Precision Contouring Control of Multi-Axis Feed Drive Systems

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Abstract

In order to improve the contouring accuracy in machine tool control, using the contour errors, which is defined as tracking error component orthogonal to desired contour curves, as feedback signals is known to be effective. This paper presents a new contouring control method for multi-axis feed drive systems. The method is applicable to any smooth contour curves and achieves better control performance with small control input variance compared to the conventional methods. The effectiveness of the proposed method is demonstrated by experimental results with the circular and non-circular contour curves.

1 Introduction

Feed drive systems are used in most of machine tool systems, and positional errors are generally defined with respect to each axis of the feed drive systems (tracking errors) in their control systems. From the viewpoint of machining, however, error components orthogonal to desired contour curves are rather important than the errors with respect to the feed drive axes. The orthogonal error components to the contour curves are called contour errors.

There have been many researches focusing on the reduction of the contour errors. Koren proposed a cross-coupling controller that uses the contour errors as feedback signals [1]. Kulkarni and Srinivasan developed an optimal cross-coupled controller based on a linear quadratic regulator (LQR) [2]. Chiu and Tomizuka presented a controller based on the technique of integrator backstepping, though implicit representation of the contour curve is needed for the controller design [3]. McNab and Tsao formulated the contour tracking as a receding horizon LQ problem with variable state weighting matrices. They proved the stability for linear trajectory case [4].

Since both the contour errors and the tracking errors are used to calculate control inputs in these methods, there may be degradation of contour tracking performance. Considering both the tracking and the contour errors simultaneously brings some difficulties in adjusting controller parameters.

In order to overcome this problem, Lo and Chung proposed a contouring control method based on a coordinate transformation for biaxial feed drive systems [5], in which tracking errors are transformed into the errors with orthogonal and tangential components to the desired contour curves. They proved the stability of their method only for straight-line trajectory motion. Since, in their method, two decoupled single-input single-output systems with respect to the orthogonal and tangential directions are obtained, controllers for both directions can be designed independently, and hence controller parameters are adjusted rather easier. Their method, however, is effective only for the case that the mismatch of the dynamics of both feed drive axes is enough small. The case is practically impossible since one-axial feed drive system reposes on the other in most of biaxial feed drive systems.

In this paper, we propose a new contouring control method based on a complete coordinate transformation of the tracking errors. The proposed method allows the feed drive systems to have the dynamics mismatch between each axis. And also, the stability of control systems is guaranteed to any smooth contour curves. The effectiveness of the proposed method is demonstrated by experimental results with circular and non-circular contour curves.

2 Problem Formulation

2.1 Definition of Contour Errors

In this paper, we consider the 3 dimensional case as shown in Fig. 1, where the curve c is the desired contour of a point of the feed drive system. The symbol Σ_w is a fixed coordinate frame whose axes correspond to the feed drive axes. The symbol $r = [r_1\ r_2\ r_3]^T$ is a desired position of the feed drive system at time t, and defined with respect to Σ_w. The actual position of the feed drive system is assumed at $x = [x_1\ x_2\ x_3]^T$. We further define a local coordinate frame Σ_l whose origin is at r and three axes are $l_i\ (i = 1,2,3)$ in the figure. The axis l_1 is in the tangential direction of c at r. The direction of l_2 is perpendicular to l_1 and in the tangential plane of the machined surface at r. The direction of l_3 is perpendicular to both l_1 and l_2 as shown in Fig. 1.

