8

Inspection Policies

System reliability can be improved by providing some standby units. Especially, even a single standby unit plays an important role in the case where failures of an operating unit are costly and/or dangerous. A typical example is the case of standby electric generators in nuclear power plants, hospitals, and other public facilities. It is, however, extremely serious if a standby generator fails at the very moment of electric power supply stoppage. Hence, frequent inspections are necessary to avoid such unfavorable situations.

Similar examples can be found in army defense systems, in which all weapons are on standby, and hence, must be checked at suitable times. For example, missiles are stored for a great part of their lifetimes after delivery. However, their reliabilities are known to decrease with time because some parts deteriorate with time. Thus, it would be important to test the functions of missiles as to whether they can operate normally. We need to check them periodically to monitor their reliabilities and to repair them if necessary.

Earlier work has been done on the problem of checking a single unit. The optimum schedules of inspections that minimize two expected costs until failure detection and per unit of time were summarized in [1]. The modified models where checking times are nonnegligible, a unit is inoperative during checking times, and checking hastens failures and failure symptoms, were considered in [2–5]. Furthermore, the availability of a periodic inspection model [6] and the mean duration of hidden faults [7,8] were derived. The downtime cost of checking intervals for a continuous production process [9,10] and two types of inspection [11,12] were proposed. The optimum inspection policies for more complicated systems were discussed in [13–20]. A good survey of optimization problems for inspection models was made in [21].

It was difficult to compute an optimum solution of the algorithm presented by [1] before high-power computers were popular. Nearly optimum inspection policies were considered in [22–28]. A continuous inspection intensity was introduced and the approximate checking interval was derived in [29,30]. Using these approximate methods, some modified inspection models were discussed and compared with other methods [31–37].

All failures cannot be detected upon inspection. The imperfect inspection models were treated in [38–41], and the parameter of an exponential failure distribution was estimated in [42]. Furthermore, optimum inspection models for a unit with hidden failure [43] were discussed in [44]. In such models, even if a unit fails, it continues to operate in hidden failure, and then, it fails. Such a type of failure is called unrevealed fault [45], pending failure [25], or fault latency [47].

Most faults occur intermittently in digital systems. The optimum periodic tests for intermittent faults were discussed in [48–50]. A simple algorithm to compute an optimum time was developed in [51], and random test for fault detection in combinational circuits was introduced in [52].

It is especially important to check and maintain standby and protective units. The optimum inspection models for standby units [53–57] and protective devices [59–61] were presented. Also, the following inspection maintenance to actual systems was done: building, industrial plant, and underwater structure [62–64]; combustion turbine units and standby equipment in dormant systems and nuclear generating stations [65–67]; productive equipment [68]; fail-safe structure [69]; manufacturing station [70]; automatic trips and warning instruments [71]; bearing [72]; and safety-critical systems [73]. Moreover, the delay time models were reviewed in [74, 75], where a defect arises and becomes a failure after its delay time, and were applied to plant maintenance [76].

This chapter reviews the results of [1] and mainly summarizes our own results of inspection models. In Section 8.1, we briefly mention the results of [1], and consider the inspection model with finite number of checks [77]. In Section 8.2, we summarize four approximate inspection policies [31–35, 78]. In Section 8.3, we derive two optimum inspection policies for a standby unit as an example of an electric generator [53]. In Section 8.4, we consider the inspection policy for a storage system required to achieve a high reliability, and derive an optimum checking number until overhaul that minimizes the expected cost rate [80–83]. In Section 8.5, we discuss optimum testing times for intermittent faults [49, 50]. Finally, in Section 8.6, we rewrite the results of a standard model for inspection policies for units that have to be operating for a finite interval [84, 85]. It is shown that the proposed partition method is a useful technique for analyzing maintenance policies for a finite interval. The inspection with preventive maintenance and random inspection is covered in Sections 7.3 and 9.3, respectively.

8.1 Standard Inspection Policy

A unit should operate for an infinite time span and is checked at successive times T_k $(k = 1, 2, \dots)$, where $T_0 \equiv 0$ (see Figure 8.1). Any failure is detected at the next checking time and is replaced immediately. A unit has a failure distribution $F(t)$ with finite mean μ whose failure rate $h(t)$ is not unchanged by any check. It is assumed that all times needed for checks and replacement

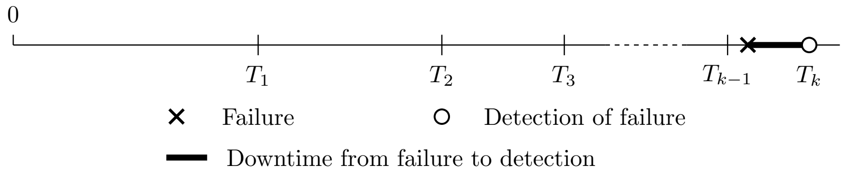

Fig. 8.1. Process of sequential inspection with checking time T_k

are negligible. Let c_1 be the cost of one check and c_2 be the loss cost per unit of time for the time elapsed between a failure and its detection at the next checking time, and c_3 be the replacement cost of a failed unit. Then, the total expected cost until replacement is

$$C_1(T_1, T_2, \dots) \equiv \sum_{k=0}^{\infty} \int_{T_k}^{T_{k+1}} [c_1(k+1) + c_2(T_{k+1} - t)]\,\mathrm{d}F(t) + c_3$$

$$= \sum_{k=0}^{\infty} [c_1 + c_2(T_{k+1} - T_k)]\overline{F}(T_k) - c_2\mu + c_3, \qquad (8.1)$$

where throughout this chapter, we use the notation $\overline{\Phi} \equiv 1 - \Phi$.

Differentiating the expected cost $C_1(T_1, T_2, \dots)$ with T_k and putting it equal to zero,

$$T_{k+1} - T_k = \frac{F(T_k) - F(T_{k-1})}{f(T_k)} - \frac{c_1}{c_2} \qquad (k = 1, 2, \dots), \qquad (8.2)$$

where f is a density function of F. The optimum checking intervals are decreasing when f is PF_2 (Pólya frequency function of order 2), and Algorithm 1 for computing the optimum inspection schedule is given in [1].

Algorithm 1

1. Choose T_1 to satisfy $c_1 = c_2 \int_0^{T_1} F(t)\mathrm{d}t$.
2. Compute T_2, T_3, ... recursively from (8.2).
3. If any $\delta_k > \delta_{k-1}$, reduce T_1 and repeat, where $\delta_k \equiv T_{k+1} - T_k$. If any $\delta_k < 0$, increase T_1 and repeat.
4. Continue until $T_1 < T_2 < \dots$ are determined to the degree of accuracy required.

