Models for Solving the Travelling Salesman Problem

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The Travelling Salesman Problem is a classic problem of Combinatorial Optimisation and involves routing around a number of cities in order to cover the minimum total distance. It is notoriously difficult to solve practical sized instances optimally. The classical Integer Programming formulation involves an exponential number of constraints.

Alternative formulations will be given which use less constraints (a polynomial number). These rely on (often ingenious) ways of introducing extra variables with a variety of real-life interpretations. The purpose of this talk will be to suggest the use of 'lateral' thinking in creating new formulations. These extra variables can be incorporated in extra 'logical constraints' which can help the solution process. The compactness of the formulations and the existence of extra variables which can be exploited in search strategies suggests they might be valuable if a Constraint Programming approach is adopted. This aspect is still to be investigated in detail.

Basically three distinct ideas are used in the different formulations.

Firstly sequence variables are introduced representing the sequence in which cities are visited. These can be used to prevent subtours by using $O(n^2)$ constraints (instead of the exponential number needed in the classical formulation). These extra variables also allow one to specify extra relations which help the solution process.

Secondly flow variables are introduced together with material balance constraints. These force the tour to be connected. $O(n^2)$ constraints are needed. If, however, the flows are split into distinct quantities, leading to a multicommodity flow formulation then $O(n^3)$ extra constraints are needed but the Linear Programming Relaxation of the model is of equal strength to that of the classical (exponential) formulation.

Thirdly staged variables are used with a third index representing the stage at which an arc is traversed. There are a number of ways of incorporating these variables into constraints to prevent subtours leading to models with $O(n)$ (remarkably) and $O(n^2)$ constraints. Again the existence of these variables allows one to specify extra conditions which could aid the solution process.

These formulations can be compared by projecting the polytopes of the Linear Programming relaxations into the same space. Remarkably the resultant polytopes are proper subsets of each other. The hierarchy of sizes of the poly-

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topes is independent of problem instance allowing one to rank the quality of the formulations in terms of the strength of the Linear Programming relaxation. This does not, however, mean that the relative qualities of the formulations will be the same if other Search and bounding procedures are used other than Linear Progaramming.

Finally the possibility of arriving at better formulations syntactically (as opposed to semantically) will be discussed.