The tracking error vector e_w, which consists of tracking errors of each feed drive axis, is defined as follows:

$$e_w = [e_{w1}\ e_{w2}\ e_{w3}]^T = r - x. \tag{2.1}$$

This error vector can be transformed into that with respect to Σ_l as follows:

$$e_l = [e_{l1}\ e_{l2}\ e_{l3}]^T = R^T e_w, \tag{2.2}$$

where R is a 3×3 rotation matrix that transforms a position with respect to Σ_l into that to Σ_w.
From the viewpoint of machining, error components orthogonal to the desired contour curve are rather important than the tacking errors of feed drive axes. Hence we propose a controller design that allows us to adjust the control performance with respect to three axes of Σ_l independently each other. In the design, we can set the controller gain for reducing the error along l_1 to smaller value than that for the other axes from the reason that the tangential error component to the contour curve is less important than the orthogonal ones. It also should be noted that the error components e_{l2} and e_{l3} are just approximate values of contour errors, because even if they are reduced to zero by some controllers, the position $\hat{x}$ in Fig. 1 moved from x is not still on the curve c. It is, however, difficult to calculate actual contour errors online because we need to solve nonlinear equations if the contour curve is not a simple one. For this reason, the control objective of the proposed system is to reduce the error components e_{l2} and e_{l3}. They may be good approximations of contour errors if the error component e_{l1} is small.

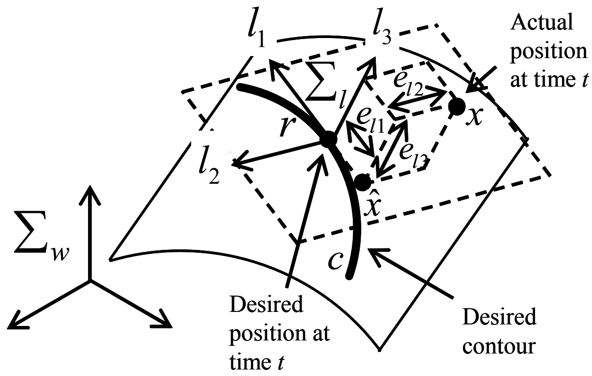

Fig. 1. Definition of tracking errors

2.2 Plant Dynamics

In this paper, we consider the feed drive system driven by servo motor systems, which are commonly used as industrial applications. The feed drive dynamics is generally represented by the following decoupled second order system:

$$\begin{aligned} &M\ddot{x} + C\dot{x} = F, \\ &M = \mathrm{diag}\{M_i\}, C = \mathrm{diag}\{C_i\}, i = 1,2,3, \\ &F = [F_1\ F_2\ F_3]^T, \end{aligned} \tag{2.3}$$

where $M_i(>0), C_i(\geq 0)$ and F_i are the mass of load, the viscous friction coefficient and the driving force on the i th drive axis, respectively. The symbol $\mathrm{diag}\{A_i\}$ is a diagonal matrix with the element A_i at the i th row. Nonlinear frictions such as Coulomb frictions are not explicitly considered in this dynamics.
The dynamics of the motors for driving the feed drive systems is described as follows:

$$\begin{aligned} &J\ddot{\theta} + D\dot{\theta} + \tau = KV, \\ &J = \mathrm{diag}\{J_i\}, \quad D = \mathrm{diag}\{D_i\}, \\ &K = \mathrm{diag}\{K_i\}, \quad i = 1,2,3, \\ &\tau = [\tau_1\ \tau_2\ \tau_3]^T, \quad V = [V_1\ V_2\ V_3]^T, \end{aligned} \tag{2.4}$$

where $J_i(>0), D_i(\geq 0), K_i(>0), \tau_i$ and V_i are the motor inertia, the motor viscous friction coefficient, the torque-voltage conversion ratio, the torque for driving the feed drive system Eq. (2.3) and the motor input voltage of the i th axis, respectively.
The relation among the force F_i, torque τ_i, position x_i, angle θ_i and pitch of the ball screw P_i are represented as follows:

$$F_i = \frac{2\pi\tau_i}{P_i}, \quad x_i = \frac{P_i\theta_i}{2\pi}. \tag{2.5}$$

3 Controller Design

We assume the followings on the controller design:

- The desired trajectory r_i, and its derivatives $\dot{r}_i$ and $\ddot{r}_i$ are available.
- The rotation matrix R, and its derivatives $\dot{R}$ and $\ddot{R}$ are available.
- The position of the feed drive system x and its derivative $\dot{x}$ are measurable.
- The plant parameters M, C, J, D, K and $P_i's$ are all available.

