# *Τα Παιδία Παίζει* The Interaction Between Algorithms and Game Theory*

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The theories of algorithms and games were arguably born within a year of each other, in the wake of two quite distinct breakthroughs by John von Neumann, in the former case to investigate the great opportunities – as well as the ever mysterious obstacles – in attacking problems by computers, in the latter to model and study rational selfish behavior in the context of interaction, competition and cooperation. For more than half a century the two fields advanced as gloriously as they did separately. There was, of course, a tradition of computational considerations in equilibria initiated by Scarf [13], work on computing Nash and other equilibria [6, 7], and reciprocal isolated works by algorithms researchers [8], as well as two important points of contact between the two fields *à propos* the issues of repeated games and bounded rationality [15] and learning in games [2]. But the current intensive interaction and cross-fertilization between the two disciplines, and the creation of a solid and growing body of work at their interface, must be seen as *a direct consequence of the Internet*.

By enabling rapid, well-informed interactions between selfish agents (as well as by being itself the result of such interactions), and by creating new kinds of markets (besides being one itself), the Internet challenged economists, and especially game theorists, in new ways. At the other bank, computer scientists were faced for the first time with a mysterious artifact that was not designed, but had emerged in complex, unanticipated ways, and had to be approached with the same puzzled humility with which other sciences approach the cell, the universe, the brain, the market. Many of us turned to Game Theory for enlightenment.

The new era of research in the interface between Algorithms and Game Theory is rich, active, exciting, and fantastically diverse. Still, one can discern in it three important research directions: *Algorithmic mechanism design, the price of anarchy*, and *algorithms for equilibria*.

If mainstream Game Theory models rational behavior in competitive settings, *Mechanism Design* (or *Reverse Game Theory*, as it is sometimes called) seeks to create games (auctions, for example) in which selfish players will behave in ways conforming to the designers objectives. This modern but already

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mathematically well-developed branch of Game Theory received a shot in the arm by the sudden influx of computational ideas, starting with the seminal paper [9]. Computational Mechanism Design is a compelling research area for both sides of the fence: Several important classical existence theorems in Mechanism Design create games that are very complex, and can be informed and clarified by our fields algorithmic and complexity-theoretic ideas; it presents a new genre of interesting algorithmic problems; and the Internet is an attractive theater for incentive-based design, including auction design.

Traditionally, distributed systems are designed centrally, presumably to optimize the sum total of the users objectives. The Internet exemplified another possibility: A distributed system can also be designed by the interaction of its users, each seeking to optimize his/her own objective. Selfish design has advantages of architectural and political nature, while central design obviously results in better overall performance. The question is, how much better? The *price of anarchy* is precisely the ratio of the two. In game-theoretic terms, it is the ratio of the sum of player payoffs in the worst (or best) equilibrium, divided by the payoff sum of the strategy profile that maximizes this sum. This line of investigation was initiated in [5] and continued by [11] and many others. That economists and game theorists had not been looking at this issue is surprising but not inexplicable: In Economics central design is not an option; in Computer Science it has been the default, a golden standard that invites comparisons. And computer scientists have always thought in terms of ratios (in contrast, economists favor the difference or "regret"): The approximation ratio of a hard optimization problem [14] can be thought of as the price of complexity; the competitive ratio in an on-line problem [4] is the price of ignorance, of lack of clairvoyance; in this sense, the price of anarchy had been long in coming.

This sudden brush with Game Theory made computer scientists aware of an open algorithmic problem: *Is there a polynomial-time algorithm for finding a mixed Nash equilibrium in a given game?* Arguably, and together with factoring, this is the most fundamental open problem in the boundary of P and NP: Even the 2-player case is open – we recently learned [12] of certain exponential examples to the pivoting algorithm of Lemke and Howson [6]. Even though some game theorists are still mystified by our fields interest efficient algorithms for finding equilibria (a concept that is not explicitly computational), many more are starting to understand that the algorithmic issue touches on the foundations of Game Theory: An intractable equilibrium concept is a poor model and predictor of player behavior. In the words of Kamal Jain "If your PC cannot find it, then neither can the market". Research in this area has been moving towards games with many players [3, 1]), necessarily under some succinct representation of the utilities (otherwise the input would need to be astronomically large), recently culminating in a polynomial-time algorithm for computing correlated equilibria (a generalization of Nash equilibrium) in a very broad class of multiplayer games [10].

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