A Method to Reconstruct Activation Wavefronts Without Isotropy Assumptions Using a Level Sets Approach

Felipe Calderero[1], Alireza Ghodrati[2], Dana H. Brooks[2], Gilead Tadmor[2], and Rob MacLeod[3]

[1] Department of Signal Theory and Communications, Technical University of Catalonia (UPC), Barcelona, Spain
[2] Department of Electrical and Computer Engineering, Northeastern University, Boston, MA, USA
[3] Nora Eccles Harrison Cardiovascular Research and Training Institute (CVRTI), University of Utah, Salt Lake City, Utah, USA

Abstract. We report on an investigation into using a Level Sets based method to reconstruct activation wavefronts at each time instant from measured potentials on the body surface. The potential map on the epicardium is approximated by a two level image and the inverse problem is solved by evolving a boundary, starting from an initial region, such that a filtered residual error is minimized. The advantage of this method over standard activation-based solutions is that no isotropy assumptions are required. We discuss modifications of the Level Sets method used to improve accuracy, and show the promise of this method via simulation results using recorded canine epicardial data.

1 Introduction

Inverse electrocardiography (ECG) estimates the electrical activity of the heart from potential measurements on the body surface. Because of smoothing and attenuation in the body, the measured potentials on the body surface can obscure significant detail about the heart's electrical activity. Thus conventional electrocardiography fails to detect heart problems in many situations [1]. A possible improvement is to model the electrical properties of the torso volume conductor and attempt to explicitly estimate features of cardiac electrical behavior; this is known as inverse electrocardiography. This problem is considered by many research groups [1, 2, 3, 4, 5]. However, the inverse problem of ECG is ill-posed and we need to add constraints to get a stable solution. The single most important feature of the heart's electrical activity is the activation wavefront, which passes through the heart muscle once per cardiac cycle and triggers, after some delay, the mechanical contraction of the muscle. The time that this wavefront passes through any given point in the heart is called the activation time. The problem of finding activation time has been studied using both activation-based models [3, 4, 5] and potential-based models [6].

A.F. Frangi et al. (Eds.): FIMH 2005, LNCS 3504, pp. 195–204, 2005.

The advantage of activation-based models is the reduction of the unknowns to the arrival time of the wavefront at each point on the epicardial and endocardial surfaces. Potential-based models instead treat the value of the potential at each point on the relevant surface at each time instant as a free variable. However, activation-based models depend on isotropy / homogeneity assumptions and a fixed shape of the temporal waveform in order to form a tractable forward model. Potential-based models are less restrictive but imply a high-order parameterization and thus require considerable smoothing (regularization). A method that is used frequently in inverse ECG is Tikhonov regularization, which indeed smooths the solution because of the type of 2-norm constraints employed. It is difficult to include the physical and geometric constraints imposed by the central physiological feature, namely wavefront behavior, except via indirect and somewhat coarse models [7, 8].

Our goal here is to investigate the possibility of estimating the activated region on the epicardium at each time instant using a Level Sets based inverse solution [9, 10]. The forward model we use is potential-based, with a very simple two-level model to characterize the potential distribution given the wavefront. The potential advantage is that we maintain some benefits of activation-based solutions without requiring isotropy assumptions.

In this work we use two constraints. The first assumes that the potentials on the heart can be effectively approximated by two values, representing the activated and inactivated regions respectively. This assumption is of course a rather crude approximation in both the activated and non-activated regions, and ignores the transition area between the two regions. But since we are looking for activation time this assumption may be useful, and we follow similar assumptions used in activation-based solutions [3, 4, 5]. The second constraint is a spatial constraint applied by the Level Sets method. Level Sets were first proposed in [9] to solve inverse problems when a constant-value inhomogeneity is enclosed in a constant-value background by evolving a boundary, starting from an initial region, such that the residual error is minimized. Modifications of the original Level Sets method were needed to improve reconstruction quality. New constraints were added to the Level Sets evolution to improve the shape of the recovered wavefront and to enhance sensitivity to regions of the epicardium whose effect on the residual error was otherwise too weak. In addition, we filtered the residual error to reduce the effect of the error introduced by the two-level quantization on the wavefront evolution.

Section 2 introduces the Level Sets Method applied to the inverse problem of electrocardiography. We first present the formulation of linear inverse problems in terms of Level Sets, as proposed in [9]. Second, we discuss practical problems implementing this method for inverse electrocardiography. In Section 3, we report on improvements obtained by adding new spatial constraints to the evolution and by filtering the residual error. Finally, Section 4 discusses our results, summarizes our conclusions, and gives some suggestions for future research.

