G^1 Continuity Triangular Patches Interpolation Based on PN Triangles

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Abstract. There are currently many methods for triangular local interpolation: Given triangular meshes P in three dimension space, the given flat triangles are based only on the three vertices and three normal vectors, or PN triangles, construct a smooth surface that interpolates the vertices of P. In this paper a completely local interpolation scheme is presented to guarantee to join patches G^1 continuously around boundary curves of each PN triangle.

1 Introduction

It is well-known that the interpolation of curved triangular patches over PN triangular meshes, each triangle is based on the three vertices and the corresponding normal vectors, is an important tool in computer aided geometric design. The common approach [1,2,3] is to firstly create a cross boundary tangent vector field for each boundary and then to construct patches that agree with these cross boundary fields. Steven [4] gave a unifying survey of the published methods. Stephen Mann [5] discusses a method for increasing the continuity between two polynomial patches by adjusting their control points. But all these methods must know the information of two adjacent patches and they are not completely local method.

Stefan Karbacher [6] present a non-linear local subdivision scheme for the refinement of triangle meshes. Overveld [7] gave an algorithm for polygon subdivision based on point-normals. Alex Vlachos [8] introduced curved point-normal (PN) triangles to replace the flat triangle. In these methods the authors only consider the point-normals of the input triangle and they are completely local. But they only construct C^0 continuity meshes. In this paper our objective is to construct a smooth G^1 continuity surface.

2 Triangular G^1 Local Interpolation Based on PN Triangles

In this paper we shall employ the quartic Bézier polynomial to define a triangular patch with linear normal patch:

$$\rho(\lambda_1,\lambda_2,\lambda_3)=\sum_{i=0}^{4}\sum_{j=0}^{4-i}\mathrm{b}_{i,j,k}\mathrm{B}^4{}_{i,j,k}(\lambda_1,\lambda_2,\lambda_3);\quad B^4{}_{i,j,k}(\lambda_1,\lambda_2,\lambda_3)=\frac{4!}{i!\,j!\,k!}\lambda_1{}^i\lambda_2{}^j\lambda_3{}^k \quad (2.1)$$

$$Q_1(\lambda_1,\lambda_2,\lambda_3)=\lambda_1 q_{1,0,0}+\lambda_2 q_{0,1,0}+\lambda_3 q_{0,0,1} \quad (2.2)$$

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$b_{i,j,k}$ and $q_{i,j,k}$ are respectively the control points of curved patches of the surface ρ_i and corresponding linear normal patches Q_l .

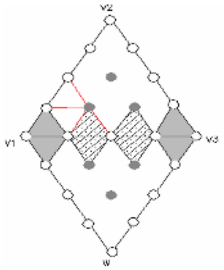


Fig. 1. Two patches meeting along a common boundary

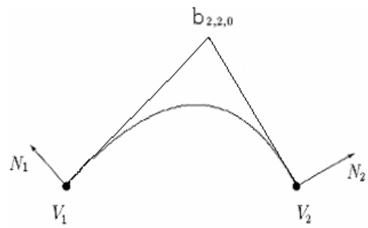


Fig. 2. Determination of $b_{2,2,0}$

As illustrated schematically in Fig.1, these two patches will share boundary edge V_1V_3. To meet with G^1 continuity, each of the four panels of four control points must be coplanar. We get G^1 continuity by adjusting the inner control points [5]. As illustrated by red lines, an inner point has four constrains, but only three freedoms. In order to solve the problem, we make the gray triangle degenerate to become a point. So for quartic Bézier polynomial control points we have:

$$b_{4,0,0} = b_{3,1,0} = b_{3,0,1} \,;\; b_{0,4,0} = b_{1,3,0} = b_{0,3,1} \,;\; b_{0,0,4} = b_{0,1,3} = b_{1,0,3} \qquad (2.3)$$

2.1 Determination of Boundary Curve

In the following parts of this section we only consider the edge $e_3\,(V_1V_2, \lambda_3 = 0)$ (see Fig. 2). We have $b_{4,0,0} = b_{3,1,0}$ and $b_{0,4,0} = b_{1,3,0}$, the next is to determine $b_{2,2,0}$. Here we decide the point $b_{2,2,0}$ by the intersection of three planes: 1) The first plane is decided by the point V_1 and the corresponding normal N_1; 2) The second plane is decided by the point V_2 and the corresponding normal N_2; 3) The third plane is decided by two lines, the one line is V_1V_2, another line is $((V_1+V_2)/2+ (N_1+N_2)/2)$.