Clearly, because the mean time to replacement time is $\sum_{k=0}^{\infty}(T_{k+1} - T_k)\overline{F}(T_k)$, the expected cost rate is, from (3.3) in Chapter 3,

$$C_2(T_1, T_2, \dots) \equiv \frac{c_1 \sum_{k=0}^{\infty} \overline{F}(T_k) - c_2\mu + c_3}{\sum_{k=0}^{\infty}(T_{k+1} - T_k)\overline{F}(T_k)} + c_2. \qquad (8.3)$$

In particular, when a unit is checked at periodic times and the failure time is exponential, *i.e.*, $T_k = kT$ $(k = 0, 1, 2, \dots)$ and $F(t) = 1 - \mathrm{e}^{-\lambda t}$, the total expected cost is

$$C_1(T) = \frac{c_1 + c_2 T}{1 - \mathrm{e}^{-\lambda T}} - \frac{c_2}{\lambda} + c_3. \tag{8.4}$$

The optimum checking time T^* to minimize (8.4) is given by a unique solution that satisfies

$$\mathrm{e}^{\lambda T} - (1 + \lambda T) = \frac{\lambda c_1}{c_2}. \tag{8.5}$$

Similarly, the expected cost rate is

$$C_2(T) = \frac{c_1 - (c_2/\lambda - c_3)(1 - \mathrm{e}^{-\lambda T})}{T} + c_2. \tag{8.6}$$

When $c_2/\lambda > c_3$, the optimum T^* is given by solving

$$1 - (1 + \lambda T)\mathrm{e}^{-\lambda T} = \frac{c_1}{c_2/\lambda - c_3}. \tag{8.7}$$

The following total expected cost for a continuous production system was proposed in [9].

$$\begin{aligned}\widetilde{C}_1(T_1, T_2, \dots) &\equiv \sum_{k=0}^{\infty} \int_{T_k}^{T_{k+1}} [c_1(k+1) + c_2(T_{k+1} - T_k)]\,\mathrm{d}F(t) + c_3 \\ &= c_1 \sum_{k=0}^{\infty} \overline{F}(T_k) + c_2 \sum_{k=0}^{\infty} (T_{k+1} - T_k)[\overline{F}(T_k) - \overline{F}(T_{k+1})] + c_3.\end{aligned} \tag{8.8}$$

In this case, Equation (8.2) can rewritten as

$$T_{k+1} - 2T_k + T_{k-1} = \frac{\overline{F}(T_{k+1}) - 2\overline{F}(T_k) + \overline{F}(T_{k-1})}{f(T_k)} - \frac{c_1}{c_2} \quad (k = 1, 2, \dots). \tag{8.9}$$

In general, it would be important to consider the availability more than the expected cost in some production systems [86, 87]. Let β_1 be the time of one check and β_3 be the replacement time of a failed unit. Then, the availability is, from **(3)** of Section 2.1.1,

$$A(T_1, T_2, \dots) \equiv \frac{\int_0^{\infty} \overline{F}(t)\,\mathrm{d}t}{\sum_{k=0}^{\infty} [\beta_1 + T_{k+1} - T_k]\overline{F}(T_k) + \beta_3}.$$

Thus, the policy maximizing $A(T_1, T_2, \dots)$ is the same one as minimizing $C_1(T_1, T_2, \dots)$ in (8.1) by replacing $c_i = \beta_i$ $(i = 1, 3)$ and $c_2 = 1$.

Next, we consider the inspection model with a finite number of checks, because a system such as missiles involves some parts that have to be replaced when the total operating times of checks have exceeded a prespecified time of quality warranty. A unit is checked at times T_k $(k = 1, 2, \dots, N-1)$ and is replaced at time T_N $(N = 1, 2, \ldots)$. The periodic inspection policy was suggested in [86], where a system is maintained preventively at the Nth check or is replaced at failure, whichever occurs first. We may consider replacement as preventive maintenance or overhaul.

In the above finite inspection model, the expected cost when a failure is detected and a unit is replaced at time T_k $(k = 1, 2, \ldots, N)$ is

$$\sum_{k=1}^{N} \int_{T_{k-1}}^{T_k} [c_1 k + c_2(T_k - t) + c_3] \, \mathrm{d}F(t)$$

and the expected cost when a unit is replaced without failure at time T_N is

$$(c_1 N + c_3)\overline{F}(T_N).$$

Thus, the total expected cost until replacement is

$$\sum_{k=0}^{N-1} [c_1 + c_2(T_{k+1} - T_k)]\overline{F}(T_k) - c_2 \int_0^{T_N} \overline{F}(t) \, \mathrm{d}t + c_3.$$

Similarly, the mean time to replacement is

$$\sum_{k=1}^{N} \int_{T_{k-1}}^{T_k} T_k \, \mathrm{d}F(t) + T_N \overline{F}(T_N) = \sum_{k=0}^{N-1} (T_{k+1} - T_k)\overline{F}(T_k).$$

Therefore, the expected cost rate is

$$C_2(T_1, T_2, \ldots, T_N) = \frac{c_1 \sum_{k=0}^{N-1} \overline{F}(T_k) - c_2 \int_0^{T_N} \overline{F}(t) \, \mathrm{d}t + c_3}{\sum_{k=0}^{N-1} (T_{k+1} - T_k)\overline{F}(T_k)} + c_2. \qquad (8.10)$$

In particular, when $T_k = kT$ $(k = 1, 2, \ldots, N)$ and $F(t) = 1 - \mathrm{e}^{-\lambda t}$, the expected cost rate is

$$C_2(T) = \frac{c_1}{T} - \frac{1}{\lambda T}(1 - \mathrm{e}^{-\lambda T})\left(c_2 - \frac{c_3 \lambda}{1 - \mathrm{e}^{-\lambda N T}}\right) + c_2. \qquad (8.11)$$

Differentiating $C_2(T)$ with respect to T and putting it to 0, we have

$$\left(\frac{c_2}{\lambda} - \frac{c_3}{1 - \mathrm{e}^{-\lambda N T}}\right)[1 - (1 + \lambda T)\mathrm{e}^{-\lambda T}] - \frac{c_3 \lambda N T \mathrm{e}^{-\lambda N T}(1 - \mathrm{e}^{-\lambda T})}{(1 - \mathrm{e}^{-\lambda N T})^2} = c_1. \qquad (8.12)$$