2 Level Sets Method

2.1 Level Sets Formulation

The Level Sets Method, as described in [10], is a curve/surface evolution technique, based on a function whose dimension is one higher than the boundary of interest. The zero level set of this function is iteratively guided by a well designed speed function to evolve to an unknown desired contour. It naturally provides an opportunity for geometrical and spatial constraints. A particular set of inverse problems, known as obstacle reconstruction problems, can be formulated in terms of Level Sets [9]. In these problems, the solution consists of an unknown region, simply or multiply connected, with some characteristic that differs from the surrounding background. The solution only has two possible reconstruction values: one for the unknown region and another for the background. Applying Level Sets, the zero level set will evolve to the boundary of this region. Hence, Level Sets evolution adds geometrical and spatial constraints, without any *a priori* assumption about the connectedness of the region. Besides that, the Level Set boundary can split and merge naturally and provide multiple connectivity without any additional complexity. On the other hand, it turns a possibly linear problem into a decidedly nonlinear problem (although non-linearity is common to all activated-based inverse methods in ECG). In addition, there is no theoretical proof on convergence (only practical results, see [9]), and the solution depends on the algorithm initialization.

We use the approach described in [9]. Let ϕ, be the function whose level set $\phi = 0$ is taken as the contour of interest (here the activation wavefront location). The general Level Sets evolution equation is:

$$\phi_t + F|\nabla\phi| = 0 \tag{1}$$

where F is the speed in the outward normal direction and ϕ_t is the time derivative of ϕ. The key issue in using Level Sets in most problems is determining the speed function F. In inverse problems F should be defined such that the solution moves toward minimizing the norm of the residual [9].

Our forward model for ECG is:

$$y = Ax + n \tag{2}$$

where A is a forward matrix, obtained here by the boundary element method (BEM), x holds the heart potentials, y holds the body surface potentials and n is white Gaussian noise. It is shown in [9] that the residual error is monotonically descending if the speed is defined as follows:

$$F = -A^T(Ax - y). \tag{3}$$

This evolution can be seen as a flow in the steepest descent direction of the residual error $||Ax - y||_2^2$.

Thus, the Level Sets evolution equation for inverse problems at iteration $n+1$ is:

$$\phi(n+1) = \phi(n) - \Delta t \cdot (A^T(A \cdot x(n) - y) \cdot |\nabla\phi(n)|) \tag{4}$$

where x is initialized and then updated in each iteration as the zero level of ϕ.

To approximate the inverse problem in electrocardiography as an obstacle reconstruction problem formulated in terms of Level Sets, we divide the heart surface in two regions: activated and inactivated areas. The potential in each area is assumed to be constant with two different values, obtained independently for each time instant from a dataset of cardiac mapping ECG data. The zero level set is evolved to estimate the boundary between activated and inactivated regions.

2.2 Level Sets Practical Implementation and Initial Results

An inverse ECG Level Sets implementation has to overcome some practical problems. First, an accurate heart geometry model is needed. In this work we used the Utah Cardiovascular Research and Training Institute (CVRTI) Heart Geometry Model (a 3D, non-uniform triangulated grid). We further interpolation the surface to improve the model, and thus, the Level Sets solution, removing large triangles in the superior region and near the apex and some non-differentiable points in the original.

Another practical problem was the initialization of the Level Sets function, because of the solution's dependence on the starting value. Our solution ensured that the activated area was included inside the initial guess, and we centered it on the activated area recovered by Tikhonov regularization at each time instant. To obtain the activated area from the Tikhonov solution, the potential histogram was computed and the middle point of its two first maximums was used as a threshold (ensuring that the maxima were different enough that one belong to the activated potentials and the other to the inactivated set). Finally, we chose the value of evolution step size to ensure that the evolution didn't stop prematurely, but rather remained sensitive to the curve boundaries.

We first applied the Level Sets method in this straight-forward manner. We used an epicardial electrocardiogram dataset recorded during tank experiments by our collaborators at CVRTI in Utah [12]. From these epicardial potentials we computed the potentials on the torso surface using the linear model in Eq. 2, and added Gaussian white noise to achieve a 30dB signal to noise ratio (SNR). A realistic homogeneous torso volume conductor forward model matrix was computed by the BEM method with dimension 711×620: 620 nodes in the heart geometry model mapped to 711 electrodes on the torso.

