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# A Multivariate Stochastic Volatility Model for Portfolio Risk Estimation

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## Abstract

A Multivariate Latent Stochastic Volatility Factor Model is introduced for the estimation of volatility and optimal allocation of stocks portfolio in a Markowitz type portfolio. Returns on a set of 5 banks among the best capitalized banks' stocks traded on the Italian stock market (BIT) between 1 January 1986 and 31 August 2011 are modeled. Computational complexities arising in the estimation step are dealt by simulation-based methods, introducing a Griddy Gibbs sampler. The association structure among time-series is captured via a factor model, which reduces the computational burden required in the estimation step.

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## Keywords

Factor model • Griddy Gibbs • Markowitz portfolio • Stochastic volatility model

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## 1 Introduction

Volatility modelling plays an important role in the analysis of financial time series. The persistence of volatility phenomenon is the most well-established effect exhibited by financial time series. Indeed, the variance of returns exhibits high serial

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autocorrelation, which becomes evident by looking at periods of high volatility, with large changes in assets returns being followed by large ones as well, and at periods of low volatility in which small changes are followed by small ones. As this observation obviously is of great interest, capturing this effect could be challenging. This is the reason why stochastic volatility (SV) models have been introduced and undergone a lot of research during the last two decades. Since the seminal papers by [14, 15], the univariate SV model has been widely used and several estimation methods have been introduced (see e.g. [3, 10, 13]). Nevertheless, as pointed out by e.g. [2], assets are linked together or influenced by common unobserved factors, which render the univariate approach too restrictive. It is then crucial to extend the univariate SV model to the multivariate case in order to capture the covariation effect. Several alternatives can be considered to describe the time evolution of the joint distribution of different assets (see e.g. [1, 7, 11]).

In this paper we aim at providing a multivariate SV model, based on a latent structure, in which covariation is accounted for via a factor model. Appropriately accounting for covariation is crucial in terms of portfolio diversification and asset allocation. Indeed, the ultimate goal of this paper is to provide indications on portfolio diversification with minimum risk, in a Markovitz framework.

Parameters estimation could be cumbersome and inference becomes therefore hard. This has led to a substantial development of sampling-based methods in order to obtain parameter estimates, such as rejection sampling, Markov Chain Monte Carlo and Monte Carlo integration (see e.g. [3, 8, 12]). To reduce the computational burden often involved in sampling procedures, we adopt a nested Griddy Gibbs as sampler. In this way, we avoid the need of simulated density that mimic the shape of the preceding one and of guaranteeing the dominance (as imposed by [10]).

The proposed SV model is then applied to a subset of 5 banks among the best capitalized banks' stocks traded on the Italian stock market (BIT) between 1 January 1986 and 31 August 2011.

The paper is organized as follows. In Sect. 2 we introduce the model, while its computational aspects are discussed in Sect. 3. Section 4 briefly introduces the data, and the obtained results. Section "Conclusions" concludes.

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## 2 Model Summary

In the following, we consider the SV parameterization introduced in [8, 9]. Let  $\mathbf{r}_t = (r_t^{(1)}, \dots, r_t^{(n)})$  an array of  $n$  stock returns, in which  $r_t^{(i)}$  is the return for the  $i$ -th asset at time  $t$  and  $\mathbf{a}_t = (a_{1t}, \dots, a_{Kt})$  the vector of  $K$  common factors at time  $t$ ,  $t = 1, \dots, T$ . The SV parameterization we consider sets

$$\mathbf{r}_t = \boldsymbol{\mu}_t + \Theta \mathbf{a}_t + \mathbf{e}_t \quad (1)$$

$$\mu_t^{(i)} = \beta_0^{(i)} + \beta_1^{(i)} r_{t-1}^{(i)} + \dots + \beta_p^{(i)} r_{t-p}^{(i)} \quad (2)$$

$$e_t^{(i)} = \sqrt{h_t^{(i)}} \epsilon_t^{(i)} \tag{3}$$

$$\log(h_t^{(i)}) = \alpha_0^{(i)} + \alpha_1^{(i)} \log(h_{t-1}^{(i)}) + v_t^{(i)}. \tag{4}$$