2.2 Construct the Boundary Normal Curves

From equation (2.2) we can get $Q_1(\lambda_1, \lambda_2, 0) = \lambda_1 q_{1,0,0} + \lambda_2 q_{0,1,0}$ (on edge e_3), $Q_l(\lambda_1, \lambda_2, 0)$ must interpolate N_1 and N_2: $q_{1,0,0} = \omega_1 N_1$; $q_{0,1,0} = \omega_2 N_2$, Where ω_1, ω_2 are positive constants. If the boundary curve is G^1 continuity, we must have

$$Q_1 \bullet \left(\frac{\partial \rho}{\partial \lambda_1} - \frac{\partial \rho}{\partial \lambda_2}\right) = 0 \,, \text{ or } \; [\lambda_1 q_{1,0,0} + \lambda_2 q_{0,1,0}] \bullet [(b_{4,0,0} - b_{3,1,0})\lambda_1^{\,3}$$

$$+ 3(b_{3,1,0} - b_{2,2,0})\lambda_1^{\,2}\lambda_2 + 3(b_{2,2,0} - b_{1,3,0})\lambda_1\lambda_2^{\,2} + (b_{1,3,0} - b_{0,4,0})\lambda_2^{\,3}] = 0 \qquad (2.4)$$

The equation equals zero means that the coefficients of each term must be zero and we have $b_{4,0,0} = b_{3,1,0}$ and $b_{0,4,0} = b_{1,3,0}$, so equation (2.4) can be simplified to the following equation:

$$(q_{1,0,0} + q_{0,1,0}) \bullet (V_1 - V_2) = 0 \tag{2.5}$$

Now the problem become to finding reasonable values for ω_1, then use equation (2.5) to compute ω_2. Here we set $\omega_1 = 1$.

2.3 Decide the Inner Control Points

The next is to decide the still unknown inner control points $b_{1,2,1}, b_{2,1,1}$ and $b_{1,1,2}$. In order to construct cross boundary G^1 continuity, we must hold (the edge e_3):

$$Q_l(\lambda_1, \lambda_2, 0) \bullet (\frac{\partial \rho}{\partial \lambda_2} - \frac{\partial \rho}{\partial \lambda_3}) = (\lambda_1 q_{1,0,0} + \lambda_2 q_{0,1,0}) \bullet (\frac{\partial \rho}{\partial \lambda_2} - \frac{\partial \rho}{\partial \lambda_3}) = 0 \tag{2.6}$$

The equation equals zero means that the coefficients of each term must be zero and associate with equation (2.5), we can get following equations:

$$N_1 \bullet (V_1 - b_{2,1,1}) = 0;\ N_2 \bullet (V_2 - b_{1,2,1}) = 0;\ \omega_2 N_2 \bullet (V_2 - b_{2,1,1}) + \omega_1 N_1 \bullet (V_2 - b_{1,2,1}) = 0 \tag{2.7}$$

Using the same computation for e_1 and e_2, we have:

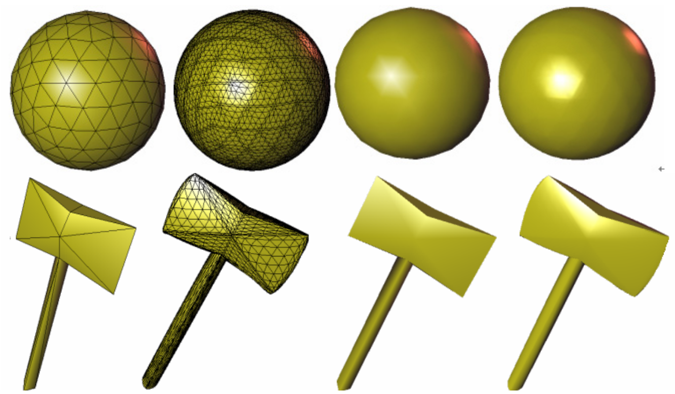
$$N_3 \bullet (V_3 - b_{1,1,2}) = 0;\ N_2 \bullet (V_2 - b_{1,2,1}) = 0;\ \omega_3 N_3 \bullet (V_3 - b_{1,2,1}) + \omega_4 N_2 \bullet (V_3 - b_{1,1,2}) = 0$$

$$N_3 \bullet (V_3 - b_{1,1,2}) = 0;\ N_1 \bullet (V_1 - b_{2,1,1}) = 0;\ \omega_5 N_1 \bullet (V_1 - b_{1,1,2}) + \omega_6 N_3 \bullet (V_1 - b_{2,1,1}) = 0 \tag{2.8}$$

Unite equation (2.7) and (2.8), we get six equations, but we have 9 unknown values. Here we optimized the fairness of the given patch by reducing its curvature [9]. A standard measure for the surface quality in geometric modeling is the thin plate energy: $E(s) \approx \int_{\Delta} F^2{}_{uu} + 2F^2{}_{uv} + F^2{}_{vv}$. where $\triangle$ denotes the domain triangle $V_1V_2V_3$. Now we can determine the free parameters by minimizing E(s).

3 Conclusion

Methods for local interpolation of triangulated, parametric data have existed for many years and received more and more attention. However it is difficult to construct cross boundary continuity surface. So the presented local methods are C^0 continuity. In this paper we present a completely local method to construct G^1 boundary continuity patches based on the point-normals of the inputted flat triangles. The examples used and shown (see Fig.3,4) demonstrate that this algorithm can produce a very smooth mesh from an initial coarse mesh model. However in order to construct G^1 boundary



Figs. 3,4. (from left to right) (a) initial mesh (b) mesh with our method (c) shading model (initial mesh) (d) shading for our scheme

continuity patches, we make the triangle attached to each vertex degenerate to become a point and introduce non-regular points. The investigation about non-regular points is not involved in this paper.

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