The results obtained were not satisfactory. Although the algorithm provided some information about the location of the activated area, the shape of the wavefront was not geometrically reasonable. A lack of geometric constraints (causing, for instance, non-physiological aberrations such as inactivated nodes inside the depolarized region) and, a lack of sensitivity (few activated nodes, in general, on the side and back of the heart) were obvious.

3 Improvement of the Level Sets Method Performance

To improve on these results, we made several modifications to the standard Level Sets algorithm. The first two modifications were rather straight-forward attempts to reinforce the spatial constraints and improve the sensitivity of the Level Sets approach. First, we adopted a "restart" method, reinitializing the Level Sets function every 10 iterations to avoid excessive deformation. In addition, to improve the sensitivity, a new constraint was added, which at each restart pushed the zero level set inwards. In other words, assuming that the next activated region would be inside the current activated region, the zero level set was forced to evolve even when the error was small at a specific node. Specifically, the Level Sets function was rebuilt as equal to a signed distance function

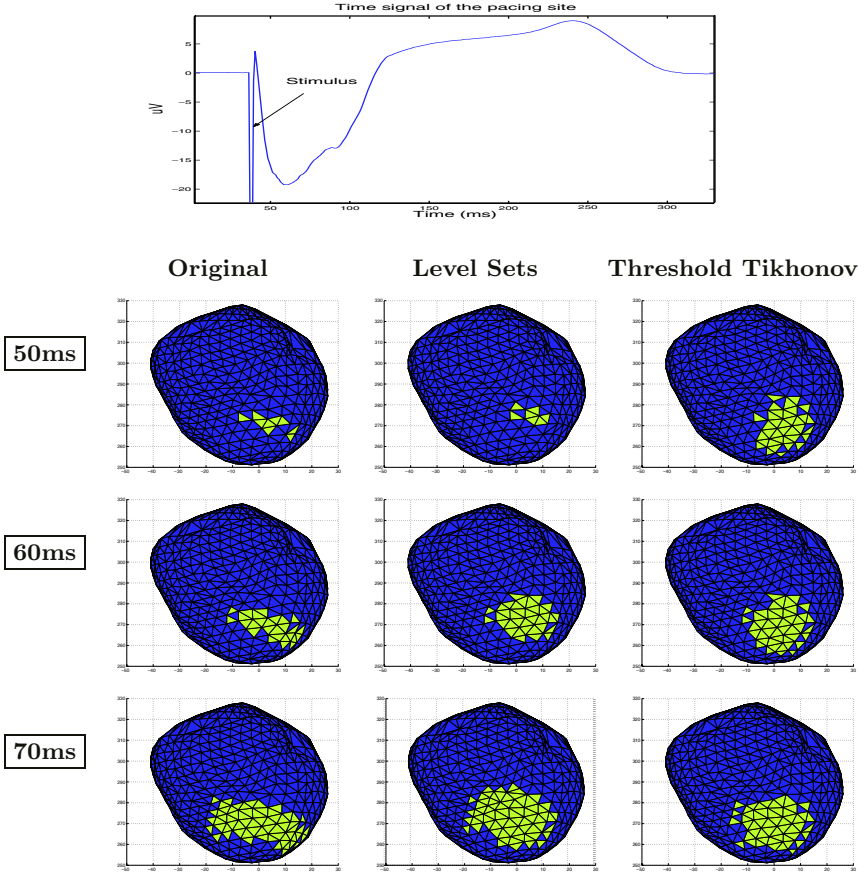


Fig. 1. Top panel: Time signal of the pacing site. Bottom: Activated and inactivated areas of the original data, and of the Level Sets and thresholded Tikhonov reconstructions. Time instants 50, 60 and 70ms, as seen on the top panel's waveform, are shown