Denoting the left-hand side of (8.12) by $Q_N(T)$, $\lim_{T \to 0} Q_N(T) = -c_3/N$ and $\lim_{T \to \infty} Q_N(T) = c_2/\lambda - c_3$. First, we prove that $Q_N(T)$ is an increasing function of T for $c_2/\lambda > c_1 + c_3$. It is noted that the first term in $Q_N(T)$ is strictly increasing in T. Differentiating $-T\mathrm{e}^{-\lambda N T}(1 - \mathrm{e}^{-\lambda T})/(1 - \mathrm{e}^{-\lambda N T})^2$ with respect to T,

$$A[\lambda NT(1-\mathrm{e}^{-\lambda T})(1+\mathrm{e}^{-\lambda NT})-(1-\mathrm{e}^{-\lambda NT})(1-\mathrm{e}^{-\lambda T}+\lambda T\mathrm{e}^{-\lambda T})],$$

where $A \equiv \mathrm{e}^{-\lambda NT}/(1-\mathrm{e}^{-\lambda NT})^3 > 0$ for $T > 0$. Denoting the quantity in the bracket of the above equation by $L_N(T)$,

$$\begin{aligned} L_1(T) &= (1-\mathrm{e}^{-\lambda T})(\lambda T - 1 + \mathrm{e}^{-\lambda T}) > 0 \\ L_{N+1}(T) - L_N(T) &= (1-\mathrm{e}^{-\lambda T})[\lambda T(1 - N\mathrm{e}^{-\lambda NT} + N\mathrm{e}^{-\lambda(N+1)T}) \\ &\quad - (1-\mathrm{e}^{-\lambda T})\mathrm{e}^{-\lambda NT}] \\ &> (1-\mathrm{e}^{-\lambda T})^2[1-(N+1)\mathrm{e}^{-\lambda NT} + N\mathrm{e}^{-\lambda(N+1)T}] > 0. \end{aligned}$$

Hence, $L_N(T)$ is strictly increasing in N. Thus, $L_N(T)$ is always positive for any N, and the second term of $Q_N(T)$ is an increasing function of T, which completes the proof. Therefore, there exists a finite and unique T_N^* $(0 < T_N^* < \infty)$ that satisfies (8.12) for $c_2/\lambda > c_1 + c_3$, and it minimizes $C_2(T)$ in (8.11).

Next, we investigate properties of T_N^*. We prove that $Q_N(T)$ is also an increasing function of N as follows. From (8.12),

$$\begin{aligned} Q_{N+1}(T) - Q_N(T) &= c_3(1-\mathrm{e}^{-\lambda T})[1-E_N(T)] \\ &\times \left[\frac{1-(1+\lambda T)\mathrm{e}^{-\lambda T}}{E_N(T)E_{N+1}(T)} + \lambda T\left(\frac{N}{E_N(T)^2} - \frac{(N+1)\mathrm{e}^{-\lambda T}}{E_{N+1}(T)^2}\right)\right], \end{aligned}$$

where $E_N(T) \equiv 1-\mathrm{e}^{-\lambda NT}$. The first term in the bracket of the above equation is positive. The second term can be rewritten as

$$\frac{N}{E_N(T)^2} - \frac{(N+1)\mathrm{e}^{-\lambda T}}{E_{N+1}(T)^2} = \frac{NE_{N+1}(T)^2 - (N+1)\mathrm{e}^{-\lambda T}E_N(T)^2}{E_N(T)^2E_{N+1}(T)^2}$$

and the numerator of the right-hand side is

$$\begin{aligned} &NE_{N+1}(T)^2 - (N+1)\mathrm{e}^{-\lambda T}E_N(T)^2 \\ &\quad = \mathrm{e}^{-\lambda T}[N(\mathrm{e}^{\lambda T}-1)(1-\mathrm{e}^{-\lambda(2N+1)T}) - (1-\mathrm{e}^{-\lambda NT})^2] > 0. \end{aligned}$$

Hence, $Q_N(T)$ is a strictly increasing function of N because $Q_{N+1}(T) - Q_N(T) > 0$. Thus, T_N^* decreases when N increases. When $N = 1$, we have from (8.12),

$$1-(1+\lambda T)\mathrm{e}^{-\lambda T} = \frac{(c_1+c_3)\lambda}{c_2} \tag{8.13}$$

and when $N = \infty$,

$$1-(1+\lambda T)\mathrm{e}^{-\lambda T} = \frac{c_1\lambda}{c_2 - c_3\lambda}. \tag{8.14}$$

Because $[(c_1+c_3)\lambda]/c_2 > c_1\lambda/(c_2 - c_3\lambda)$, we easily find that $T_\infty^* < T_N^* \le T_1^*$, where T_1^* and T_∞^* are the respective solutions of (8.13) and (8.14).

Table 8.1. Optimum checking time T_N^* when $c_1 = 10$, $c_2 = 1$, and $c_3 = 100$

N	$\lambda = 1.0 \times 10^{-3}$				$\lambda = 1.1 \times 10^{-3}$				$\lambda = 1.2 \times 10^{-3}$			
	m											
	1.0	1.1	1.2	1.3	1.0	1.1	1.2	1.3	1.0	1.1	1.2	1.3
1	564	436	355	309	543	423	347	307	526	412	341	307
2	396	315	259	223	380	304	251	219	367	294	245	217
3	328	268	224	193	314	258	216	188	303	249	210	185
4	289	243	206	178	277	233	198	173	267	225	192	169
5	264	228	195	170	253	218	188	165	243	210	181	161
6	246	217	189	165	236	208	181	160	226	200	174	156
7	233	210	184	162	223	200	176	157	214	192	170	154
8	222	204	181	161	212	194	173	156	204	186	167	153
9	214	200	179	160	204	190	171	155	196	182	165	152
10	207	196	178	160	197	187	170	155	189	179	163	152

The condition of $c_2/\lambda > c_1 + c_3$ means that the total loss cost until the whole life of a unit is higher than the sum of costs of checks and replacements. This would be realistic in the actual field.

Example 8.1. We compute the optimum checking time T_N^* that minimizes $C_2(T)$ in (8.11) when $F(t) = 1 - \exp(-\lambda t^m)$ $(m \geq 1)$. When $m = 1$, it corresponds to an exponential case. Table 8.1 shows the optimum time T_N^* for $\lambda = 1.0 \times 10^{-3}$, 1.1×10^{-3}, 1.2×10^{-3}/hour, $m = 1.0$, 1.1, 1.2, 1.3 and $N = 1, 2, \ldots, 10$ when $c_1 = 10$, $c_2 = 1$, and $c_3 = 100$. This indicates that T_N^* decreases when λ, m, and N increase, and that a unit should be checked once every several weeks. ∎

8.2 Asymptotic Inspection Schedules

The computing procedure for obtaining the optimum inspection schedule was specified in [1]. Unfortunately, it is difficult to compute Algorithm 1 numerically, because the computations are repeated until the procedures are determined to the required degree by changing the first checking time. To avoid this, a nearly optimum inspection policy that depends on a single parameter p was suggested in [22]. This policy was used for Weibull and gamma distribution cases [23, 24]. Furthermore, the procedure of introducing a continuous intensity $n(t)$ of checks per unit of time was proposed in [29, 30]. This section summarizes four approximate calculations of optimum checking procedures.

