# **Ultraprecision Wide-angle Profile Measurement with Air-bearing Cylinder Slant Probes**

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### **Abstract**

This chapter introduces a newly-developed profile measuring method with air-bearing cylinder slant probes to measure the profile in the wide range of angles with high accuracy. The air-bearing cylinder can be moved by electric-pneumatic regulator accurately with low friction. Since the measuring force can be controlled in a very low value, high measuring accuracy can be obtained. And the combination of multiple measurements by using slant probes enables to achieve high measuring accuracy in the points with steep angles. The airbearing cylinder slant probe can be also applied to general 3D-CMM to improve the measuring accuracy.

#### **1 Introduction**

Demands for CCD camera lens of cellular phones or pick-up lens for CD or DVD players have been increasing in these days. Some of these lens profile have steep angle of over 70 degrees to achieve high quality and multi-functions. In many cases, it is very difficult not only to machine lens or lens molds with steep angle, but also to measure the profile with high repeatability and accuracy.

In this chapter, a new profile measuring method with an air-bearing cylinder slant probe or probes is introduced to achieve high repeatability and accuracy in the profile measurement of the surface with steep angles.

# **2 Difficulties with Wide-Angle Profile Measurement**

#### **2.1 Probe Structure**

**Fig. 1** shows the schematic view of the profile measuring probe for ultraprecision optics with an air-bearing cylinder. The air-bearing cylinder has large stiffness in the radial direction and much small stiffness in the axial direction, and it can be moved in the axial direction with no friction by air pressure. By controlling the air pressure in a low value with the high resolution electro-pnumatic regulator, the measuring force can be controlled precisely in about 100mgf or lower. Gravity cancel port is used when the probe is installed vertically or slantly in the vertical direction to cancel the gravity force to the probe shaft.

The tip of the probe shaft has a sphere ball which is contacted with the surface of the measured object. The movement of the probe shaft can be measured with the displacement sensor attached to the opposite side of the probe shaft.



**Fig. 1.** Schematic view of a profile measuring probe

#### **2.1 Kinematic Analysis**

**Fig. 2** shows the kinematic model of the profile measuring probe when the tilt surface whose angle is  $\phi$  is measured with the measuring force *Fy*. The tip sphere ball of the probe whose radius is  $r$  is contact with the surface of the measured object. The probe shaft is supported by two air bearings, whose lengths are  $l_1$  and  $l_2$ , and the lengths between the center of the tip ball of the probe and the endface of the air bearings are *a* and *b*.



**Fig. 2.** Kinematic model of the profile measuring probe

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During the measurement, the probe shaft is affected by the normal drag *N* and friction force  $\mu$ *N* from the measured object, so the probe shaft is tilted and slipped.

Assuming that the probe shaft is slipped between the length of *p* along the tangential line from the contact point *P* to  $P'$  and tilted in the angle of  $\theta$ , the cross points  $\hat{A}$ ,  $\hat{B}$ between the probe shaft and the first and second airbearings move to *A', B'* shown in **Fig. 3**. The probe shaft is balancing during the measurement among the forces  $N$ ,  $\mu N$ ,  $Fy$  and thrust forces at the air bearings,  $f_{x1}$ ,  $f_{x2}$ . The case that no change in acceleration (constant velocity) during the measurement is discussed.



**Fig. 3.** Slip and tilt model of the probe shaft

First the forces and moments generated at the air bearings are calculated. Assuming that the forces only in the radial direction are generated, the spring constants of the first and second air bearings along X axis,  $k_{x1}$ ,  $k_{x2}$ , are expressed as follows:

$$
k_{x1} = \rho_1 l_1 \tag{1}
$$

$$
k_{x2} = \rho_2 l_2 \tag{2}
$$

where  $\rho_1$  and  $\rho_2$  are the spring constant per unit length of the first and second air bearings.