Since the matrix R is a function of the desired trajectory, we can calculate it and its derivatives beforehand. Hence the second assumption is not a strict one.
We propose the following control:

$$V = H\left\{\ddot{r} - R\left(-K_{vl}\dot{e}_l - K_{pl}e_l - \ddot{R}^T e_w - 2\dot{R}^T\dot{e}_w\right)\right\} + E\dot{x},$$
$$H = \mathrm{diag}\left\{\frac{M_i + J_i\left(2\pi/P_i\right)^2}{2\pi K_i/P_i}\right\}, \tag{3.1}$$
$$E = \mathrm{diag}\left\{\frac{C_i + D_i\left(2\pi/P_i\right)^2}{2\pi K_i/P_i}\right\},$$

where K_{vl} and K_{pl} are the so-called velocity and position feedback gain matrices, and we assume that they are also the diagonal matrices with all positive elements. Considering Eqs. (2.1)-(2.5) and (3.1), we have the following relation:

$$HR\left(\ddot{e}_l + K_{vl}\dot{e}_l + K_{pl}e_l\right) = 0. \tag{3.2}$$

Since H and R are nonsingular matrices, we have

$$\ddot{e}_l + K_{vl}\dot{e}_l + K_{pl}e_l = 0. \tag{3.3}$$

From the above equation, it is concluded that by appropriately assigning the feedback gain matrices K_{vl} and K_{pl} in Eq. (3.1), we can achieve $e_l \to 0$ as $t \to \infty$. And also we can independently adjust the error convergence speed along each axis of Σ_l in Fig. 1 since the matrices K_{vl} and K_{pl} are both diagonal. Setting the feedback gains with respect to e_{l2} and e_{l3} larger than that for e_{l1}, we may reduce the contour errors faster than the tracking error tangential to the desired contour curve.
In order to comparatively see the effectiveness of the proposed design, we consider the following non-contouring control using the error signals on Σ_w:

$$V = H\left(\ddot{r} + K_{vw}\dot{e}_w + K_{pw}e_w\right) + E\dot{x}, \tag{3.4}$$

where K_{vw} and K_{pw} are the 3×3 velocity and position feedback gain matrices on Σ_w. The matrices are assumed also to be diagonal with positive elements. Then we can have the following error dynamics on Σ_w:

$$\ddot{e}_w + K_{vw}\dot{e}_w + K_{pw}e_w = 0. \tag{3.5}$$

Both in Eqs. (3.3) and (3.5), we can have decoupled systems and assign control system poles to any places on the complex plane. For symplifying the analysis, we consider the case that the desired contour is a straight-line (i.e., $\dot{R} = \ddot{R} = 0$) and some gains are increased for reducing the contour errors. It is possible to assign greater values only to the second and third diagonal elements in K_{vl} and K_{pl} in Eq. (3.3) from the reason that they directly relate to the contour errors. This assignment is not possible in Eq. (3.5), because the relation between the controller gains and their effect to the contour errors is not obvious. The dynamics Eq. (3.3) can be transformed into the following one on Σ_w with Eq. (2.2):

$$\ddot{e}_w + RK_{vl}R^T\dot{e}_w + RK_{pl}R^T e_w = 0. \tag{3.6}$$

Note that only the feedback gain matrices are different in Eqs. (3.5) and (3.6). Assuming that the first diagonal elements of K_{vl} and K_{pl}, which relates only to e_{l1}, are set smaller than the others in K_{vl}, K_{pl}, K_{vw} and K_{pw}, we can make the Frobenius norms of $RK_{vl}R^T$ and $RK_{pl}R^T$ in Eq. (3.6) smaller than those of K_{vw} and K_{pw} in Eq. (3.5). This means that the proposed design may achieve the similar contouring control performance with smaller feedback gains on Σ_w, which may provide a wider stability margin.