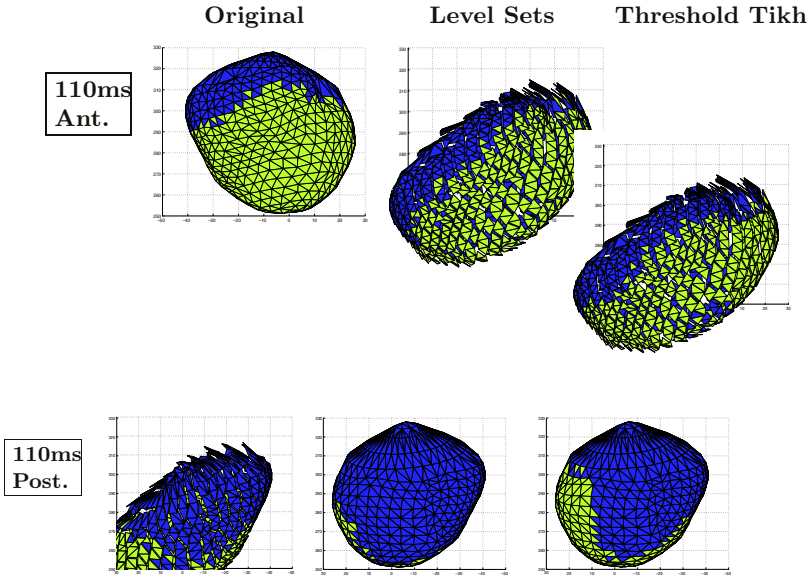


Fig. 2. Same format as bottom panel of previous figure, except that two views (Anterior, top, and Posterior, bottom) are shown for the same time instant, 110ms (using the time markings on the waveform shown in previous figure), later in QRS

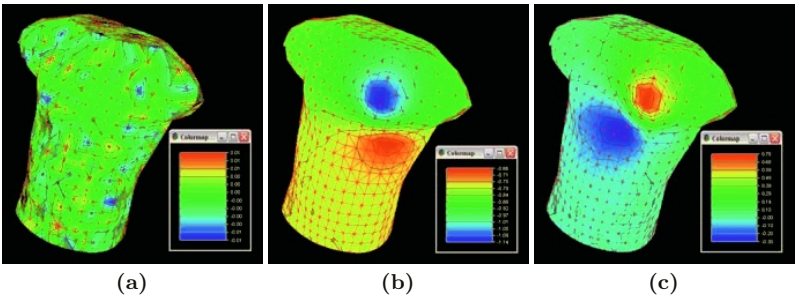
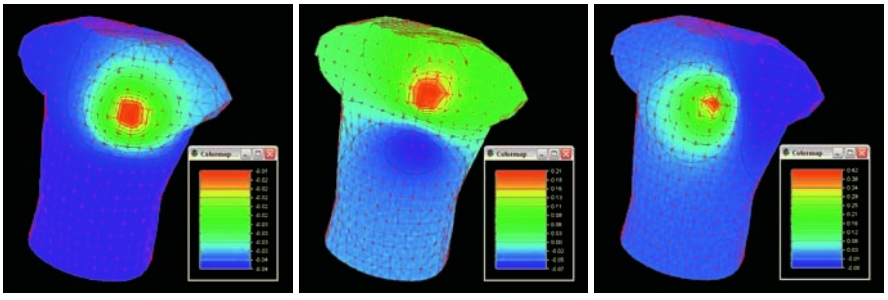


Fig. 3. (a) Front view of the residual error of the Tikhonov solution. (b) Front view of the residual error induced only by the two-level approximation of the original potentials. (c) Front view of Level Sets residual error. Time instant 70ms.

from each node to a zero level set. We shrank the zero level set first by simply taking the inward values at the border between positive and negatives points as the zero level set for the reinitialization.

In Fig. 1 and 2, we show the inverse solutions for different time instants of the electrocardiogram dataset described in Section 3. The time waveform at the pacing site is shown in the top panel of Fig. 1. The bottom panel of this figure contains maps at three time instants. Fig. 2 shows the same comparison for anterior and posterior views at a later time instant in QRS. From these results, we can conclude that Level Sets solution, after these modifications, is slightly better than Tikhonov in terms of shape information, capturing the anisotropy of the propagating front. This improvement is especially visible in the earlier time



(a) **(b)** **(c)**

Fig. 4. (a) Front view of the first singular vector of the matrix U, from the Singular Value Decomposition: $A = U\Sigma V^T$. (b) Front view of the second singular vector of the matrix U. (c) Front view of the third singular vector of the matrix U. Time instant 70ms