Assuming that the forces in X direction and the moments around the center of the probe tip ball affected at the air bearings are  $f_{x1}$ ,  $f_{x2}$ , and  $M_1$ ,  $M_2$  respectively, the following equations can be expressed by using the equation (1) and (2).

$$
f_{x1} = \int_{a}^{a+l_1} \rho_1(p \cos \phi - y \sin \theta) dy
$$
  
=  $k_{x1} \left[ p \cos \phi - \left( a + \frac{l_1}{2} \right) \sin \theta \right]$  (3)

$$
f_{x2} = \int_0^{b+l_2} \rho_2 (p \cos \phi - y \sin \theta) dy
$$
  
=  $k_{x2} \left[ p \cos \phi - \left( b + \frac{l_2}{2} \right) \sin \theta \right]$  (4)

$$
M_1 = \int_a^{a+l_1} \rho_1(p\cos\phi - y\sin\theta) y dy
$$
  
=  $k_{x1}$   $\bigg[ p\cos\phi \bigg( a + \frac{l_1}{2} \bigg) - \bigg( a^2 + a l_1 + \frac{l_1^2}{3} \bigg) \sin\theta \bigg]$  (5)

$$
M_2 = \int_0^{b+l_2} \rho_2 (p \cos \phi - y \sin \theta) y dy
$$
  
=  $k_{x2} \left[ p \cos \phi \left( b + \frac{l_2}{2} \right) - \left( b^2 + bl_2 + \frac{l_2^2}{3} \right) \sin \theta \right]$  (6)

Here the equations of motion are following,

$$
0 = -f_{x1} - f_{x2} + N \sin \phi + \mu N \cos \phi \tag{7}
$$

$$
0 = F_y - N\cos\phi + \mu N\sin\phi \tag{8}
$$

$$
0 = M_1 + M_2 + r \cdot \mu N + c \cdot F_y \sin \theta \tag{9}
$$

The normal drag *N* is solved from the equation (8);

$$
N = \frac{F_y}{\cos\phi - \mu \sin\phi}
$$
 (10)

Under the approximation that the stiffness of the air bearings in the axial direction is ignorable to that in the radial direction, the tilt angle  $\theta$  and the slip length  $p$  are expressed from the equations  $(3)$  to  $(10)$  as follows;

$$
k_{x1} = \rho_1 l_1
$$
 (1) 
$$
\theta = \frac{\{A(\sin \phi + \mu \cos \phi) + \mu (k_{x1} + k_{x2})r\}F_y}{\{(k_{x1} + k_{x2})B - A^2\}(\cos \phi - \mu \sin \phi)}
$$
 (11)

$$
p = \frac{\{B(\sin\phi + \mu\cos\phi) + \mu Ar\}F_{y}}{\{(k_{x1} + k_{x2})B - A^2\}(\cos\phi - \mu\sin\phi)\cos\phi}
$$
 (12)

where some parameters are replaced as follows;

*p*

$$
A \equiv k_{x1}\left(a + \frac{l_1}{2}\right) + k_{x2}\left(b + \frac{l_2}{2}\right) \tag{13}
$$

$$
B = k_{x1} \left( a^2 + aI_1 + \frac{l_1^2}{3} \right) + k_{x2} \left( b^2 + bI_2 + \frac{l_2^2}{3} \right) \tag{14}
$$

When the same two air bearings are used, the spring constants  $k_{x1}$ ,  $k_{x2}$  and lengths  $l_1$ ,  $l_2$  are the same  $k_x$  and *l* respectively. The equations (11), (12) can be simplifed;

$$
\theta = C \cdot \xi \cdot \Phi \tag{15}
$$

$$
p = D \cdot \xi \cdot \frac{\Phi}{\cos \phi} \cdot a \tag{16}
$$

where some parameters are replaced as follows;

$$
C = \frac{\left(\frac{b}{a}\right) + \left(\frac{l}{a}\right) + 1}{\left\{\left(\frac{b}{a}\right) - 1\right\}^2 + \frac{1}{3}\left(\frac{l}{a}\right)^2}
$$
(17)

$$
D = \frac{\left(\frac{b}{a}\right)^2 + 1 + \left\{\left(\frac{b}{a}\right) + 1\right\}\left(\frac{l}{a}\right) + \frac{2}{3}\left(\frac{l}{a}\right)^2}{\left\{\left(\frac{b}{a}\right) - 1\right\}^2 + \frac{1}{3}\left(\frac{l}{a}\right)^2}
$$
(18)

$$
\zeta = \frac{F_y}{k_x a}
$$
\n(19)\n
$$
\equiv k_2 \cdot \frac{\cos \phi - \mu \sin \phi}{\tan \phi (\sin \phi + \mu \cos \phi)}
$$

$$
\Phi = \frac{\sin \phi + \mu \cos \phi}{\cos \phi - \mu \sin \phi}
$$
 (20)