In this factor model, the matrix  $\Theta$  is a constant  $(n \times K)$  matrix of factor loading with  $K < n$ ,  $\boldsymbol{\mu}_t = (\mu_t^{(1)}, \dots, \mu_t^{(n)})'$ , the errors  $\mathbf{e}_t \sim N(\mathbf{0}, \mathbf{E})$  are serially and mutually independent of all other error terms. The errors  $\epsilon_t^{(i)}$  and  $v_t^{(i)}$ ,  $i = 1, \dots, n$  are serially and mutually independent  $N(0, 1)$  and  $N(0, \sigma_v^{2,(i)})$ ,  $E(\mathbf{a}_t) = \mathbf{0}$ ,  $Cov(\mathbf{a}_t) = I$ . We have also  $|\alpha_1^{(i)}| < 1$ , so that the factor log-volatility processes are stationary. Furthermore,  $\beta^{(i)} = (\beta_1^{(i)}, \dots, \beta_p^{(i)})$  are fixed regression parameters.

It follows from this model that the marginal distribution of the returns is multivariate Gaussian with mean  $\boldsymbol{\mu}_t$  and covariance  $\Theta \mathbf{H}_t \Theta' + \mathbf{E}$ , where  $\mathbf{H}_t = \text{diag}(h_t^{(1)}, \dots, h_t^{(n)})$ . Of course, it should also be noted that more complicated dynamics could be introduced in the latent SV process and for our purpose there is no need to estimate  $\mathbf{a}_t$ .

Jacquier et al. [9] proposed to use MCMC methods to estimate model parameters. However, the method has not been implemented for a multivariate financial portfolio. In the following we provide computational details to obtain parameter estimates.

### 3 Computational Details

Let us denote with  $\omega^{(i)} = (\sigma_v^{2,(i)}, \alpha_0^{(i)}, \alpha_1^{(i)})'$  and  $h^{(i)} = (h_1^{(i)}, \dots, h_T^{(i)})$ ,  $r^{(i)} = (r_1^{(i)}, \dots, r_T^{(i)})$ ,  $i = 1, \dots, n$ . The likelihood function can be written as:

$$f(r^{(i)} | \beta^{(i)}, \omega^{(i)}) = \int_{\mathbf{R}^T} f(r^{(i)} | X^{(i)}, \beta^{(i)}, h^{(i)}) f(h^{(i)} | \omega^{(i)}) dh^{(i)}. \tag{5}$$

To tackle this estimation problem we use the MCMC method as in [8].

Let's describe the steps of the algorithm that implements the MCMC method. *First step:* for each return  $i$  the optimal lag  $p$  of the Eq. (2) is selected, independently from the other returns, according to the AIC criterion, fixing  $v_t^{(i)} = 0$ .

*Second step:* for each return  $i$  the following likelihood function, related to the Eqs. (1)–(4) with  $v_t^{(i)} = 0$ , is maximized to find the MLE parameters as in [6]:

$$-L(\beta^{(i)}, \omega^{(i)}) = T^{-1} \sum_{t=1}^n a_t^{(i)} / \tilde{h}_t^{(i)} + \log(\tilde{h}_t^{(i)}) \tag{6}$$

where

$$\begin{aligned} a_t^{(i)} &= r_t^{(i)} - \mu_t^{(i)} \\ \tilde{h}_t^{(i)} &= \log(h_t^{(i)}) = \alpha_0^{(i)} + \alpha_1^{(i)} \tilde{h}_{t-1}^{(i)} \end{aligned} \tag{7}$$

These parameter estimations are chosen as the initial values for the conditional posterior distributions used by the MCMC iterations.

*Third step:* Steps 1,2 are repeated for each intrinsic values  $i$  corresponding to the return  $i$ , given by  $P/E^{(i)} \times EPS^{(i)}$ , where  $P/E^{(i)}$  is the Price to Earnings ratio and  $EPS^{(i)}$  is the Earning per share.

As the intrinsic value is considered to give indication on the return, the obtained parameter estimates are chosen as the prior distribution parameters used to define the conditional posterior distributions needed by the MCMC iterations.

Let's call these prior parameters as follow:

$\beta^{0,(i)} = (\beta_1^{0,(i)}, \dots, \beta_p^{0,(i)})'$  and their variances  $A^{0,(i)} = \text{diag}(\sigma_{\beta_1^{0,(i)}}^2, \dots, \sigma_{\beta_p^{0,(i)}}^2)$  for the mean parameters,  $\omega^{0,(i)} = (\sigma_v^{2,0,(i)}, \alpha_0^{0,(i)}, \alpha_1^{0,(i)})'$  and  $C^{0,(i)} = \text{diag}(\sigma_{\alpha_0^{0,(i)}}^2, \sigma_{\alpha_1^{0,(i)}}^2)$  for the volatility parameters.