4 Experiment

We have designed both the control systems in Eqs. (3.3) and (3.5) for an X-Y table, which is driven by DC servo motors and ball screw drives as shown in Fig. 2, and experimentally compared the control performances. All the 3-dimensional vectors and matrices used in the previous sections are reduced to 2-dimensional ones in the following design. The 2-dimensional definitions of error signals are shown in Fig. 3, where the contour error is e_{l2}. In the 2 dimensional case, the matrix R in Eq. (2.2) is

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{4.1}$$

The nominal parameter values of the X-Y table are shown in Table 1. The position of the X-Y table is measured by linear scales attached to each drive axis, and the sensor resolution is $0.1[\mu\mathrm{m}]$. The velocities of each drive axis are computed by the backward difference operation of the position measurements.
In the experiment, the circular contour curve as shown in Fig. 4 (a) is employed, namely,

$$r_1 = L_c\cos\omega t, \quad r_2 = L_c\sin\omega t, \tag{4.2}$$

where $L_c = 3[\mathrm{mm}], \omega = 2\pi/5[\mathrm{rad/s}]$. The control time T is 10 [s]. The actual contour error e_c can be calculated as follows:

$$e_c = \sqrt{x_1^2 + x_2^2} - L_c. \tag{4.3}$$

The closed loop poles of the dynamics for e_{l1} are set to $q_{l1} = -30$ [1/s] as repeated poles, while those for e_{l2} are changed from $q_{l2} = -30$ to -70 by -20 [1/s] as also the repeated poles. Namely the following feedback gain matrices are used for the error dynamics Eq. (3.3).

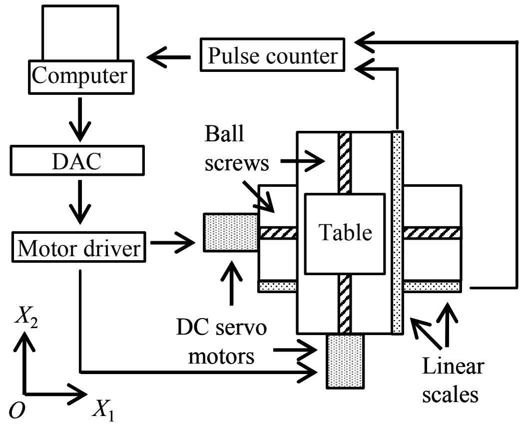

Fig. 2. Experimental system

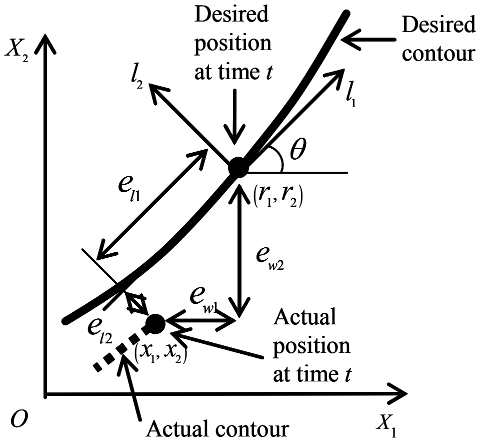

Fig. 3. Definition of errors for the experimental system

Table 1. Parameter Values

Parameters	Values
P_1 & P_2	$0.005\,[\mathrm{m}]$
$M_1(P_1/2\pi)^2$: load inertia moment of axis 1	$1.30\,[\mathrm{kgm^2}]$
$M_2(P_2/2\pi)^2$: load inertia moment of axis 2	$0.77\,[\mathrm{kgm^2}]$
C_1 & C_2	$0\,[\mathrm{Ns/m}]$
J_1 & J_2	$0.05\,[\mathrm{Kgm^2}]$
D_1 & D_2	$0.31\,[\mathrm{Nm/(rad/s)}]$
K_1 & K_2	$1.42\,[\mathrm{Nm/V}]$

