Possibilities for Obtaining the Derivative of a Received Signal Using Computer-Driven Second Order Oscillators

Andreea Sterian and Ghiocel Toma*

Politehnica University, Department of Physics, Bucharest, Romania

Abstract. As it is known, a first step in modeling dynamic phenomena consists in measuring with higher accuracy some physical quantities corresponding to the dynamic system. However, for suddenly-emerging phenomena ,the data acquisition can't be restricted at sampling procedures for a received signal (corresponding to a certain physical quantity). A significant quantity is represented by the derivative (the slope) of the received signals, because all dynamical models must take it into consideration. Usually the derivative of a received signal is obtained by filtering the received signal and by dividing the difference between the filtered values of the signal at two different moments of time at the time difference between these time moments. Many times these filtering and sampling devices consists of low-pass filters represented by asymptotically stable systems, sometimes an integration of the filter output over a certain time interval being added. However, such a structure is very sensitive at random variations of the integration period, and so it is recommended the signal which is integrated to be approximately equal to zero at the end of the integration period. It will be shown that the simplest structure with such properties is represented by an oscillating second order computer-driven system working on a time period.

1 Introduction

It is known that the derivative of a received signal is usually obtained by filtering the received signal (using low-pass filters) and by dividing the difference between the filtered values of the signal at two different moments of time at the time difference between these time moments. The time difference Δt is very small and it is usually set by oscillators having a higher accuracy, and so it can be considered as constant.

Usually the filtering device consists of low-pass filters represented by asymptotically stable systems, sometimes an integration of the filter output over a certain time interval being added. However, such a structure is very sensitive at random variations of the integration period, and so it is recommended the signal

* This work is partly supported by the European Commision under contract EVG1-CT-2002-0062 (OPTSDET).

O. Gervasi et al. (Eds.): ICCSA 2005, LNCS 3482, pp. 585–591, 2005.

which is integrated to be approximately equal to zero at the end of the integration period. So we must try to use oscillating systems for filtering the received signal (just in this case the filtered signal and its slope are approximately zero at the end of a certain time interval). However, for avoiding instability of such oscillating systems we must add certain electronic devices (gates) controlled by computer commands, so as to restore initial null conditions for the oscillating system. Before designing such a structure, we must notice that filtering and sampling devices consisting of low-pass filters of first or second order have the transfer function

$$H(s) = \frac{1}{T_0 s + 1} \tag{1}$$

(for a first order system) and

$$H(s) = \frac{1}{T_0^2 s^2 + 2bT_0 s + 1} \tag{2}$$

(for a second order system). They attenuate an alternating signal of angular frequency $\omega >> \omega_0 = 1/T_0$ about ω/ω_0 times (for a first order system) or about $(\omega/\omega_0)^2$ times (for a second order system). The response time of such systems at a continuous useful signal is about $4 - 6T_0$ ($5T_0$ for the first order system and $4T_0/b$ for the second order system). If the signal given by the first or second order system is integrated over such a period, a supplementary attenuation for the alternating signal of about $4 - 6\omega/\omega_0$ can be obtained.

However, such structures are very sensitive at the random variations of the integration period (for unity-step input, the signal, which is integrated, is equal to unity at the sampling moment of time). Even if we use oscillators with a very high accuracy, such random variations will appear due to the fact that an electric current charging a capacitor usually performs the integration. This capacitor must be charged at a certain electric charge Q necessary for further conversions; this electric charge can't be smaller than a certain value Q_{lim}, while it has to supply a minimum value I_{min} for the electric current necessary for conversions on the time period t_{conv} required by these conversions, the relation

$$Q_{lim} = I_{min} t_{conv} \tag{3}$$

being valid. So the minimum value $I_{int}(min)$ for the electric current charging the capacitor in the integrator system is determined by the relation

$$I_{int}(min) = \frac{Q_{lim}}{t_{int}} \tag{4}$$

where t_{int} is the integration period required by the application (knowing the sampling frequency f_s, we can approximately establish t_{int} using the relation $t_{int} = 1/f_s$). So the current charging the capacitor can't be less than a certain value. thus random variations of the integration period will appear due to the fact that the random phenomena are generated when a nonzero electric current is switched off.