(1) Periodic Inspection

When a unit is checked at periodic times kT $(k = 1, 2, \ldots)$, the total expected cost is, from (8.1),

$$C_1(T) = \frac{c_1}{T}[E\{D\} + \mu] + c_2 E\{D\} + c_3, \tag{8.15}$$

where $E\{D\} \equiv \sum_{k=0}^{\infty} \int_0^T [F(t+kT) - F(kT)]\,\mathrm{d}t$, which is the mean duration of time elapsed between a failure and its detection.

Suppose that $F(t)$ has the piecewise linear approximation:

$$F(t+kT) - F(kT) = \frac{t}{T}[F((k+1)T) - F(kT)] \quad (0 \le t \le T). \tag{8.16}$$

Then, $E\{D\} = T/2$; *i.e.,* the mean duration of undetected failure is half the time between the checking times. The result is also given when the failure times between successive checking times are independent and distributed uniformly. In this case, the optimum checking time is $\widetilde{T}_1 = \sqrt{(2c_1\mu)/c_2}$. This time is also derived from (8.5) by putting $\mathrm{e}^{\lambda T} \approx 1 + \lambda T + (\lambda T)^2/2$ approximately and $\lambda = 1/\mu$.

(2) Munford and Shahani's Method

The asymptotic method for computing the optimum schedule was proposed in [22]. When a unit is operating at time T_{k-1}, the probability that it fails in an interval $(T_{k-1}, T_k]$ is constant for all k; *i.e.,*

$$\frac{F(T_k) - F(T_{k-1})}{\overline{F}(T_{k-1})} \equiv p \qquad (k = 1, 2, \dots). \tag{8.17}$$

This represents that the probability that a unit with age T_{k-1} fails in interval $(T_{k-1}, T_k]$ is given by a constant p. Noting that $F(T_1) = p$, Equation (8.17) can be solved for T_k, and we have

$$\overline{F}(T_k) = q^k \qquad \text{or} \qquad T_k = \overline{F}^{-1}(q^k) \qquad (k = 1, 2, \dots), \tag{8.18}$$

where $q \equiv 1 - p$ $(0 < p < 1)$. Thus, from (8.1), the total expected cost is

$$C_1(p) = \frac{c_1}{p} + c_2 \sum_{k=1}^{\infty} T_k q^{k-1} p - c_2\mu + c_3. \tag{8.19}$$

We seek p that minimizes $C_1(p)$ in (8.19). It was assumed in [28] that p is not constant and is an increasing function of the checking number.

(3) Keller's Method

An inspection intensity $n(t)$ is defined as follows [29]: $n(t)\mathrm{d}t$ denotes the probability that a unit is checked at interval $(t, t+\mathrm{d}t)$ (see Figure 8.2). From this definition, when a unit is checked at times T_k, we have the relation

$$\int_0^{T_k} n(t)\,\mathrm{d}t = k \qquad (k = 1, 2, \dots). \tag{8.20}$$

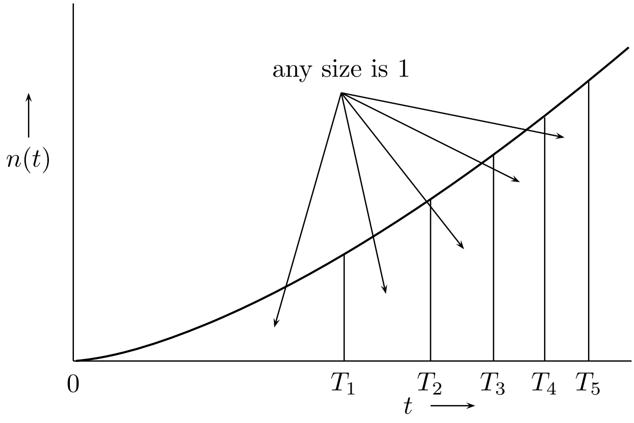

Fig. 8.2. Inspection intensity $n(t)$

Furthermore, suppose that the mean time from the failure at time t to its detection at time $t+a$ is half of a checking interval, the same as obtained in case **(1)**. Then, we have

$$\int_t^{t+a} n(u)\,\mathrm{d}u = \frac{1}{2}$$

which can be approximately written as

$$\int_t^{t+a} n(u)\,\mathrm{d}u \approx an(t) = \frac{1}{2}$$

and hence, $a = 1/[2n(t)]$. By the same arguments, we can easily see that the next checking interval, when a unit was checked at time T_k, is $1/n(T_k)$ approximately.

Therefore, the total expected cost in (8.1) is given by

$$\begin{aligned} C(n(t)) &= \int_0^\infty \left[c_1 \int_0^t n(u)\,\mathrm{d}u + \frac{c_2}{2n(t)}\right] \mathrm{d}F(t) + c_3 \\ &= \int_0^\infty \overline{F}(t) \left[c_1 n(t) + \frac{c_2 h(t)}{2n(t)}\right] \mathrm{d}t + c_3, \end{aligned} \tag{8.21}$$

where $h(t) \equiv f(t)/\overline{F}(t)$ which is the failure rate. Differentiating $C(n(t))$ with $n(t)$ and putting it to zero,

$$n(t) = \sqrt{\frac{c_2 h(t)}{2c_1}}. \tag{8.22}$$

Thus, from (8.20), the optimum checking time is given by the equation:

$$k = \int_0^{T_k} \sqrt{\frac{c_2}{2c_1} h(t)}\, \mathrm{d}t \qquad (k = 1, 2, \dots). \tag{8.23}$$

The inspection intensity $n(t)$ was also obtained in [36] by solving the Euler equation in (8.21), and using $n(t)$, the optimum policies for models with imperfect inspection were derived in [88].

In particular, when $F(t) = 1 - \mathrm{e}^{-\lambda t}$, the interval between checks is constant, and is $\sqrt{2c_1/(\lambda c_2)}$ which agrees with the result of case **(1)**. It is of great interest that a function $\sqrt{2c_1/(\lambda c_2)}$ evolves into the same form as an optimum order time of a classical inventory control model [89], by denoting c_1 and c_2 as the ordering cost per order and holding cost per unit of time, respectively, and λ as the constant demand rate for an inventory unit.

(4) Nakagawa and Yasui's Method

When T_n is sufficiently large, we may assume approximately [79]

$$T_{n+1} - T_n + \varepsilon = T_n - T_{n-1}. \tag{8.24}$$

It is easy to see that if f is PF_2 then $\varepsilon \geq 0$ because the optimum checking intervals are decreasing [1]. Further substituting the relation (8.24) into (8.2),

$$\frac{c_1}{c_2} - \varepsilon = \frac{\int_{T_{n-1}}^{T_n} [f(t) - f(T_n)]\, \mathrm{d}t}{f(T_n)} \geq 0 \tag{8.25}$$

because $f(t) \geq f(T_n)$ for $t \leq T_n$ and large T_n. Thus, we have $0 \leq \varepsilon \leq c_1/c_2$.