Moreover the prior distributions are hypothesized multivariate normal for  $\beta^{(i)} \sim N(\beta^{0,(i)}, A^{0,(i)})$  and  $\alpha^{(i)} \sim N(\alpha^{0,(i)}, C^{0,(i)})$ , inverted Chi squared for  $\sigma_v^{2,(i)}$ , that's to say  $T\lambda/\sigma_v^{2,(i)} \sim \chi_T^2$ , with  $\lambda$  a scale parameter.

*Fourth step:* We consider simulation-based methods. The MCMC Gibbs sampling estimation of the model (1)–(4), after combining the prior distributions with the likelihood using the Bayes' rule, consists in drawing random samples from the conditional posterior distributions

$$\begin{aligned} f(\beta^{(i)} | r^{(i)}, x^{(i)}, h^{(i)}) &\sim N(\beta^{*,(i)}, A^{*,(i)}) \\ f(h_t^{(i)} | r^{(i)}, x^{(i)}, h^{(i)}, \beta^{(i)}, \omega^{(i)}) & \\ f(\alpha^{(i)} | h^{(i)}, \sigma_v^{2,(i)}) &\sim N(\alpha^{*,(i)}, C^{*,(i)}) \\ f(d/\sigma_v^{2,(i)} | h^{(i)}, \alpha^{(i)}) &\sim \chi_{2T-1}^2 \end{aligned} \quad (8)$$

in a sequence from initial value of the conditioning variables, with step by step substitutions of the new sampled values to the previous ones, until a number of iteration  $g$  is reached.

That's is to say, at the MCMC Gibbs iteration  $j$  with  $j = 1, \dots, g$ :

1. we draw a random sample  ${}_{[j]}\beta^{(i)}$  from:  $f(\beta^{(i)} | r^{(i)}, x^{(i)}, {}_{[j-1]}h^{(i)})$
2. we draw a random sample  ${}_{[j]}h_t^{(i)}$  from:  $f(h_t^{(i)} | r^{(i)}, x^{(i)}, {}_{[j-1]}h_{-t}^{(i)}, {}_{[j]}\beta^{(i)}, {}_{[j-1]}\omega^{(i)})$
3. we draw a random sample  ${}_{[j]}\alpha^{(i)}$  from:  $f(\alpha^{(i)} | {}_{[j]}h^{(i)}, {}_{[j-1]}\sigma_v^{2,(i)})$
4. we draw a random sample  ${}_{[j]}\sigma_v^{2,(i)}$  from:  $f(d/\sigma_v^{2,(i)} | {}_{[j]}h^{(i)}, {}_{[j]}\alpha^{(i)})$

This completes a MCMC Gibbs iteration and current parameters values are

$$({}_{[j]}\beta^{(i)}, {}_{[j]}h_{1,\dots,t}^{(i)}, {}_{[j]}\alpha^{(i)}, {}_{[j]}\sigma_v^{2,(i)})$$

In this way we obtain random samples  $\{{}_{[j]}\beta^{(i)}\}_{j=g_0,\dots,g}$ ,  $\{{}_{[j]}h^{(i)}\}_{j=g_0,\dots,g}$ ,  $\{{}_{[j]}\alpha^{(i)}\}_{j=g_0,\dots,g}$ ,  $\{{}_{[j]}\sigma_v^{2,(i)}\}_{j=g_0,\dots,g}$  that can be used to make inference.