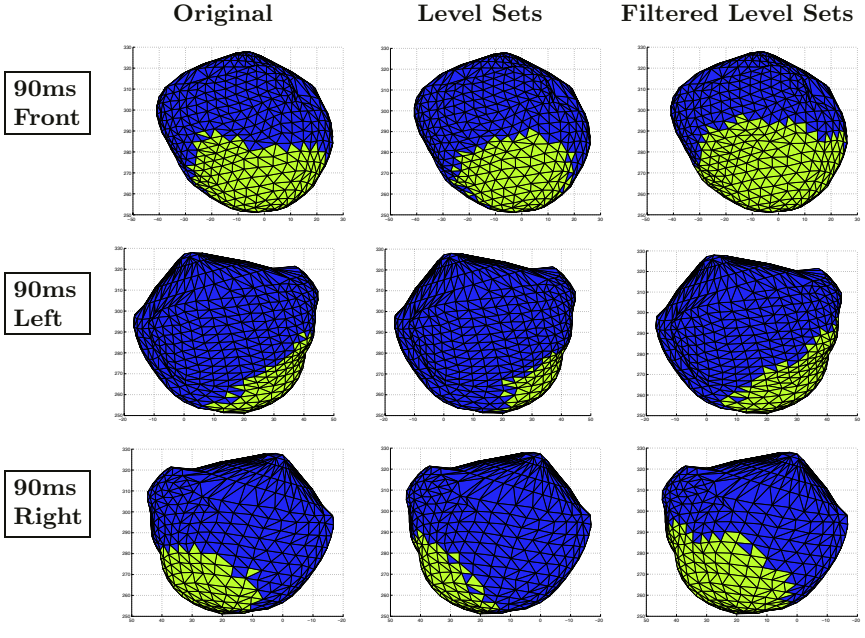


Fig. 5. Original activated and inactivated areas, Level Sets and filtered Level Sets ($k = 3$) activated and inactivated areas. Data from ECG dataset, time instant 90ms from front and side views, respectively

instants, when the propagating front has a characteristic elliptical shape. The solution also preserves some physiological behavior of the depolarization wavefront: closed activated area and no isolated activated or inactivated nodes inside inactivated or activated regions respectively. In this sense, Tikhonov fails for

later time instants, as seen in Fig. 2. In addition, Tikhonov is more sensitive to threshold level variation than Level Sets, due to its smoothing. A small threshold variation can cause a large change in the area of the Tikhonov-estimated activated region, while the Level Sets solution would hardly change. On the other hand, the Level Sets method was rather insensitive to some regions, especially in the back and sides of the heart. The Tikhonov solution behaved slightly better in those areas, as it can be seen in the posterior view of Fig. 2.

Studying this effect, we noted that the residual error was highly spatially correlated, unlike the Tikhonov error. This is due to the systematic error introduced by the two-level approximation. We illustrate this effect for one time instant in Fig. 3 [13]. We can see that the Level Sets residual error varies smoothly and slowly in space, so that it is mainly a low spatial frequency phenomenon. We decreased the effect of this error on the inverse solution by minimizing a filtered version of the residual error, where the low spatial frequencies of the residual error were removed. The idea is to concentrate the Level Sets iterations on matching the components of the data that are in a subspace orthogonal to these low-frequency components, since this is the error due to mismatch of the activation region rather than simply the effect of the thresholding itself.

In Fig. 4 [13], the three singular vectors of A corresponding to the three largest singular values are shown as torso maps. We observe that the dominant residual error components are similar to these first singular vectors. Hence, we can apply the Level Sets method by projecting the residual error onto the subspace spanned only by the singular vectors corresponding to indices higher than some small value k, i.e., calculating the speed function of the Level Sets evolution equation with a filtered version of the forward matrix, where the first k singular values of A have been set to zero.

In Fig. 5 we show the results for one time instant when we filter the residual to remove components in the subspace spanned by the first 3 singular vectors. We note that the activated area, especially in the right view, more closely approximates the original. We believe this is because of enhanced sensitivity to regions of the heart farthest from the anterior electrodes due to removal of the low-frequency threshold-induced residual error.

4 Discussion, Conclusions, and Future Work

The purpose of this work was to develop and evaluate an initial attempt at a Level Sets method that can be applied to non-invasive electrocardiography to reconstruct activation wavefronts on the epicardium. The principle attractive features are that we avoid any isotropy assumptions and develop a framework within which we can use spatial and geometric information that the physiology of the problem might provide. Essentially, our method is close to standard activation-based methods in terms of how we model the source. However, those methods depend on isotropy assumptions that Level Sets skips because it calculates a coarse model of epicardial potentials and uses that in a potential-based forward model.