From the above discussion, we can specify the computation for obtaining the asymptotic inspection schedule.

Algorithm 2

1. Choose an appropriate ε from $0 < \varepsilon < c_1/c_2$.
2. Determine a checking time T_n after sufficient time for required accuracy.
3. Compute T_{n-1} to satisfy

$$T_n - T_{n-1} - \varepsilon = \frac{F(T_n) - F(T_{n-1})}{f(T_n)} - \frac{c_1}{c_2}.$$

4. Compute $T_{n-1} > T_{n-2} > \dots$ recursively from (8.2).
5. Continue until $T_k < 0$ or $T_{k+1} - T_k > T_k$.

Example 8.2. Suppose that the failure time has a Weibull distribution with a shape parameter m; *i.e.,* $F(t) = 1 - \exp[-(\lambda t)^m]$.

(1) *Periodic inspection.* The optimum checking time is

$$\lambda \widetilde{T}_1 = \left[\frac{2\lambda c_1}{c_2} \Gamma\left(1 + \frac{1}{m}\right)\right]^{1/2}.$$

Table 8.2. Comparisons of Nakagawa, Barlow, Munford, and Keller policies when $F(t) = 1 - \exp[-(\lambda t)^2]$, $1/\lambda = 500$, and $c_1/c_2 = 10$

k	Nakagawa $T_n = 1500$ $\varepsilon = 5$	Nakagawa $T_n = 1500$ $\varepsilon = 4.5$	Barlow	Barlow	Munford $p = 0.215$	Keller
1	219.6	207.1	205.6	205.6	246.0	177.8
2	318.7	308.9	307.6	307.6	347.9	282.3
3	402.0	393.5	392.3	392.3	426.1	369.9
4	476.4	468.7	467.5	467.5	492.0	448.1
5	544.8	537.6	536.4	536.5	550.1	520.0
6	608.7	601.9	600.7	600.8	602.6	587.2
7	669.1	662.6	661.5	661.6	650.9	650.8
8	726.6	720.4	719.2	719.4	695.8	711.4
9	781.7	775.8	774.6	774.8	738.0	769.5
10	834.8	829.1	827.8	828.2	777.9	825.5
11	886.1	880.6	879.3	879.7	815.9	879.6
12	935.8	930.5	929.1	929.7	852.2	932.2
13	984.1	979.0	977.4	978.3	887.0	983.3
14	1031.1	1026.2	1024.5	1025.6	920.5	1033.1

(2) *Munford and Shahani's method.* From (8.19), we obtain p that minimizes

$$g(p) = \frac{\lambda c_1}{p c_2} + \left(\log \frac{1}{q}\right)^{1/m} \sum_{k=1}^{\infty} k^{1/m} q^{k-1} p$$

and the optimum checking intervals are

$$\lambda T_k = \left(k \log \frac{1}{q}\right)^{1/m} \qquad (k = 1, 2, \dots).$$

(3) *Keller's method.* From (8.23),

$$T_k = \left[(m+1)k\sqrt{\frac{c_1}{2m\lambda^m c_2}}\right]^{2/(m+1)} \qquad (k = 1, 2, \dots).$$

In particular, when $m = 1$, $T_k = k\sqrt{2c_1/(\lambda c_2)}$.

Table 8.2 shows the comparisons of the methods of Barlow et al., Munford et al., Keller, and Nakagawa et al., when $m = 2$, $1/\lambda = 500$, $c_1/c_2 = 10$. Nakagawa and Yasui's method gives a fairly good approximation of Barlow's one. In particular, when we choose $\varepsilon = 4.5$, the results are almost the same as the sequence of optimum checking times. The computation of Keller's method is very easy, and this method would be very useful for obtaining checking times in the actual field. ∎

8.3 Inspection for a Standby Unit

In this section, we consider an inspection policy for a single standby electric generator. We check a standby generator frequently to guarantee the upper bound of the probability that it has failed at the time of the electric power supply stoppage, but to reduce unnecessary costs do not check it too frequently.

The details of the model are described as follows.

(1) The failure time of a standby generator has a general distribution $F(t)$ and its failure is detected only at the next checking time.
(2) A failed standby generator, which was detected at some check, undergoes repair immediately and its repair time has a general distribution $G(t)$.
(3) The time required for the check is negligible and a standby generator becomes as good as new upon inspection or repair.
(4) The next checking time is scheduled at constant time T $(0 < T \leq \infty)$ after either the prior check or the repair completion.
(5) Costs c_0 and c_1 are incurred for each repair and check, respectively, and cost c_2 is incurred for the failure of a generator when the electric power supply stops, where $c_2 > c_0 \geq c_1$.
(6) The policy terminates with the time of electric power supply stoppage, which occurs according to an exponential distribution $(1 - \mathrm{e}^{-\alpha t})$.

Under the assumptions above, we consider two optimization problems: (a) an optimum checking time T^* that minimizes the expected cost until the time of electric power supply stoppage, and (b) the largest $\overline{T}$ such that the probability that a generator has failed at the time of electric power supply stoppage is not greater than a prespecified value ε.

To obtain the expected cost of the inspection model as described above, we derive the expected numbers of checks and repairs of a standby electric generator, and the probability that it has failed at the time of electric power supply stoppage.

As an initial condition, it is assumed for convenience that a generator goes into standby and is good at time 0. Furthermore, for simplicity of equations, we define $D(t) \equiv 0$ for $t < T$ and $\equiv 1$ for $t \geq T$; *i.e.*, $D(t)$ is a degenerate distribution at time T.

Let $H(t)$ be the distribution of the recurrence time to the state that a standby generator is good upon inspection or repair completion. Then, we have

$$H(t) = \int_0^t \overline{F}(u)\,\mathrm{d}D(u) + \left[\int_0^t F(u)\,\mathrm{d}D(u)\right] * G(t), \tag{8.26}$$

where the asterisk represents the Stieltjes convolution. Equation (8.26) can be explained by the first term on the right-hand side being the probability that a standby generator is good upon inspection until time t, and the second term the probability that a failed generator is detected at a check and its repair is completed until time t.

In addition, let $M_0(t)$ and $M_1(t)$ be the expected numbers of repairs of a failed generator and of checks of a standby generator during $(0,t]$, respectively. Then, the following renewal-type equations are given by

$$M_0(t) = \int_0^t F(u)\,\mathrm{d}D(u) + H(t) * M_0(t) \tag{8.27}$$

$$M_1(t) = D(t) + H(t) * M_1(t). \tag{8.28}$$

Thus, forming the Laplace–Stieltjes (LS) transforms of (8.26), (8.27), and (8.28), respectively, we have

$$H^*(s) = \mathrm{e}^{-sT}[\overline{F}(T) + F(T)G^*(s)] \tag{8.29}$$

$$M_0^*(s) = \frac{\mathrm{e}^{-sT}F(T)}{1 - H^*(s)}, \qquad M_1^*(s) = \frac{\mathrm{e}^{-sT}}{1 - H^*(s)}, \tag{8.30}$$

where throughout this section, we denote the LS transform of the function by the corresponding asterisk; *e.g.,* $G^*(s) \equiv \int_0^\infty \mathrm{e}^{-st}\mathrm{d}G(t)$ for $s > 0$.