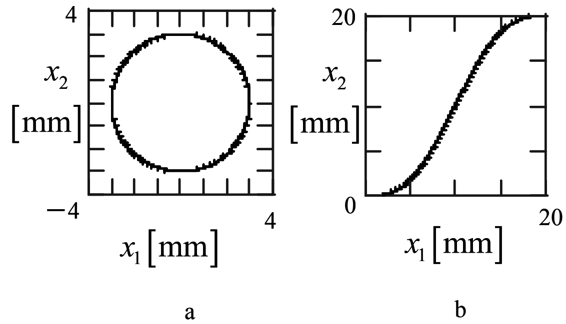

Fig. 4. Desired contour curves used in experiments

$$K_{vl} = \begin{bmatrix} -2q_{l1} & -2q_{l2} \end{bmatrix}^T, \quad K_{pl} = \begin{bmatrix} q_{l1}^2 & q_{l2}^2 \end{bmatrix}^T. \qquad (4.4)$$

On the other hand, in the experiment with the system Eq. (3.5), both the poles for the dynamics of e_{w1} and e_{w2} are changed from $q_{w1} = q_{w2} = -30$ to -70 by -20 [1/s], and hence the following feedback gain matrices are used:

$$K_{vw} = \begin{bmatrix} -2q_{w1} & -2q_{w2} \end{bmatrix}^T, \quad K_{pw} = \begin{bmatrix} q_{w1}^2 & q_{w2}^2 \end{bmatrix}^T. \qquad (4.5)$$

As mentioned above, only in the proposed method, we can increase the controller gain for reducing the contour error as in Eq. (4.4), because the relation between the controller gain and the size of the contour error is obvious. Since the relation is not obvious in Eq. (3.5), we need to increase the controller gains for reducing tracking errors in both feed drive axes as in Eq. (4.5).

The experimental results are shown in Fig. 5, where (a)-(c) and (d) - (f) are the results by the conventional and the proposed methods, respectively.

Comparing the results (a) and (d), and (b) and (e), respectively, we can see that almost the same contour error and control input profiles are obtained. However comparing the results (c) and (f), we can confirm that the control input variance for the system Eq. (3.3) is much smaller than that for Eq. (3.5), and as a result, the size of the contour error in (c) is greater than that in (b) and (f).

The stability of the control system with the proposed methods is guaranteed for any contour curve, though this is not possible in some existent methods. In order to verify the effectiveness to non-linear and non-circular contour curves, we have also applied both the control systems to the following trajectory:

$$r_1 = L_n \frac{t}{T}, \quad r_2 = L_n \left\{ \frac{t}{T} - \frac{1}{2\pi} \sin\left(\frac{2\pi t}{T} \right) \right\}, \qquad (4.6)$$

