A Quantity-Time-Based Dispatching Policy for a VMI System

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Abstract. In a Vendor-Managed Inventory (VMI) system, the supplier or the distributor is authorized to coordinate and consolidate the inventories at the retailers. The advantage of VMI is that the bullwhip effect can be minimized and the stock-out situations can also be reduced. Moreover, it provides a framework for synchronizing transportation decisions and hence reduce the transportation cost significantly. In this paper, we present an analytic model for quantity-time-based dispatching policy. The model discussed here takes into the account of the inventory cost, the transportation cost, the dispatching cost and the re-order cost. Since a new inventory cycle begins whenever there is a dispatching of products, the long-run average costs of the model can be obtained by using the renewal theory. We also derive a closed form solution of the optimal dispatching policy.

1 Introduction

In this paper, we consider a Vendor-Managed Inventory (VMI) system consisting of a vendor, a manufacturer and groups of retailers at different regions, see Figure 1. An analytic model of similar framework focusing on the Emergency Lateral Transshipment (ELT) has been studied by Ching [3]. Recent development in supply chain management focus on the coordination of different functional specialties and the integration of inventory control and transportation logistics, see Thomas [12] for instance. VMI is a supply chain initiative where the supplier or the distributor is responsible for all decisions regarding inventories at the retailers. Usually demands should be shipped immediately, but the vendor has the right of not delivering small orders to a region until an accumulated amount or an agreeable dispatching time. VMI requires the retailers to share the demands information with the supplier so as to allow making inventory replenishment decisions. This is usually achieved by using online data-retrieval systems and Electronic Data Interchange (EDI) technology, see for instance Chopra and Meindl [5] and Dyer and Nobeoka [6]. As a result, through the sharing of demands information the bullwhip effect can be reduced [5, 9]. The bullwhip effect is the distortion of demands information transferred from the downstream retailers to the upstream suppliers, see Lee and Padmanabhan [8]. The current focus of

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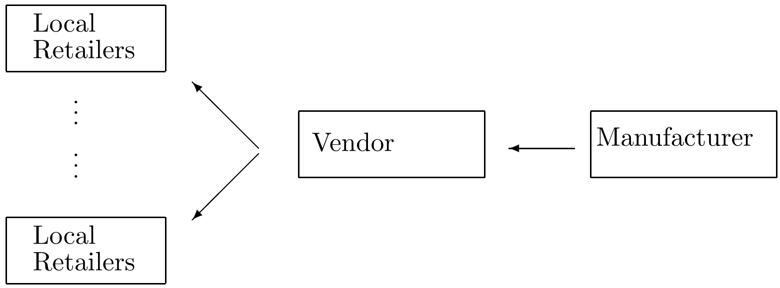

Fig. 1. The Supply Chain

VMI is the value of information sharing within a supply chain. Significant savings can also be achieved by carefully incorporating shipment consolidation and dispatching with stock replenishment decision in a VMI system, Higginson and Bookbinder [7]. Here shipment consolidation refers to the management of combining small shipments together in order to take the advantage of the decreased per unit transportation cost. Simulation is a useful tool for studying freight consolidation, Masters [10]. Other analytical approaches such as queueing theory and Markov decision process have been proposed to solve the consolidation models, see Higginson and Bookbinder [7] and Minkoff [11].