Next, let $P(t)$ denote the probability that a standby generator has failed at time t; *i.e.,* a standby generator, which is not good, will be detected at the next check or a failed generator, which was detected at the prior check, is now under repair. Then, the probability that a standby generator is good at time t is given by

$$\overline{P}(t) = \overline{F}(t)\overline{D}(t) + H(t) * \overline{P}(t).$$

Forming the LS transform of $P(t)$, we have

$$1 - P^*(s) = \frac{\int_0^T s\mathrm{e}^{-st}\overline{F}(t)\,\mathrm{d}t}{1 - H^*(s)}. \tag{8.31}$$

We consider the total expected cost until the time of electric power supply stoppage. Note that the inspection model of a standby generator may involve at least the following three costs: the costs c_0 and c_1 incurred by each repair and each check, respectively, and the cost c_2 incurred by failure of a standby generator when the electric power supply stops.

Suppose that the electric power supply stops at time t. Then, the total expected cost during $(0,t]$ is given by

$$\widetilde{C}(t) = c_0 M_0(t) + c_1 M_1(t) + c_2 P(t).$$

Thus, dropping the condition that the electric power supply stops at time t from assumption (6), we have the expected cost:

$$C_1(T) \equiv \int_0^\infty \widetilde{C}(t)\alpha\mathrm{e}^{-\alpha t}\,\mathrm{d}t = c_0 M_0^*(\alpha) + c_1 M_1^*(\alpha) + c_2 P^*(\alpha)$$

which is a function of T. Using (8.30) and (8.31), $C_1(T)$ can be written as

$$C_1(T) = \frac{\mathrm{e}^{-\alpha T}[c_0 F(T) + c_1] - c_2 \int_0^T \alpha \mathrm{e}^{-\alpha T}\overline{F}(t)\,\mathrm{d}t}{1 - \mathrm{e}^{-\alpha T}[\overline{F}(T) + F(T)G^*(\alpha)]} + c_2. \tag{8.32}$$

It is evident that

$$C_1(0) \equiv \lim_{T\to 0} C_1(T) = \infty, \qquad C_1(\infty) \equiv \lim_{T\to\infty} C_1(T) = c_2 F^*(\alpha)$$

which represents the expected cost for the case where no inspection is made.

We seek an optimum checking time T_1^* that minimizes the expected cost $C_1(T)$ given in (8.32). Differentiating $\log C_1(T)$ with respect to T, we have, for large T,

$$\frac{\mathrm{d}[\log C_1(T)]}{\mathrm{d}T} \approx \alpha \mathrm{e}^{-\alpha T}\left[\frac{c_2 G^*(\alpha) - c_0 - c_1}{c_2 F^*(\alpha)} - G^*(\alpha)\right].$$

Thus, if the quantity in the bracket on the right-hand side is positive; *i.e.,*

$$c_2 G^*(\alpha)[1 - F^*(\alpha)] > c_0 + c_1, \tag{8.33}$$

then there exists at least some finite T such that $C_1(\infty) > C_1(T)$, and hence, it is better to check a standby generator at finite time T.

In general, it is difficult to discuss analytically an optimum checking time T^* that minimizes $C_1(T)$. In particular, consider the case where $F(t) = 1 - \mathrm{e}^{-\lambda t}$ and $G(t) \equiv 1$ for $t \geq 0$; *i.e.,* the failure time is exponential and the repair time is negligible. Then, the resulting cost is

$$C_1(T) = \frac{\mathrm{e}^{-\alpha T}[c_0(1-\mathrm{e}^{-\lambda T}) + c_1] + c_2[1-\mathrm{e}^{-\alpha T} - \frac{\alpha}{\alpha+\lambda}(1-\mathrm{e}^{-(\alpha+\lambda)T})]}{1 - \mathrm{e}^{-\alpha T}}. \tag{8.34}$$

Differentiating $C_1(T)$ with respect to T and setting it equal to zero,

$$c_0 \mathrm{e}^{-\lambda T}\left[1+\frac{\lambda}{\alpha}(1 - \mathrm{e}^{-\alpha T})\right] + c_2\left[1 - \mathrm{e}^{-\lambda T} - \frac{\lambda}{\alpha+\lambda}(1 - \mathrm{e}^{-(\alpha+\lambda)T})\right] = c_0 + c_1, \tag{8.35}$$

where the left-hand side is strictly increasing in the case of $c_2 > [(\alpha+\lambda)/\alpha]c_0$, and conversely, nonincreasing in the case of $c_2 \leq [(\alpha+\lambda)/\alpha]c_0$. Further note that the left-hand side is c_0 as $T \to 0$ and $[\alpha/(\alpha+\lambda)]c_2$ as $T \to \infty$.

Therefore, we have the following results from the above discussion.

(i) If $c_2 > [(\alpha+\lambda)/\alpha](c_1 + c_0)$ then there exists a finite checking time T_1^* that satisfies (8.35), and the resulting cost is

$$C_1(T^*) = c_2 - c_1 - c_0 - \left(c_2 - c_0\frac{\alpha+\lambda}{\alpha}\right)\mathrm{e}^{-\lambda T^*}. \tag{8.36}$$

(ii) If $c_2 \leq [(\alpha+\lambda)/\alpha](c_1 + c_0)$ then $T_1^* = \infty$; *i.e.,* no inspection is made, and $C_1(\infty) = c_2[\lambda/(\alpha+\lambda)]$.

Note that the inequality of $c_2 > [(\alpha+\lambda)/\alpha](c_1+c_0)$ has been already derived from (8.33).

It is also of interest to make the probability as small as possible by checks, that a standby generator has failed at the time of electric power supply stoppage. If the probability is prespecified, we can compute a checking time $\overline{T}_1$ such that $P^*(\alpha) \leq \varepsilon$; *i.e.,*

$$\frac{\int_0^T \mathrm{e}^{-\alpha t}\,\mathrm{d}F(t) - \mathrm{e}^{-\alpha T}F(T)G^*(\alpha)}{1-\mathrm{e}^{-\alpha T}[\overline{F}(T)+F(T)G^*(\alpha)]} \leq \varepsilon. \tag{8.37}$$

For instance, if the repair time is negligible, *i.e.,* $G^*(\alpha) = 1$, then the left-hand side of (8.37) is strictly increasing in T. Hence, there exists a unique checking time $\overline{T}$ that satisfies

$$\frac{\int_0^T F(t)\alpha\mathrm{e}^{-\alpha t}\,\mathrm{d}t}{1-\mathrm{e}^{-\alpha T}} = \varepsilon \tag{8.38}$$

for sufficiently small $\varepsilon > 0$.