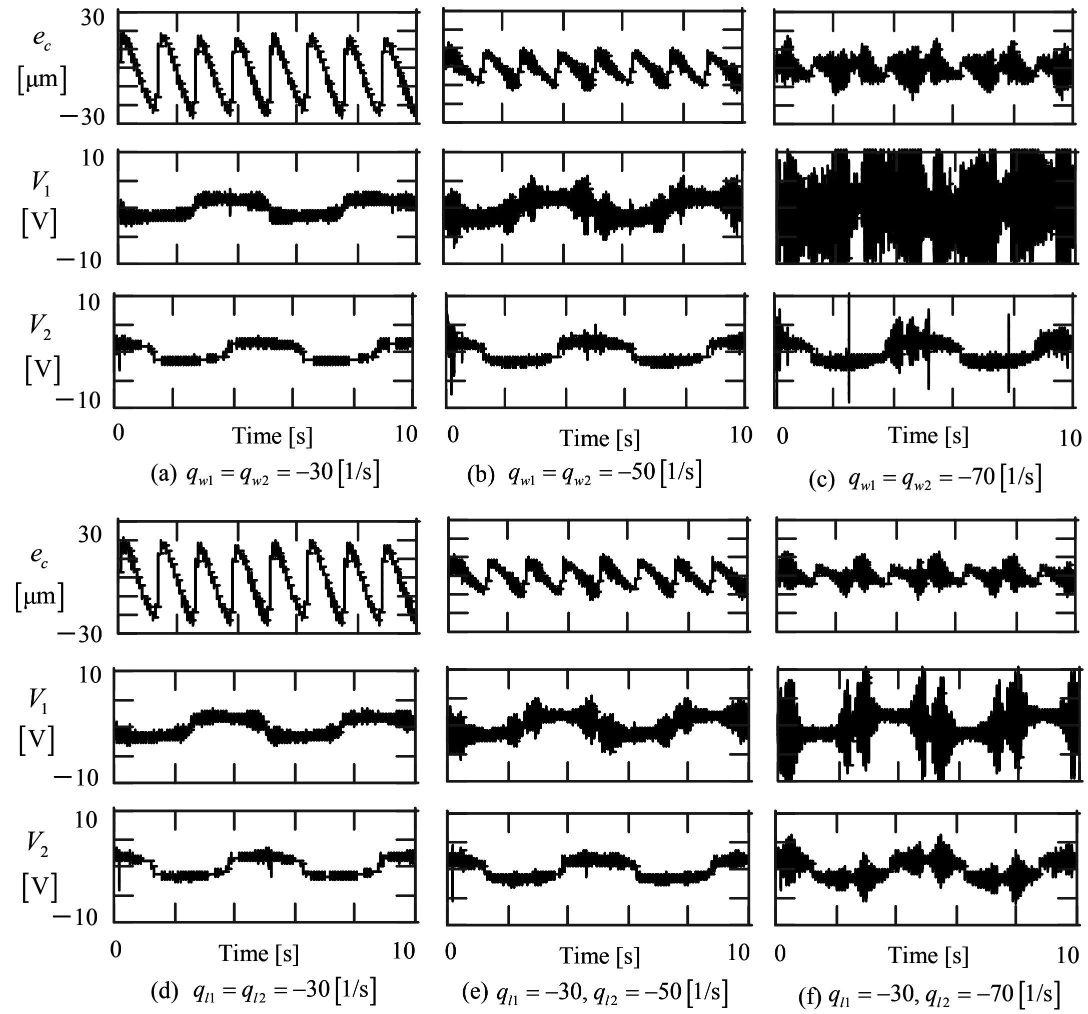

Fig. 5. Experimental results (Circular trajectory case, (a)-(c): Conventional, (d)-(f): Proposed)

where $L_n = 20$ [mm] and $T = 10$ [s]. The desired contour curve is shown in Fig. 4 (b). The actual contour error size is calculated by solving the following minimization problem:

$$|e_c| = \min_t \left\{ (r_1 - x_1)^2 + (r_2 - x_2)^2 \right\}. \tag{4.7}$$

The control results are shown in Fig. 6, where (a)-(c) and (d)-(f) are the results by the conventional and the proposed methods, respectively. Comparing (b) and (e), we can see that the variance of the control inputs is greater in (b). Also in the result (c), the variance of the control inputs is much greater than that in (f), and as a result, the size of the contour error in (c) is also greater than those in (b) and (f).
From these experimental results, we can conclude that the proposed method achieves the better control performance with smaller controller gain, because we can adjust the controller gain for reducing the contour error independently from the tracking error tangential to the contour curve.