Our estimations are the point estimation sample means of the previous random samples after eliminating the first  $g_0 - 1$  values, that's to say

$$\begin{aligned} \hat{\beta}^{(i)} &= \frac{1}{g-g_0} \sum_{j=g_0, \dots, g} [j] \beta^{(i)} \\ \hat{h}^{(i)} &= \frac{1}{g-g_0} \sum_{j=g_0, \dots, g} [j] h_t^{(i)} \\ \hat{\alpha}^{(i)} &= \frac{1}{g-g_0} \sum_{j=g_0, \dots, g} [j] \alpha^{(i)} \\ \hat{\sigma}_v^{2,(i)} &= \frac{1}{g-g_0} \sum_{j=g_0, \dots, g} [j] \sigma_v^{2,(i)} \end{aligned}$$

The value of  $g_0$  is chosen so that the estimation  $(\hat{\beta}^{(i)}, \hat{h}^{(i)}, \hat{\alpha}^{(i)}, \hat{\sigma}_v^{2,(i)})$  of the parameters  $(\beta^{(i)}, h^{(i)}, \alpha^{(i)}, \sigma_v^{2,(i)})$  is stable in the sense that after  $g_0$  the means obtained by adding one by one the successive random sample of the Gibbs are almost identical.

In the Eq. (8) the Bayes' rule gives  $A^{*,(i)} = (\sum_{t=1}^T x_{0,t}^{(i)} x_{0,t}^{(i)'} + (A^{0,(i)})^{-1})^{-1}$ ,

$$x_{0,t}^{(i)} = x_t^{(i)} / \sqrt{h_t^{(i)}}$$

$$\beta^{*,(i)} = A^{*,(i)} \left( \sum_{t=1}^T x_{0,t}^{(i)} r_{0,t}^{(i)} + (A^{0,(i)})^{-1} \beta^{0,(i)} \right), r_{0,t}^{(i)} = r_t^{(i)} / \sqrt{h_t^{(i)}}$$

$$C^{*,(i)} = \left( \sum_{t=2}^T y_t^{(i)} y_t^{(i)'} / \sigma_v^{2,(i)} + (C^{0,(i)})^{-1} \right)^{-1}, y_t^{(i)} = (1, \ln(h_t^{(i)}))'$$

$$\alpha^{*,(i)} = C^{*,(i)} \left( \sum_{t=2}^T y_t^{(i)} \ln(h_t^{(i)}) / \sigma_v^{2,(i)} + (C^{0,(i)})^{-1} \alpha^{0,(i)} \right)$$

$d = T\lambda + \sum_{t=2}^T v_t^{2,(i)}$ ,  $r_t^{(i)}$  is the compound return,  $x_{j,t}^{(i)}$  is its lagged value.

The posterior distribution  $f(h_t^{(i)} | r^{(i)}, x^{(i)}, h^{(i)}, \beta^{(i)}, \omega^{(i)})$  is a non standard one even if its density is known up to a normalizing constant [8].

Therefore a nested Gibbs sampler of type Griddy is implemented in the following way:

1. a grid of values for  $h_t^{(i)}$  is selected, say,  $h_{t,1}^{(i)} \leq h_{t,2}^{(i)} \leq \dots \leq h_{t,m}^{(i)}$ ; its posterior distribution is evaluated on this values to obtain  $w_s = f(h_{t,s}^{(i)} | r^{(i)}, x^{(i)}, h_{-t}^{(i)}, \beta^{(i)}, \omega^{(i)})$ ,  $s = 1, \dots, m$ .
2. The  $w_s$  are used to obtain an approximation of the inverse cumulative distribution function of  $f(h_{t,s}^{(i)} | r^{(i)}, x^{(i)}, h_{-t}^{(i)}, \beta^{(i)}, \omega^{(i)})$ .
3. A uniform random variable between 0 and 1 is drawn and transformed via the preceding step 2 to obtain a random drawn for  $h_t^{(i)}$ .

*Fifth step:* the estimation  $(\hat{v}_{i,i}(T+1 | \mathcal{Q}_T))_{i \in \{1, \dots, n\}}$  of the volatility matrix  $(v_{i,i}(T+1 | \mathcal{Q}_T))_{i \in \{1, \dots, n\}}$ , that's to say  $v_{i,i}(T+1) = h_{T+1}^{(i)}$ , will be obtained in the following way, at the iteration  $j$  of the Gibbs sampler,  $j = g_0, \dots, g$ :

1. we draw a random sample  $v_{T+1}^{(i)}$  from  $N(0, [j] \sigma_v^{2,(i)})$  and the Eq. (4) with  $[j] \beta^{(i)}$  is used to compute  $[j] h_{T+1}^{(i)}$ ;