These random variations can't be avoided if we use asymptotically stable filters. By the other hand, an improvement in an electrical scheme used for integrators in analog signal processing (see [1], [2]) can't lead to a significant increasing in accuracy, as long as such electronic devices perform the same task (the system has the same transfer function). There are also known techniques for reducing the switching noise in digital systems, but such procedures can be applied only after the analog signal is filtered and sampled, so as to be prepared for further processing. So we must give attention to some other kind of transfer functions and to analyze their properties in case of filtering and sampling procedures.

Mathematically, an ideal solution consists in using an extended Dirac function for multiplying the received signal before the integration (see [3]), but is very hard to generate thus extended Dirac functions (a kind of acausal pulses) using nonlinear differential equations (see [4] for more details). So we must use some simple functions for solving our problem.

2 The Necessity of Using Oscillating Systems for Filtering the Received Signal

As it has been shown, first or second order stable systems are not suitable for filtering the received signal in case of integration and sampling procedures. They do not have the accuracy required by the operation

$$\frac{u(t_2) - u(t_1)}{t_2 - t_1} = \frac{u(t_2) - u(t_1)}{\Delta t} \tag{5}$$

We need a system having the following property: starting to work from initial null conditions, for a unity step input it must generate an output and a derivative of this output equal to zero at a certain moment of time (the condition for the derivative of the output to be equal to zero has been added so as the slope and the first derivative of the slope of the signal which is integrated to be equal to zero at the sampling moment of time, when the integration is interrupted). It is quite obvious that the single second order system possessing such properties is the oscillating second order system having the transfer function

$$H_{osc} = \frac{1}{T_0^2 s^2 + 1} \tag{6}$$

receiving a step input and working on the time interval $[0, 2\pi T_0]$. For initial conditions equal to zero, the response of the oscillating system at a step input with amplitude A will have the form

$$y(t) = A\left(1 - \cos\left(\frac{t}{T_0}\right)\right) \tag{7}$$

By integrating this result on the time interval $[0, 2\pi T_0]$, we obtain the result $2\pi A T_0$, and we can also notice that the quantity which is integrated and its slope are equal to zero at the end of the integration period. Thus the influence of the random variations of the integration period (generated by the switching phenomena) is practically rejected.

Analyzing the influence of the oscillating system upon an alternating input, we can observe that the oscillating system attenuates about $(\omega/\omega_0)^2$ times such an input.

The use of the integrator leads to a supplementary attenuation of about $[(1/(2\pi)(\omega/\omega_0)]$ times. The oscillations having the form

$$y_{osc} = a\sin(\omega_0 t) + b\cos(\omega_0 t) \tag{8}$$

generated by the input alternating component have a lower amplitude and give a null result after an integration over the time interval $[0, 2\pi T_0]$.

As a conclusion, such a structure provides practically the same performances as a structure consisting of an asymptotically stable second order system and an integrator (response time of about $6T_0$, an attenuation of about $(1/6)(\omega/\omega_0)^3$ times for an alternating component having frequency ω) moreover being less sensitive at the random variations of the integration period. It is the most suitable for the operation

$$\frac{u(t_2) - u(t_1)}{\Delta t} \tag{9}$$

where $\Delta t = t_2 - t_1$. For restoring the initial null conditions after the sampling procedure (at the end of the working period) some electronic devices must be added. In the next section it will be shown that these devices must be represented by computer-driven electronic gates which must discharge certain capacitors at the end of each period of the oscillating system (a period corresponding to a working time).