Until now, we have assumed that a standby generator becomes as good as new upon inspection. Next, we make the same assumption as the previous ones except that the failure rate of a standby generator remains undisturbed by any inspection. This assumption would be more plausible than the previous model in practice, however, the analysis becomes more difficult. Then, the expected cost until the time of electric power supply stoppage is [53]

$$C_2(T) = \frac{\begin{array}{c} c_0\sum_{k=1}^{\infty}\mathrm{e}^{-\alpha kT}[\overline{F}((k-1)T)-\overline{F}(kT)] \\ +\,c_1\sum_{k=1}^{\infty}\mathrm{e}^{-\alpha kT}\overline{F}((k-1)T) - c_2[1-F^*(\alpha)]\end{array}}{1-G^*(\alpha)\sum_{k=1}^{\infty}\mathrm{e}^{-\alpha kT}[\overline{F}((k-1)T)-\overline{F}(kT)]} + c_2. \tag{8.39}$$

It is evident that $C_2(0) = \infty$ and $C_2(\infty) = c_2F^*(\alpha)$. Furthermore, for large T,

$$\frac{\mathrm{d}[\log C_2(T)]}{\mathrm{d}T} \approx \alpha\mathrm{e}^{-\alpha T}\left[\frac{c_2G^*(\alpha)-c_0-c_1}{c_2F^*(\alpha)} - G^*(\alpha)\right].$$

Thus, if $c_2G^*(\alpha)[1-F^*(\alpha)] > c_0 + c_1$, then there exists at least some finite T such that $C_2(\infty) > C_2(T)$, which agrees with the results of the previous model.

It is very difficult to obtain analytically an optimum time T_2^* that minimizes $C_2(T)$ in (8.39). It is noted, however, that the expected cost $C_2(T)$ agrees with (8.34) in the special case of $F(t) = 1-\mathrm{e}^{-\lambda t}$ and $G(t) \equiv 1$ for $t \geq 0$.

Example 8.3. We give a numerical example where $\overline{F}(t) = (1+\lambda t)\mathrm{e}^{-\lambda t}$ and $\overline{G}(t) = (1+\theta t)\mathrm{e}^{-\theta t}$, both of which are the gamma distribution with shape parameter 2. Table 8.3 shows the optimum checking times T_1^* and T_2^* for the mean failure time $2/\lambda$ and cost c_2, when $c_0 = 30$ dollars, $c_1 = 3$ dollars, $1/\theta = 12$ hours, and $1/\alpha = 1460$ hours; *i.e.,* the electric power supply stops 6 times a year on the average. It has been shown that both of the checking

Table 8.3. Dependent of mean failure time $2/\lambda$ and cost c_2 in optimum checking times T_1^* and T_2^* when $c_0 = 30$, $c_1 = 3$, $1/\theta = 12$, and $1/\alpha = 1460$

	$c_2 = 150$		$c_2 = 250$		$c_2 = 350$	
$2/\lambda$	T_1^*	T_2^*	T_1^*	T_2^*	T_1^*	T_2^*
1200	292	480	249	308	224	241
1600	368	535	311	354	279	280
2000	439	594	369	399	330	318
2400	507	656	424	445	379	356
2800	572	720	477	491	425	393
3200	635	783	528	537	469	430
3600	697	848	578	582	512	467
4000	757	914	626	628	554	503

times are increasing if $2/\lambda$ is increasing and are decreasing if c_2 is increasing. In addition, T_1^* becomes greater than T_2^* when c_2 and $2/\lambda$ are large enough. ∎

8.4 Inspection for a Storage System

A system such as missiles is in storage for a long time from delivery to the actual usage and has to hold a high mission reliability when it is used [90]. After a system is transported to each firing operation unit via the depot, it is installed on a launcher and is stored in a warehouse for a great part of its lifetime, and waits for its operation. Therefore, missiles are often called dormant systems.

However, the reliability of a storage system goes down with time because some kinds of electronic and electric parts of a system degrade with time [91–95]. The periodic inspection of stored electronic equipment was studied and how to compute its reliability after ten years of storage was shown in [96]. We should test and maintain a storage system at periodic times to hold a high reliability, because it is impossible to check whether a storage system can operate normally.

In most inspection models, it has been assumed that the function test can clarify all system failures. However, a missile is exposed to a very severe flight environment and some kinds of failures are revealed only in such severe conditions. That is, some failures of a missile cannot be detected by the function test on the ground. To solve this problem, we assume that a system is divided into two independent units: Unit 1 becomes new after every test because all failures of unit 1 are detected by the function test and are removed completely by maintenance, but unit 2 degrades steadily with time from delivery to overhaul because all failures of unit 2 cannot be detected by any test. The reliability of a system deteriorates gradually with time as the reliability of unit 2 deteriorates steadily.

This section considers a system in storage that is required to achieve a higher reliability than a prespecified level q $(0 < q \leq 1)$. To hold the reliability, a system is tested and is maintained at periodic times NT $(N = 1, 2, \dots)$, and is overhauled if the reliability becomes equal to or lower than q. A test number N^* and the time $N^*T + t_0$ until overhaul, are derived when a system reliability is just equal to q. Using them, the expected cost rate $C(T)$ until overhaul is obtained, and an optimum test time T^* that minimizes it is computed. Finally, numerical examples are given when failure times of units have exponential and Weibull distributions. Two extended models were considered in [82, 97], where a system is also replaced at time $(N + 1)T$, and may be degraded at each inspection, respectively.

A system consists of unit 1 and unit 2, where the failure time of unit i has a cumulative hazard function $H_i(t)$ $(i = 1, 2)$. When a system is tested at periodic times NT $(N = 1, 2, \dots)$, unit 1 is maintained and is like new after every test, and unit 2 is not done; *i.e.,* its hazard rate remains unchanged by any tests.

From the above assumptions, the reliability function $R(t)$ of a system with no inspection is

$$R(t) = \mathrm{e}^{-H_1(t) - H_2(t)}. \tag{8.40}$$

If a system is tested and maintained at time t, the reliability just after test is

$$R(t_{+0}) = \mathrm{e}^{-H_2(t)}.$$

Thus, the reliabilities just before and after the Nth test are, respectively,

$$R(NT_{-0}) = \mathrm{e}^{-H_1(T) - H_2(NT)}, \qquad R(NT_{+0}) = \mathrm{e}^{-H_2(NT)}. \tag{8.41}$$

Next, suppose that the overhaul is performed if a system reliability is equal to or lower than q. Then, if

$$\mathrm{e}^{-H_1(T) - H_2(NT)} > q \geq \mathrm{e}^{-H_1(T) - H_2[(N+1)T]} \tag{8.42}$$

then the time to overhaul is $NT + t_0$, where t_0 $(0 < t_0 \leq T)$ satisfies

$$\mathrm{e}^{-H_1(t_0) - H_2(NT + t_0)} = q. \tag{8.43}$$

This shows that the reliability is greater than q just before the Nth test and is equal to q at time $NT + t_0$.