The measurement error  $\delta$  in the normal axis caused by the tilt and slip of the probe shaft is expressed as follows;

$$
\delta = p \sin \phi + L(1 - \cos \theta)
$$
  
\n
$$
\approx \left[ D \tan \phi \cdot \Phi \cdot \xi + \frac{1}{2} \left( \frac{L}{a} \right) (C \cdot \xi \cdot \Phi)^2 \right] \cdot a
$$
 (21)

In the equation  $(21)$ , the first term is the error caused by the the steep surface near 90 degree. tilt of the probe shaft and the second term is the error caused by the slip of the probe shaft. Since the probe is designed as the stiffness of the air bearings bigger and the measuring force smaller to achieve high accuracy, the absolute value of  $\xi$  is thought to be much smaller than 1. When the tilt angle of the measured object  $\phi$  is relatively big, the measurement error d is mainly caused by the tilt of the probe shaft. The equation (21) is simplified as follows;

$$
\delta \approx D \tan \phi \cdot \Phi \cdot \xi \cdot a
$$
  
=  $D \cdot \xi \cdot a \cdot \tan \phi \cdot \Phi$   
=  $k_1 \cdot \tan \phi \frac{\sin \phi + \mu \cos \phi}{\cos \phi - \mu \sin \phi}$  (22)

When the measured surface is ideally smooth and the friction force is almost zero ( $\mu$  = 0), the equation (22) is simplied as follows;

$$
\delta = k_1 \cdot \tan^2 \phi \tag{23}
$$

When the measurement error at the measured surface angle  $\phi$  on the measured surface can be expressed as follows; of 45 degree is defined  $\delta_{45}$ ,

$$
k_1 = \frac{\delta_{45}}{\tan^2 45^\circ} = \delta_{45} \tag{24}
$$

The equation (23) can be expressed as follows;

$$
\delta = \delta_{45} \cdot \tan^2 \phi \tag{25}
$$

The measurement error  $\delta$  caused by the tilt of the probe shaft increases in proportion to  $(\tan \phi)^2$ , and has very big value in the steep surface near 90 degree.

From the equations of  $(19)$  and  $(22)$ , the relation between the measuring force  $F<sub>v</sub>$  and the measured surface angle  $\phi$  can be expressed as follows;

$$
F_y = k_x a \cdot \xi = \frac{k_x \cdot \delta}{D \tan \phi \cdot \Phi}
$$
  
=  $k_2 \cdot \frac{\cos \phi - \mu \sin \phi}{\tan \phi (\sin \phi + \mu \cos \phi)}$  (26)

In the same way at the discussion of the measurement error  $\delta$ . when the measured surface is ideally smooth and the measuring force at the measured surface angle  $\phi$  of 45 degree is defined  $F_{45}$ , the equation (26) can be expressed as follows;

$$
F_y = \frac{F_{45}}{\tan^2 \phi} \tag{27}
$$

The measuring force  $F_y$ , with which the measurement error  $\delta$ is less than the desirable measurement accuracy, is in proportion to  $(\tan \phi)^{-2}$ , and must be controlled almost zero in

#### **2.2 Maximum Scanning Speed Analysis**

To conduct a profile measurement successfully, the probe shaft must be always contact with the surface of the measured object. Too fast scanning leads the probe tip to jumping from the surface of the measured object, and it is impossible to measure accurately along the surface profile. The following discussion is the analysis of the maximum scanning speed in which the profile measurement can be successfully conducted along the surface of the measured object.

