm has ceased to function adequately in the exploration of an aspect of nature to which that paradigm itself had previously led the way."

Paradigm shifts are not easily come by. Their underlying ideas must be connected, justified, validated, formed in to a persuasive whole, through the detailed and selfless work of many people. Some of this work may be anticipatory, brave, but wrong, and in putting together this volume the editors have been all too aware of this. In particular cases, we have preferred to err on the side of caution. Again, this is a usual feature of paradigm shifts, and we hope our readers (and contributors) will understand this. We do believe **CiE** to provide a home for those exploring the developing real-world relevance of computability and complexity, and we hope this volume is a first sign of what we can achieve as a more coherent scientific community.

It is appropriate to give Thomas Kuhn the (almost) final word (pp.167–168):

> "The very existence of science depends upon investing the power to choose between paradigms in the members of a special kind of community. Just how special that community must be if science is to survive and grow may be indicated by the very tenuousness of humanity's hold on the scientific enterprise. Every civilization of which we have records has possessed a technology, an art, a religion, a political system, laws, and so on. In many cases those facets of civilization have been as developed as our own. But only the civilizations that descend from Hellenic Greece have possessed more than the most rudimentary science. The bulk of

> scientific knowledge is a product of Europe in the last four centuries. No other place and time has supported the very special communities from which scientific productivity comes."

Even if Europe is now but one part of an interconnecting global scientific community, computability continues to be an area in which the European contribution is something quite special.