2. we draw a random sample  $\epsilon_{T+1}^{(i)}$  from  $N(0, 1)$  to obtain  $e_{T+1}^{(i)} = \sqrt{[j]h_{T+1}^{(i)}}\epsilon_{T+1}^{(i)}$  and the Eq. (2) with  $[j]\alpha^{(i)}$  is used to compute  $[j]r_{T+1}^{(i)}$ ;  
 In this way we obtain a random sample  $\{[j]h_{T+1}^{(i)}\}_{j=g_0, \dots, g}$  and a random sample  $\{[j]r_{T+1}^{(i)}\}_{j=g_0, \dots, g}$  that can be used to make inference.  
 Our estimation is the point forecast of the previous two random samples, that's to say  $\hat{h}_{T+1}^{(i)} = \frac{1}{g-g_0} \sum_{j=g_0, \dots, g} [j]h_{T+1}^{(i)}$  and  $\hat{r}_{T+1}^{(i)} = \frac{1}{g-g_0} \sum_{j=g_0, \dots, g} [j]r_{T+1}^{(i)}$ .

*Sixth step:* To estimate the off-diagonal elements of  $V$ , we consider the multivariate Latent Factor model (see [4]).

*Seventh step:* The Markowitz problem can be foretasted at time  $T + 1$  using the preceding estimation of  $v_{i,j}$  and  $r^{(i)}$  that we called  $\hat{v}_{i,j}(T + 1)$  and  $\hat{r}_{T+1}^{(i)}$ , by solving through a quadratic programming method, the following:

$$\min_{\gamma \in \mathbf{R}} \{ \gamma' \hat{V}(T + 1) \gamma : \gamma' \cdot \underline{1} = 1, \gamma' \cdot \hat{r}_{T+1} = R_p, \gamma \geq 0 \} \quad (9)$$

where  $\gamma = (\gamma_1, \dots, \gamma_n)$  and  $R_p \in [\min_{\{i:i=1, \dots, n\}} \hat{r}_{T+1}^{(i)}, \max_{\{i:i=1, \dots, n\}} \hat{r}_{T+1}^{(i)}]$ .

## 4 Data and Results

The model is applied to a subset of the entire stocks' universe among the series of data regarding the best capitalized 5 banks' stocks traded on the Italian stock market (BIT) between 1 January 1986 and 31 August 2011. Data are shown in Fig. 1.

This figure shows the histograms and the fitted normal density. Even if the empirical distributions of the returns are symmetry and uni-modal, the fitted normal curves are not enough similar to the empirical counterparts in the tails. Tests of Jarque–Bera for normality reject the null hypothesis  $H_0$  of normality at 95 %.

Moreover tests Ljung–Box for the squared of residuals  $e_t^{(i)}$  of Eqs. (1)–(2), reject the null hypothesis  $H_0$  of no ARCH effects at 95 %. Therefore it is necessary to include the Eqs. (3)–(4), that's to say the SV model part, in order to obtain an unconditional distribution of the residuals  $e_t^{(i)}$  that has heavier tails, an excess of kurtosis with respect to a gaussian distribution. This is in agreement with the financial data in hand.

With respect to the model (1)–(2) setting  $v_t^{(i)} \equiv 0$ , where  $r_t^{(i)}$  is the intrinsic value, the explanatory variables  $x_{j,t}^{(i)}$  are the lagged values  $r_{t-j}^{(i)}$  and the exogenous variable  $z_t^{(i)}$  is the market index, we give the optimal lags in Table 1, which minimize the AIC.

The posterior distribution parameters, calculated as means of the last  $g-g_0 = 100$  iterations of sampling from the MCMC posterior conditional distribution are as follows (Tables 2 and 3):

From Table 2 it can be seen that  $\hat{\beta}_0^{(i)} \simeq 0$  so there is no constant term in the mean Eqs. (1)–(2). As  $\hat{\beta}_k^{(i)} \neq 0, k > 0$  it can be said that return is dependent on its past