Assuming that the measured surface is ideal periodic sine wave surface shown in **Fig. 4**, whose amplitude is *A*, angular frequency is  $\omega$ , wave length is  $d$ , the mass of the probe shaft is *m*, the measuring force is  $F_y$ , and the acceralation of the probe shaft in the amplitude direction is *a*, the equation of the motion is as follows;

$$
ma = F_v \tag{.28}
$$

On the other hand, the position  $\nu$  in the amplitude direction

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$$
y = A \sin \omega t \tag{29}
$$

$$
\frac{d^2y}{dt^2} = -A\omega^2 \sin \omega \tag{30}
$$

Here it is necessary that the acceralation of the probe shaft  $a$  object and probe calculate the profile. is always more than the maximum acceralation of the measured object to keep the probe tip contact with the measured surface.

$$
\left. \frac{d^2 y}{dt^2} \right|_{\text{max}} = A \omega^2 \le a = \frac{F_y}{m}
$$
 (31)

In case of satisfying the equation, the maximum angle **Probes** frequency  $\omega_{max} = 2\pi f_{max}(f_{max}:$  maximum scanning frequency), the following equation is satisfied;

$$
A = \frac{F_y}{4\pi^2 m} \cdot \frac{1}{f_{\text{max}}} \tag{32}
$$

The maximum scanning speed *vmax* can be expressed with the measuring data pitch *d* as follows;

$$
v_{\text{max}} = f_{\text{max}} \cdot d = \frac{d}{2\pi} \cdot \sqrt{\frac{F_y}{mA}}
$$
(33)

To improve the maximum scanning speed, it is necessary to make the measuring force  $F<sub>v</sub>$  bigger or the mass of the probe shaft lighter. It is impossible in the conventional way to keep the measuring force  $F<sub>v</sub>$  bigger in the measurement of a steep angle to maintain high measuring accuracy. So it is necessary to achieve high measuring accuracy to keep the mass of the probe shaft m lighter and shortly design the probe smaller and lighter. This means that it is necessary to use special micro machining method or special light material to design and manufacture the probe or to make a highresolution controlling unit to keep the measuring force extremely light. It is so complicated that the cost of the total measuring system becomes up.



**Fig. 4.** Profile measurement of the ideal periodic sine wave surface

### **2.3 Conventional Improvements**

Some improvements for the wide-angle profile measurement have been proposed as follows, but no way is the radical solution.

- Controlling the relative angle between measuring object and probe with the rotational table and
- Controlling the measuring force as the normal force of the measured object is kept constant.
- Scanning only along the downward path.

# **3 Wide-angle Profile Measurement with Slant**

#### **3.1 Concepts and Procedures**

In the conventional measurement of axis-symmetrical aspherical shape, a probe is attached pararell to the axis of the measured object. In the slant probe measurement, a probe or several probes are attached with an angle of the axis, for example, 30 degree to 90 degree. The area which is measured with one probe is limited within the area which can be measrued good, for example, from –45 degree to 45 degree or from -60 degree to 60 degree. In the whole surface measurement, the probe is dettached and attached again in the different angle or several probes which are attached in the different angle from the beginning of the measurement are used. The whole surface profile are evaluated by connecting the multiple measured data with different angles.

**Fig. 5** shows the advantage of the slant probe measurement compared with the conventional measurement. In the slant probe measurement, it is possible to achieve high accuracy in the profile measurement of a steep surface in a poor accuracy in the conventional way, for example, 60 degree to 75 degree or a steeper surface which cannot be measured by the conventional way, for example, 75 degree to 90 degree and over 90 degree. In the conventional way, it is necessary to control the measuring force in a small value according to the increase of the surface angle to achieve high measuring acuracy. In the slant probe measurement, it is not necessary to control the measuring force in a low value to achieve high measuring accuracy in the measurement of a steep angle surface. This makes the free room for the design of the probe.

It is also possible to increase the measuring force Fy to improve the maximum scanning speed vmax, so it is not necessary to use a compact and light-weight probe and highresolution controlling unit of the measuring force. This contributes the total cost down.



**Fig. 5.** Advantage of the slant probe measurement compared with the conventional measurement

### **3.2 Single Slant Probe Measurement**

Single slant probe system is the simplest hardware set-up, in which several measurements are conducted with the probe attached angles changed, and the total profile is evaluated by connecting the multiple measured data.

However, it is necessary in each measurement to evaluate the initial probe attached angle in high accuracy and to conduct the centering procedure to search the center point of the measured object, so it is troublesome.