There are two types of dispatching policies: the quantity-based dispatching policy and the time-based dispatching policy, see for instance Higginson and Bookbinder [7]. A quantity-based policy dispatches whenever there is an accumulated load of size q. In this model, one has to determine the optimal dispatching size q and the optimal number of dispatches in each replenishment cycle. On the other hand, a time-based dispatching policy dispatches an accumulated load in every period of T. In this model, one has to determine the optimal quantity of replenishment Q and the optimal dispatching period T in each replenishment cycle. The time-based shipment consolidation have became a part of the transportation contract among the members of a supply chain and Delivery Time Guarantee (DTG) is a common marketing strategy in the competition of marketplaces, see Ching [4]. A VMI model based on time-based dispatching policy has been proposed and studied by Cetinkaya and Lee [2], they also discussed both advantages and disadvantages of the time-based and quantity-based dispatching policies. They remark that it is interesting to consider a model for the case of quantity-time-based dispatching policy. Here we propose an analytic model based on the simplified framework of [2] for the quantity-time-based dispatching policy. Our model takes into the account of the inventory cost, the transportation cost, the dispatching cost and the re-order cost. We remark that in modern E-business supply chain, inventory handling and transportation of products are the major costs, see Chopra and Meindl [5]. The dispatching cost is associated with the consolidation of shipment and the re-order cost corresponds to the inventory replenishment. In our model, for simplicity of discussion we assume that the demands of the retailers at a region is a simple Poisson process, the vendor applies a (q, Q, T) policy for replenishing the inventory and the lead time of the replenishment is assumed be negligible. The definition of a (q, Q, T) policy will be introduced shortly in Section 2. Since a new inventory cycle begins whenever there is a dispatching of products, the long-run average costs of the model can be obtained by using the renewal theory [1]. Moreover, closed form solution of optimal dispatching policy is also obtained.

The rest of the paper is organized as follows. In Section 2, we present the model for the quantity-time-based dispatching policy. In Section 3, we give a cost analysis of the model and derive the optimal dispatching policy with a numerical example. Finally, concluding remarks are given in Section 4 to conclude the paper and address further research issues.

2 The Quantity-Time-Based Dispatching Model

In this section, we give a model for quantity-time-based dispatching policy. In order to keep the models mathematically tractable, we consider models based on the simplified model discussed in [2]. Let us first define the following notations.

(i) λ^{-1}, the mean inter-arrival time of one unit of demand
(ii) I, the unit inventory cost per unit of time
(iii) D, the dispatching cost
(iv) F, the unit transportation cost
(v) C, the re-order cost
(vi) q, the size of a dispatching (quantity-based model)
(vii) r, the number of dispatches in a cycle (quantity-based model)
(viii) Q, the replenishment quantity (time-based model)
(ix) T, the dispatching period (time-based model)

Under this policy, a (q, Q, T) inventory replenishment is assumed. This means that the size of the replenishment is such that to clear the shortage and bring the inventory level back to Q. Moreover, a dispatching decision is made at the time $\min\{T_q, T\}$ where T_q is the time when a demands of size q is reached. The objective of this problem is to find the optimal values of q, Q and T such that the average long-run cost is minimized. The followings are the assumptions of the model.

(A1) The inventory level is under continuous review.
(A2) The vendor dispatches a load regularly for every period of T. If a size of demands q is accumulated before the planned dispatching time T, the vendor dispatches a load immediately.
(A3) The lead time of inventory replenishment is assumed to be negligible.
(A4) At the time of a dispatch, if the available inventory is not enough to clear the demand, we assume that the vendor can immediate replenish its stock from the manufacturer.