As we describe, we introduced some modifications to improve the initial performance of the Level Sets algorithm. Adding new geometrical constraints increased spatial consistency and sensitivity. We also introduced a high-pass filtering of the residual error to remove part of the two-level quantization error, which helped to improve the sensitivity in the side and back areas of the heart. The geometrical constraints are visible in the results: evolution of the boundary and a closed activation area. Moreover the anisotropy of the wavefront is generally captured better than with a thresholded Tikhonov solution. The shape of the activated area recovered is better at some time instants than others but generally reflects the anisotropy induced by fiber direction. Finally, the method here uses the actual data to obtain the threshold levels; an independent method needs to be developed to estimate these values without *a priori* knowledge or to allow them to remain constant in time.

We are currently looking at several remaining aspects of this study. The residual filtering approach we used was just a first attempt, and we believe that a more careful study can lead to a more effective implementation. In addition there is a tradeoff between enhanced sensitivity and loss of robustness when filtering more singular vectors; thus we need an algorithm to estimate an appropriate number of singular vectors to remove. An idea of primary interest is to introduce more geometric physiological information into the model by incorporating fiber direction information, even from a different heart. The Level Sets speed function provides a perfect vehicle to include this *a priori* information. Relaxing the quantization of the heart potentials by defining a transition area (dividing the epicardial surface in three regions instead of only two), and/or modeling this region as an analytical function such as an arc-tangent [3], and anchoring the evolution around breakthrough's calculated using the Critical Point Theorem [4], are some other approaches to better incorporate known physiological constraints.

Acknowledgment

The work was supported in part by the National Institutes of Health, National Center for Research Resources, grant number 1-P41-RR12553-3.

References

1. Dana H. Brooks, Robert S MacLeod, *Electrical Imaging of the Heart: Electrophysical Underpinnings and Signal Processing Opportunities*, IEEE Sig. Proc. Mag., 14(1):24-42, 1997.
2. Rudy Y, Messinger-Rapport B : *The inverse problem of electrocardiography. Solutions in terms of epicardial potentials*, CRC Crit. Rev. Biomed. Eng., vol. 16, pp. 215-68, 1988.
3. G. J. M. Huiskamp and A. van Oosterom, *The depolarization sequence of the human heart surface computed from measured body surface potentials*, IEEE Trans. Biomed. Eng., vol. BME35, pp. 1047-1058, 1988.
4. G.J.M. Huiskamp and F.S. Greensite, *A new method for myocardial activation imaging*, IEEE Trans. Biomed. Eng., vol. 44, pp. 433-446, 1997.

5. Pullan A.J., Cheng L.K., Nash M.P., Bradley C.P., Paterson D.J., *Noninvasive electrical imaging of the heart: theory and model development*, Annals of Biomedical Eng., 29(10):817-36, October 2001.
6. T. Oostendorp and R.S. Macleod and A. van Oosterom, *Non-invasive determination of the activation sequence of the heart: Validation with invasive data*, Proc. IEEE Int. Conf. Eng. in Med. and Biol. Soc. 1997.
7. D.H. Brooks and G.F. Ahmad and R.S. MacLeod and G.M. Maratos, *Inverse Electrocardiography by Simultaneous Imposition of Multiple Constraints*, IEEE Trans Biomed Eng., Vol. 46, Number 1, Pages: 3–18, 1999.
8. B. Messnarz, B. Tilg, R. Modre, G. Fischer, F. Hanser, *A new spatiotemporal regularization approach for reconstruction of cardiac transmembrane potential patterns*, IEEE Trans Biomed Eng. ,Volume: 51 , Issue: 2 , Pages: 273–281, Feb. 2004.
9. Santosa F., *A Level-Set Approach for Inverse Problems Involving Obstacles*, ESAIM: Control, Optimisation and Calculus of Variations, Vol.1, pp. 17-33, January 1996.
10. Osher S., and Sethian J.A., *Fronts Propagating with Curvature-Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations*, Journal of Computational Physics, 79, pp. 12-49, 1988.
11. Sethian J.A., *Level Set Methods and Fast Marching Methods*, Cambridge University Press (Second Edition), 1999.
12. MacLeod R.S., Ni Q., Punske B., Ershler P.R., Yilmaz B., Taccardi B., *Effects of Heart Position on the Body-Surface ECG*, J. Electrocardiol., 33 Suppl: 229-237, 2000.
13. R.S. MacLeod and C.R. Johnson, *Map3d: Interactive scientific visualization for bioengineering data*, Proc. Int. Conf. IEEE Eng. Med. Bio. Soc. 1993, Pages: 30–31.