5 Conclusions

We have proposed a new contouring method for multi-axis feed drive systems. The advantages are that the method can be applied to any smooth contour curve and the stability of the control systems is guaranteed if the plant dynamics is known. We have experimentally confirmed the effectiveness of the proposed method by comparisons with the conventional controllers. The proposed method achieves the

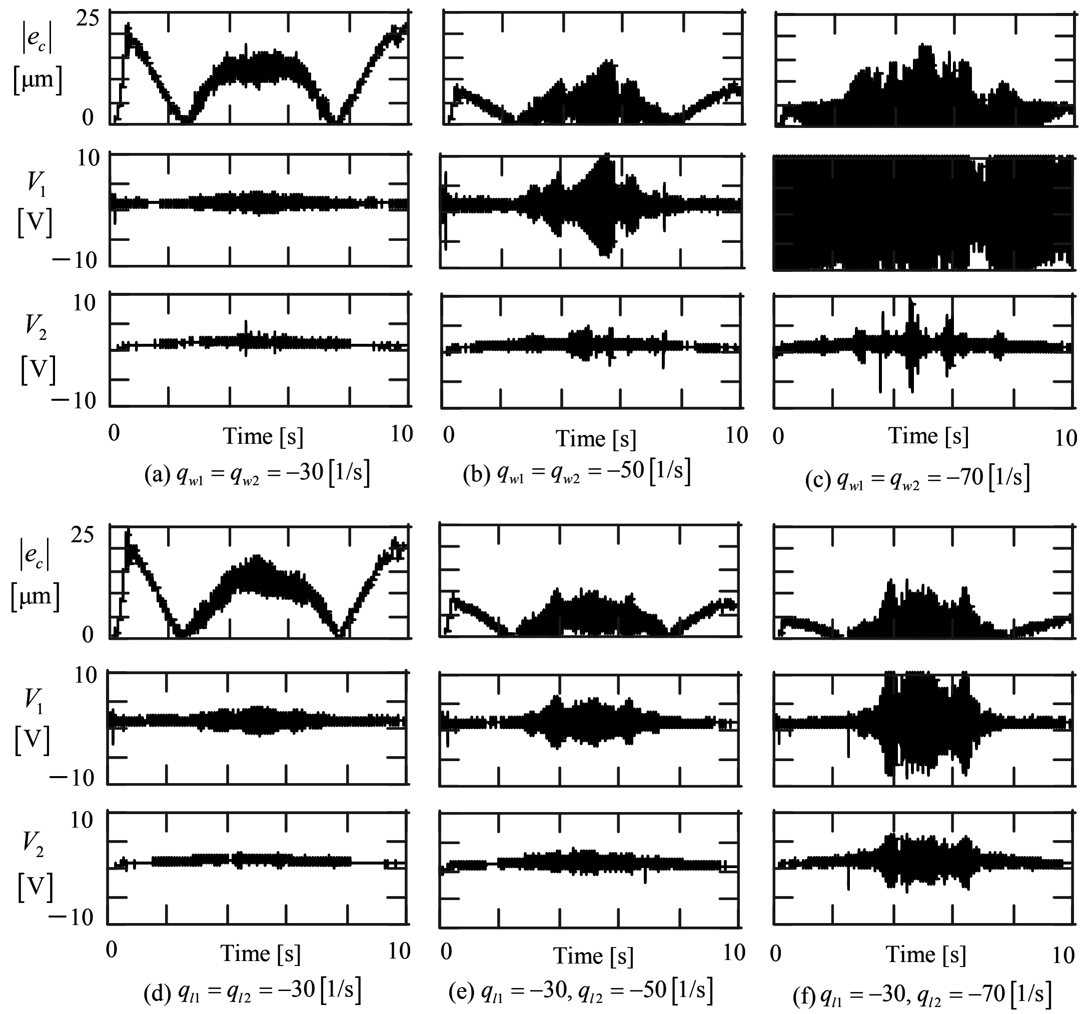

Fig. 6. Experimental results (Non-circular trajectory case, (a)-(c): Conventional, (d)-(f): Proposed)

better control performance with small control input variance because only the controller gains for reducing the contour errors can be increased.

In the current controller design, nonlinear frictions such as Coulomb frictions are not explicitly considered. Adding a nonlinear friction compensator to the proposed controller is expected to achieve the further better control performance. Furthermore, it is also expected to develop robust contouring controllers with respect to plant modeling errors.

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