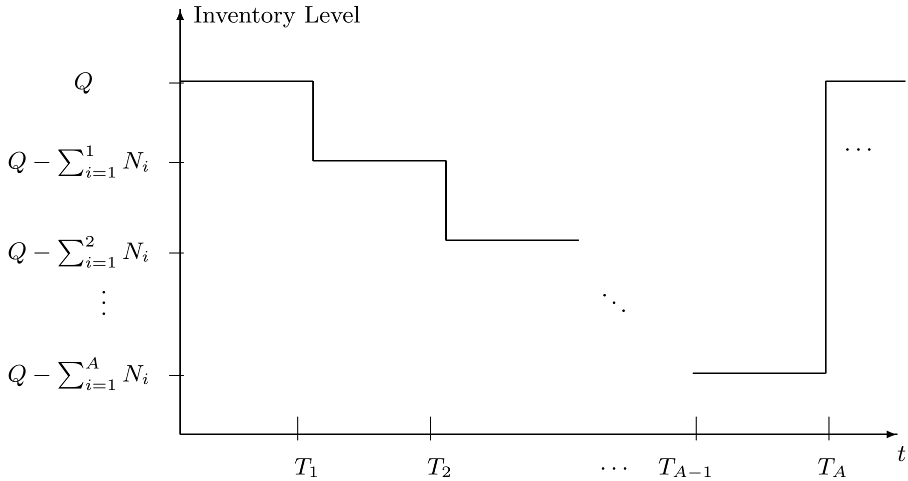

Fig. 2. The Inventory Level of a Cycle

A realization of the inventory levels in a replenishment cycle is shown in Figure 2. Here $N_i = N(T_i - T_{i-1})$ is the number of demands in the time interval $[T_{i-1}, T_i]$ and $T_i (i = 1, 2, \ldots, A)$ are the instants where a dispatch takes place. We note that $T_i = \min\{S_q, T\}$ where S_q is the time instant when the size of the demands is q. We remark that all T_i and S_q are random variables. At the time instant T_A (for certain A), the system is out of stock and an order is placed and arrived at once (as we assume zero lead time). Here

$$A = \inf \left\{ a : \sum_{i=1}^{a} N(T_i - T_{i-1}) > Q \right\}$$

and A is a random variable representing the number of dispatch in a replenishment cycle. Moreover, the random variable $N(T)$ follows the Poisson distribution with mean λT. We aim at obtaining the optimal values of q, Q and T such that the average long-run cost of the system is minimized.

3 A Cost Analysis

In this section, we derive the expected long-run cost of the system by using renewal theory [1]. We note that a new inventory cycle begins whenever there is a dispatching of products, therefore the long-run average costs can be obtained by using the renewal theory. We will first derive the expected size of a dispatch $E(q_d)$ and an approximation for the expected number of dispatches $E(A)$ in each replenishment cycle. We then derive an approximate average cost.

We let d_T be the probability of dispatching a load at a planned dispatching time T, then

$$d_T = P(N(T) < q) = \sum_{i=0}^{q-1} \frac{(\lambda T)^i e^{-\lambda T}}{i!}.$$

Therefore the expected quantity of a dispatch is

$$E(q_d) = q(1-d_T) + E(N(T))d_T = q(1-d_T) + \lambda T d_T.$$

The expected time of a single dispatch is given by

$$E(T_q)(1-d_T) + Td_T = \frac{q(1-d_T)}{\lambda} + Td_T.$$

Meanwhile, since all stocks will be used up in a replenishment cycle, we have

$$E(\text{Number of dispatches}) \times E(\text{dispatching quantity}) > Q.$$

Also, since the stock is sufficient for the demand before a replenishment cycle ends, we have

$$Q > [E(\text{Number of dispatches}) - 1] \times E(\text{dispatching quantity}).$$

Therefore an upper bound and a lower bound of the expected number of dispatches are given by

$$N_{max} = \frac{Q}{q(1-d_T) + \lambda T d_T} + 1$$

and

$$N_{min} = \frac{Q}{q(1-d_T) + \lambda T d_T}$$

respectively. In view of the above bounds, we can approximate $E(A)$ by using N_{min}.

We then derive an approximate average long-run cost for the quantity-time-based model. Again we apply the renewal reward theorem, the average long-run cost is given by

$$C(q,Q,T) = \frac{\text{Replenishment Cycle Cost}}{\text{Replenishment Cycle Length}}.$$

(i) The expected inventory cost per cycle is given by

$$\begin{aligned}
& I \times \sum_{i=1}^{E(A)} [i \times E(T_i) \times E(N([T_i - T_{i-1}]))] \\
&= I \times \sum_{i=1}^{E(A)} \left\{ i \times \left[\frac{q(1-d_T)}{\lambda} + Td_T \right] \times [q(1-d_T) + \lambda T d_T] \right\} \\
&= I \times \frac{E(A)[E(A)+1]}{2} \times \left[\frac{q(1-d_T)}{\lambda} + Td_T \right] \times [q(1-d_T) + \lambda T d_T] \\
&= \frac{IQ}{2\lambda} \left\{ Q + q(1-d_T) + \lambda T d_T \right\}.
\end{aligned} \quad (1)$$

(ii) The expected dispatching cost per cycle is given by

$$D \times E(A) = \frac{DQ}{q(1-d_T)+\lambda T d_T}.$$

(iii) The expected transportation cost per cycle is given by

$$\begin{aligned} F \times E(A) \times E(N(T_i - T_{i-1})) &= F \times \frac{Q[q(1-d_T)+\lambda T d_T]}{q(1-d_T)+\lambda T d_T} \\ &= FQ. \end{aligned}$$

(iv) The expected re-order cost per cycle is given by C.
(v) The expected length of a replenishment cycle is Q/λ.