A system, in which a rotational stage is used to change the probe attached angle automatically in high accurcy, is useful, but it causes to the total cost up. Even if an automatic rotational stage is used, the reevaluation and recentering procedures are necessary when the positioning accuracy is not high enough.

#### **3.3 Multi-probe Measurement**

By using multiple probes, measurement procedures can be much simplified, because the reevaluation is conducted only when the probe is attached at the first time, and the centering is conducted only when the measured object is attached. When you want to measure the profile of wide-angle optics, for example, it is possible to measure the whole surface from the angle of –90 degree to 90 degree by installing three measuring probes in the angle of –45 degree, 0 degree and 45 degree, and each probe measurement is conducted from the angle of –45 degree to 45 degree. The area where multiple measurements are overlapped can be used to check if the measurement process is good, or to connect data by the root mean square method when the initial probe attached angle could not be evaluated accurate enough. The number of probes, attached angles and measuring area, etc. should be decided according to the maximum angle of the measured object and the characteristics of the probes. **Fig. 6** shows the example of installing two probes in the angle of 0 degree and 45 degree.

#### **3.4 On-machine Measurement**

When an axis-symmetrical aspherical shape is measured on the machine, the axis of the measured object is precisely equal to the axis of the spindle because the measured object is machined by rotating the workpiece spindle. So the three dimensional profile measurement can be conducted by using only one slant probe and the rotational angle positioning table (C axis).



**Fig. 6.** An example of slant probe system with two probes in the angle of 0 degree and 45 degree

#### **3.5 Application of Air-bearing Cylinder Probe to 3D-CMM**

Slant probes have not been used in the conventional profile measurement for ultraprecision optics. On the other hand, the slant probe measurement with a touch probe is popular in the 3D-CMM, but the air-bearing cylinder probe whose measuring force can be precisely controlled by the air pressure has not been used in the 3D-CMM yet. To improve the measuring accuracy of 3D-CMM, the usage of the airbearing cylinder probe instead of a touch probe is much effective. **Fig. 7** shows the samples of the probe head proposed for the higher-accuracy 3D-CMM with air-bearing cylinders.



accuracy 3D-CMM with air-bearing cylinders

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A profile measurement with a slant probe was conducted on the lens cutting and grinding machine. **Fig. 8** shows the schematic view of the experimental set-up. The machine had three linear stages  $(X-Y-Z)$  axes) whose positioning resolution was 10nm. A profile measuring probe was attached in the angle of 45 degree to the Z axis in the X-Z plane on the machine. The measuring force of the probe was controlled in a low value of about 100mgf(0.98mN) by the electro-pnumatic regulator. A measured object whose shape was a sphere was attached on the workpiece spindle with the vacuum chuck. Profile measurements were conducted by sending NC data from PC to the NC controller to control the relative position between the measured object and the probe tip.

The centering procedure was conducted at the point with the angle of 0 degree to Z axis to adjust the tip ball of the probe with the center of the measured object. A profile measurement was conducted at the surface tilt angle from – 50 degree to 50 degree with the 45-degree slant probe. This means the measurement from –5 degree to 95 degree against Z axis. Measured data was fitted to the ideal shape of sphere by the least mean square method and the profile error was calculated.



**Fig. 8.** Schematic view of the experimental set-up



**Fig. 9.** Measured profile data with a slant probe

# **4 Experiments 4.2 Experimental Results**

**Fig. 9** shows the measured profile data. Steep surface which is difficult or impossible to be measured by the conventional 4.1 Experimental Set-up and Procedures<br>method with a pararell probe could be measured successfully.

Fig. 10 shows the profile error deviated from the sphere shape along the axis perpendicular to the probe shaft (-45 degree), because the measured data in the area near 90 degree can not be well fitted by calculating along X axis and the measured data in the area near 0 degree also can not be well fitted by calculating along Z axis.

## **5 Conclusion**

In this chapter, a new profile measuring method by using airbearing cylinder slant probes was introduced. This makes it possible to measure the profile with a steep angle in a good accuracy. It is expected that this measuring method is applied for the profile measurement of micro lenses or lens molds for CCD camera or DVD players, and the air-bearing cylinder probes make it possible to improve the measuring accuracy with 3D-CMM.

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**Fig. 10.** Profile error calculated from the measured data