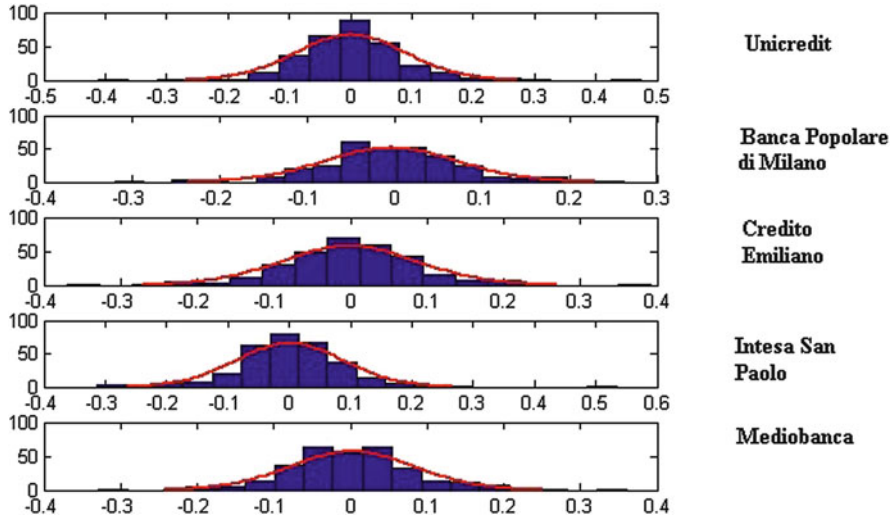


Fig. 1 Data description

Table 1 Optimal AIC lags

	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Opt. lag
Unicredit	-722.66	-722.05	-720.16	-719.8	-718.43	1
BPM	758.76	758.63	760.19	761.65	760.82	2
Credito Emiliano	530.94	531.82	533.74	535.72	536.67	1
Intesa	219.81	175.05	167.33	161.86	102.65	5
Mediobanca	606.37	607.66	599.84	588.03	580.52	5

Table 2  $\hat{\beta}^{*,(i)} = (\hat{\beta}_0^{(i)}, \dots, \hat{\beta}_p^{(i)})'$  posterior distribution parameters (stock  $i$ )

$i$	$\hat{\beta}_0^{(i)}$	$\hat{\beta}_1^{(i)}$	$\hat{\beta}_2^{(i)}$	$\hat{\beta}_3^{(i)}$	$\hat{\beta}_5^{(i)}$	$\hat{\beta}_5^{(i)}$
1	-0.0012	0.0322	0	0	0	0
2	-0.0039	-0.2394	-0.1512	0	0	0
3	-0.0018	-0.1528	0	0	0	0
4	-0.0026	-0.6606	-0.0108	-0.0783	0.1415	0.3238
5	0.0016	-0.0050	-0.0874	-0.1363	-0.1751	-0.2782

values. Moreover, from Table 3 it can be seen that the volatility is dependent from its past as each  $\hat{\alpha}_1^{(i)} \neq 0$  in the Eq. (4). Lastly  $\hat{\sigma}_v^{2,(i)} > 0$  is not negligible so the SV model, which introduces  $v_t^{(i)}$ , is capable to improve a pure ARCH model for the volatility.

The estimated variance-covariance (*risk*) matrix is given in Table 4.

As expected, it can be seen in Table 4 that  $0.47 < \hat{v}_{i,j} < 0.70, i \neq j$ , so there are positive correlations among the series. Indeed the series belong to the same sector.

**Table 3**  $\hat{\omega}^{(i)} = (\hat{\sigma}_v^{2,(i)}, \hat{\alpha}_0^{(i)}, \hat{\alpha}_1^{(i)})'$  posterior distribution parameters (stock  $i$ )

$i$	$\hat{\sigma}_v^{2,(i)}$	$\hat{\alpha}_0^{(i)}$	$\hat{\alpha}_1^{(i)}$
1	2.3661	-0.00023698	0.98773
2	3.1517	-0.0063111	0.9798
3	2.2379	-0.0028054	0.99625
4	1.6529	-0.044743	0.86632
5	2.1015	-0.077245	0.7201

**Table 4** Covariance risk matrix estimation

$\begin{pmatrix} \hat{v}_{1,1} & \cdots & \hat{v}_{1,n} \\ \vdots & \ddots & \vdots \\ \hat{v}_{n,1} & \cdots & \hat{v}_{n,n} \end{pmatrix}_{T+1 \Omega_T}$	=	2.3211	0.6257	0.59352	0.69577	0.6879
		0.6257	1.6836	0.47016	0.55623	0.54937
		0.59352	0.47016	1.7268	0.56166	0.60608
		0.69577	0.55623	0.56166	1.6892	0.6187
		0.6879	0.54937	0.60608	0.6187	1.6936

**Table 5** Latent factor loadings estimates

Series	$\hat{\theta}_1$	$\hat{\theta}_2$
Unicredit	0.8176	0.33716
BPM	0.65679	0.26308
Credito Emiliano	0.37386	0.85376
Intesa	0.70746	0.34807
Mediobanca	0.66952	0.41672

It can be seen that the variances dominate the covariances which are all positive. Thus, it is possible to find portfolios that have lower risk than either single asset. Among those ones we choose the minimum risk portfolio.