3 The Necessity of Comparing the Value of the Derivative over Two Working Periods

The most simple structure having the transfer function

$$H = \frac{1}{T_0^2 s^2 + 1} \tag{10}$$

consists of some operational amplifier for lower frequency, with resistors R_0 connected at the (-) input and capacitors C_0 connected between the output and the (-) input (the well-known negative feedback) together with computer-driven electronic gates (for discharging these capacitors at the end of each working period); no resistors and capacitors are connected between the (+) connection and the "earth" (as required by the necessity of compensating the influence of the polarizing currents at the input of the amplifiers), so as to avoid instabilities of operational amplifiers at higher frequencies. In figure 1 is represented such a structure, having the period of oscillation of about $15\mu s$ (the capacity of C_1 and C_2, corresponding to C_0, being set to $69pF$). However, tests have shown that,

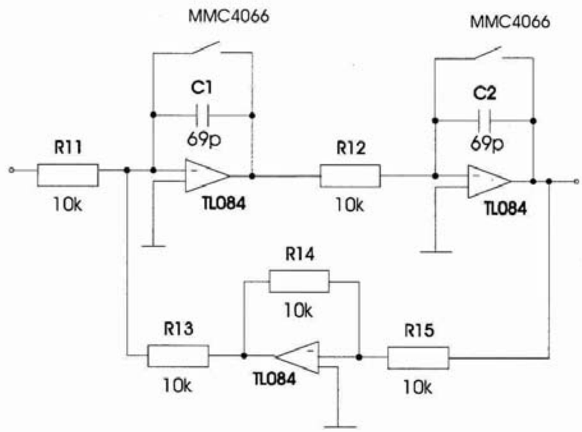

Fig. 1. Circuit for medium frequency

for a constant input A, the output is about $0.35A$ at the end of a period)instead of zero). So the capacitors were replaced by some others, having a capacity 10 times greater than the capacity of C_1 and C_2 in figure 1. Thus the time period became equal to $150\mu s$. For a constant input A, the output of the oscillating system is represented in figure 2. It can be noticed that the output is about $0.1A$ at the end of a complete oscillation. The output of the oscillating system can be integrated over a period using a similar device (based on an operational amplifier with a resistor R_i connected at the (-) input and a capacitor C_i connected on the negative feedback loop), at the end of the period the integrated signal being sampled. For a robust integration, we must chose as sampling moment of time the moment when the output is equal to zero (thus the working time interval presents a small difference as related to a period of the oscillating system, a scan be noticed studying figure 2). The time constants T_i - for the integrating system - and T_0 -for the oscillating system - have the form

$$T_i = R_i C_i, \;\; T_0 = R_0 C_0 \tag{11}$$

If the resistors R_0, R_i and the capacitors C_0, C_i are made of the same material, the coefficient for temperature variation will be the same for resistors and will be also the same for capacitors. Thus the ratio

$$A\frac{2\pi T_0}{T_i} = A\frac{2\pi R_0 C_0}{R_i C_i} = 2\pi A \left(\frac{R_0}{R_i}\right)\left(\frac{C_0}{C_i}\right) \tag{12}$$

(the result of the integration) is insensitive at temperature variations (for more details, see [4]).

However, for determining the derivative of the received signal we can't simply use the ratio

$$\frac{u(t_2) - u(t_1)}{t_2 - t_1} = \frac{u(t_2) - u(t_1)}{\Delta t} \tag{13}$$