Hence the expected cost is

$$C(q,Q,T) = \frac{IQ + Iq(1-d_T) + I\lambda T d_T}{2} + \frac{D\lambda}{q(1-d_T)+\lambda T d_T} + F\lambda + \frac{C\lambda}{Q}. \quad (2)$$

If we denote

$$V = q(1-d_T) + \lambda T d_T > 0$$

then (2) can be rewritten as

$$C(Q,V) = \frac{IQ + IV}{2} + \frac{D\lambda}{V} + \lambda F + \frac{C\lambda}{Q}. \quad (3)$$

From (3) we have

$$\begin{cases} \frac{\partial C(Q,V)}{\partial Q} &= \frac{I}{2} - \frac{C\lambda}{Q^2} \\ \frac{\partial C(Q,V)}{\partial V} &= \frac{I}{2} - \frac{D\lambda}{V^2} \\ \frac{\partial^2 C(Q,V)}{\partial Q^2} &= \frac{2C\lambda}{Q^3} \\ \frac{\partial^2 C(Q,V)}{\partial V^2} &= \frac{2D\lambda}{V^3}. \end{cases} \quad (4)$$

We note that the cost function $C(Q,V)$ is strictly convex for positive Q and V. Thus the unique global minimum for positive Q and V can be obtained by solving

$$\begin{cases} \frac{\partial C(Q,V)}{\partial Q} = \frac{I}{2} - \frac{C\lambda}{Q^2} = 0 \\ \frac{\partial C(Q,V)}{\partial V} = \frac{I}{2} - \frac{D\lambda}{V^2} = 0. \end{cases} \quad (5)$$

The optimal pair is then given by

$$(Q^*, V^*) = (\sqrt{\frac{2C\lambda}{I}}, \sqrt{\frac{2D\lambda}{I}}).$$

Therefore the optimal solution for minimizing $C(q,Q,T)$ is given by (q^*, Q^*, T^*), where

$$Q^* = \sqrt{\frac{2C\lambda}{I}}$$

and q^*, T^* satisfy the equation

$$q(1-d_T) + \lambda T d_T = \sqrt{\frac{2D\lambda}{I}}$$

where $q \in N$ and $T \in (0, \infty)$. One possible choice of the optimal solution is the following:

$$(q^*, Q^*, T^*) = (\sqrt{\frac{2C\lambda}{I}}, \sqrt{\frac{2D\lambda}{I}}, \sqrt{\frac{2D}{\lambda I}}).$$

We note that if we set q^* to be large enough, then d_T will tend to 1 and

$$T^* = \sqrt{\frac{2D}{\lambda I}}.$$

Similarly if we set T^* to be large enough, d_T will tend to zero, then

$$q^* = \sqrt{\frac{2D\lambda}{I}}.$$

Example 1. Suppose that $\lambda = 10, D = 50$ and $I = 5$ then we have

$$V^* = \sqrt{\frac{2D\lambda}{I}} \approx 14.14.$$

In Table 1, we give some possible values of q and T such that $q(1-d_T) + \lambda T d_T$ is close to 14.14.

Table 1. Solutions for q and T

q	T	$q(1-d_T) + \lambda T d_T$
15	1.20	12.684
15	1.25	13.187
16	1.30	13.709
18	1.35	14.126
23	1.40	14.150
14	1.45	14.206
14	1.50	14.363

4 Concluding Remarks

In this paper, we discuss a Vendor-Managed Inventory (VMI) system where the vendor is authorized to coordinate and consolidate the inventory at the retailers. We present an analytic model for the quantity-time-based dispatching policy. Moreover, closed form solution of optimal dispatching policy is also obtained. For ease of discussion, the effect of the lead time in the inventory replenishment was not included in our model. It will be interesting to extend our model to include the lead time.

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