In order to provide further insights on the association structure we may look at the estimated latent factors (Table 5). The first latent factor can be associated with Unicredit, BPM and Intesa San Paolo, whilst the second one is mainly related to Credito Emiliano. This can be seen in Fig. 2, where we call Component  $i$  the  $\hat{\theta}_i$ ,  $i = 1, 2$  and the numbers are the stocks in the same order of Table 5.

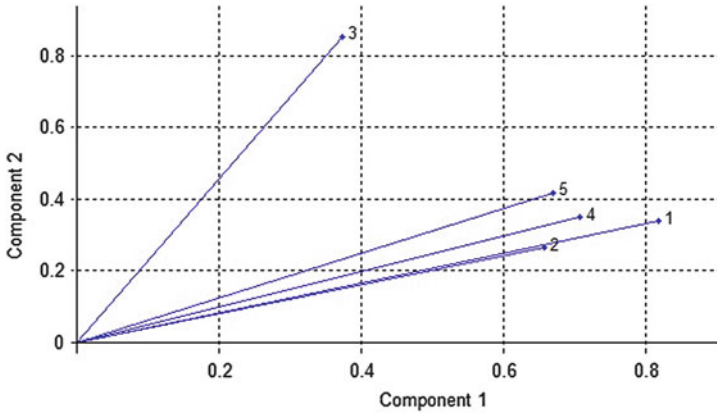
The possible double clustering suggested by the Fig. 2 could drive another portfolio diversification in order to take into consideration the different dependency (loading) each stock's return has of the common latent factors.

Of course, as a central issue, we have to solve the quadratic programming of Markowitz in order to optimize our portfolio. We propose the following optimal (with minimum risk) fractions to invest in each stock as the numerical solution of the optimization problem given in the Eq. (9) (Table 6).

By depicting the optimal fractions in Fig. 3, it can be seen that a good diversification among the stocks is obtained.

The optimal fractions  $\hat{\gamma}_{(i)}$  give a risk of 0.90724 and a monthly portfolio return of 0.035194. Different portfolio choice are possible that can get greater return or

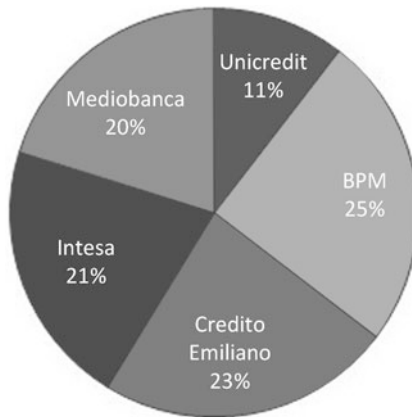




**Fig. 2** Latent factor loadings

**Table 6**  $\hat{\gamma}_{(i)}$  optimal (with minimum risk) fractions

Series	Unicredit	BPM	Credito Emiliano	Intesa	Mediobanca
$\hat{\gamma}_{(i)}$	0.10474	0.2493	0.23337	0.21036	0.20222



**Fig. 3** Optimal fractions

better latent shocks warranty at the expense of a greater risk. Thus in a risk averse view, an investor should choose the minimum risk portfolio suggested.

## Conclusions

We have conducted an empirical investigation of stochastic volatility of major Italian banks, by introducing a computational feasible algorithm based on simulation techniques. The proposed estimation methodology is easily implementable and this is an important step forward on multivariate volatility estimation, since the likelihood function of stochastic volatility models is not easily calculable. The procedure proposed in this work attempts to combine the simplicity of the factor model with the sophistication of stochastic volatility procedures. Open problems remain, primarily in the modelling of multivariate heavy-tailed or skewed error distributions, as well as the computational burden required in the estimation step in the modelling of high dimensional data. In time, further significant developments can be achieved by introducing a time-varying latent structure such as parsimonious hidden Markov models, which are able to reduce the curse of dimensionality of the considered problem and account for well-know stylized facts arising in the stock returns modelling.

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