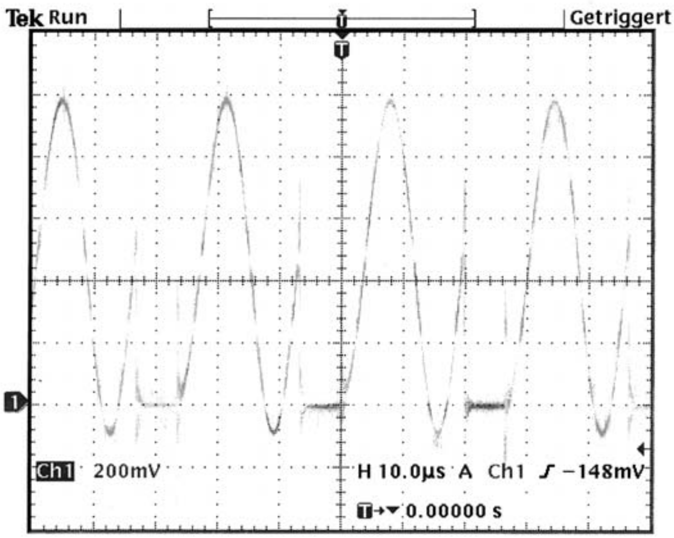

Fig. 2. Output of circuit for working period of 150 μs

while it is quite possible for the received signal to begin to change its value, with a constant slope, at a time moment within the working period $[0, 2\pi T_0]$ of the oscillating system. Thus we can't just consider the result obtained over two successive working periods (presented above) as the value of the derivative. We have to wait another working period, and then we must compare the values

$$\frac{u(t_2) - u(t_1)}{t_2 - t_1} = \frac{u(t_2) - u(t_1)}{\Delta t}, \quad \frac{u(t_3) - u(t_2)}{t_3 - t_2} = \frac{u(t_3) - u(t_2)}{\Delta t} \tag{14}$$

and only when the result of these two operations are almost equal we can assign their result to the value of the derivative of the received signal.

4 An Extension of the Notion of Observability

As it has been shown in the previous paragraph, in the conditions of a step input the output of an oscillating second order system possesses two components: a step component and an alternating component of angular frequency ω_0. The fact that this output and its derivative are equal to zero at the sampling moment of time can be connected with the notion of observability in systems theory. At the sampling moment of time, both the state variables $y(t)$ and dy/dt are equal to zero (are unobservable) and thus the signal which is integrated and its slope are equal to zero at this moment of time (the whole sampling structure is practically insensitive at the random variations of the integration period). However, there is a major difference between the notion of observability in this case and the usual notion of stability (considered for analog linear systems): in our case the state variables are analyzed from the observability point of view only at certain moments of time (at the sampling moments of time).

5 Conclusions

This paper has presented a possibility of obtaining the derivative of the received electrical signal using a filtering device consisting of an oscillating second order system and an integrator. The oscillating systems is working on a time period for filtering a received electrical signal, with initial null conditions. The output of this oscillating system is integrated over this time period (at the end of this period the integrated signal being sampled). In the conditions of a unity-step input, the output of the oscillating system (the quantity which is integrated) is practically equal to zero at the sampling moment of time (when the integration is interrupted). The necessity of using two such oscillating systems if we intend to process the received signal in a continuous manner has been also presented. The method can be used for obtaining the derivative of the optoelectronic signal in case of phase detection for vibration measurements (see [5]).

References

1. Sun, Y., Fidler, J. K.: Synthesis and Performance Analysis of Universla Minimum -Component Integrator Based IFLF-OTA Grounded Capacitor Filter, IEE Proc.Circ.Dev.Syst., **143** 4 (1996) 107–114
2. Smith, S.L., Simenco, E.: Low Voltage Integrators for High-Frequency CMOS Filters Using Current-Mode Techniques, -IEEE Trans.Circ. Syst. II, An. and Dig. Signal Processing, **43**.1 (1996) 39–48
3. Toma, C. : An extension of the notion of observability at filtering and sampling devices, Proceedings of the International Symposium on Signals, Circuits and Systems Iasi SCS 2001, Romania 233–236
4. Toma, C. : Device and method for filtering high-frequency electrical signals, Romanian License of Invention 105124 (1989)
5. Sterian, A., Toma, C.: Phase detection for vibration measureemnts based on test-functions, SPIE Proceedings **5503**, 6th International Conference on Vibration Measurements based on Laser Technoques - Advances and Applications AIVELA 2004, Italy, 164–168