Let c_1 and c_2 be the test and the overhaul costs, respectively. Then, denoting the time interval $[0, NT + t_0]$ as one cycle, the expected cost rate until overhaul is given by

$$C(T, N) = \frac{Nc_1 + c_2}{NT + t_0}. \tag{8.44}$$

We consider two particular cases where the cumulative hazard functions $H_i(t)$ are exponential and Weibull ones. A test number N^* that satisfies (8.42), and t_0 that satisfies (8.43), are computed. Using these quantities, we compute the expected cost $C(T, N)$ until overhaul and seek an optimum test time T^* that minimizes it.

(1) Exponential Case

When the failure time of units has an exponential distribution, *i.e.*, $H_i(t) = \lambda_i t$ $(i = 1, 2)$, Equation (8.42) is rewritten as

$$\frac{1}{Na+1}\log\frac{1}{q} \leq \lambda T < \frac{1}{(N-1)a+1}\log\frac{1}{q}, \tag{8.45}$$

where $\lambda \equiv \lambda_1 + \lambda_2$ and $a \equiv H_2(T)/[H_1(T)+H_2(T)] = \lambda_2/\lambda$ $(0 < a < 1)$ which represents an efficiency of inspection [90], and is widely adopted in practical reliability calculation of a storage system.

When a test time T is given, a test number N^* that satisfies (8.45) is determined. Particularly, if $\log(1/q) \leq \lambda T$ then $N^* = 0$, and N^* diverges as λT tends to 0. In this case, Equation (8.43) is

$$N^*\lambda_2 T + \lambda t_0 = \log\frac{1}{q}. \tag{8.46}$$

From (8.46), we can compute t_0 easily. Thus, the total time to overhaul is

$$N^*T + t_0 = N^*(1-a)T + \frac{1}{\lambda}\log\frac{1}{q} \tag{8.47}$$

and the expected cost rate is

$$C(T, N^*) = \frac{N^*c_1 + c_2}{N^*(1-a)T + \frac{1}{\lambda}\log\frac{1}{q}}. \tag{8.48}$$

When a test time T is given, we compute N^* from (8.45) and $N^*T + t_0$ from (8.47). Substituting these values into (8.48), we have $C(T, N^*)$. Changing T from 0 to $\log(1/q)/[\lambda(1-a)]$, because λT is less than $\log(1/q)/(1-a)$ from (8.45), we can compute an optimum T^* that minimizes $C(T, N^*)$. In the particular case of $\lambda T \geq \log(1/q)/(1-a)$, $N^* = 0$ and the expected cost rate is $C(T, 0) = c_2/t_0 = \lambda c_2/\log(1/q)$.

Example 8.4. Table 8.4 presents the optimum number N^* and the total time $\lambda(N^*T + t_0)$ to overhaul for λT when $a = 0.1$ and $q = 0.8$. For example, when λT increases from 0.203 to 0.223, $N^* = 1$ and $\lambda(N^*T + t_0)$ increases from 0.406 to 0.424. In accordance with the decrease in λT, both N^* and $\lambda(N^*T + t_0)$ increase as shown in (8.45) and (8.47), respectively.

Table 8.5 gives the optimum number N^* and time λT^* that minimize the expected cost $C(T, N)$ for c_2/c_1, a and q, and the resulting total time $\lambda(N^*T^* + t_0)$ and the expected cost rate $C(T^*, N^*)/\lambda$ for $c_1 = 1$. These indicate that λT^* increases and $\lambda(N^*T^* + t_0)$ decreases when c_1/c_2 and a increase, and both λT^* and $\lambda(N^*T^* + t_0)$ decrease when q increases. ∎

Table 8.4. Optimum inspection number N^* and total time to overhaul $\lambda(N^*T+t_0)$ for λT when $a = 0.1$ and $q = 0.8$

λT	N^*	$\lambda(N^*T + t_0)$
$[0.223, \infty)$	0	$[0.223, \infty)$
$[0.203, 0.223)$	1	$[0.406, 0.424)$
$[0.186, 0.203)$	2	$[0.558, 0.588)$
$[0.172, 0.186)$	3	$[0.687, 0.725)$
$[0.159, 0.172)$	4	$[0.797, 0.841)$
$[0.149, 0.159)$	5	$[0.893, 0.940)$
$[0.139, 0.149)$	6	$[0.976, 1.026)$
$[0.131, 0.139)$	7	$[1.050, 1.102)$
$[0.124, 0.131)$	8	$[1.116, 1.168)$
$[0.117, 0.124)$	9	$[1.174, 1.227)$
$[0.112, 0.117)$	10	$[1.227, 1.280)$

Table 8.5. Optimum inspection time λT^*, total time to overhaul $\lambda(N^*T+t_0)$, and expected cost rate $C(T^*)/\lambda$

c_2/c_1	a	q	N^*	λT^*	$\lambda(N^*T^* + t_0)$	$C(T^*, N^*)/\lambda$
10	0.1	0.8	8	0.131	1.168	15.41
50	0.1	0.8	19	0.080	1.586	43.51
10	0.5	0.8	2	0.149	0.372	32.27
10	0.1	0.9	7	0.062	0.552	32.63

(2) Weibull Case

When the failure time of units has a Weibull distribution; *i.e.*, $H_i(t) = (\lambda_i t)^m$ $(i = 1, 2)$, Equations (8.42) and (8.43) are rewritten as

$$\left\{\frac{1}{a[(N+1)^m - 1] + 1}\log\frac{1}{q}\right\}^{1/m} \le \lambda T < \left\{\frac{1}{a[N^m - 1] + 1}\log\frac{1}{q}\right\}^{1/m} \tag{8.49}$$

$$(1-a)t_0^m + a(NT + t_0)^m = \frac{1}{\lambda^m}\log\frac{1}{q}, \tag{8.50}$$

respectively, where $\lambda^m \equiv \lambda_1^m + \lambda_2^m$ and

$$a \equiv \frac{H_2(T)}{H_1(T) + H_2(T)} = \frac{\lambda_2^m}{\lambda_1^m + \lambda_2^m}.$$

When an inspection time T is given, N^* and t_0 are computed from (8.49) and (8.50). Substituting these values into (8.44), we have $C(T, N^*)$, and changing T from 0 to $[\log(1/q)/(1-a)]^{1/m}/\lambda$, we can compute an optimum T^* that minimizes $C(T, N^*)$.

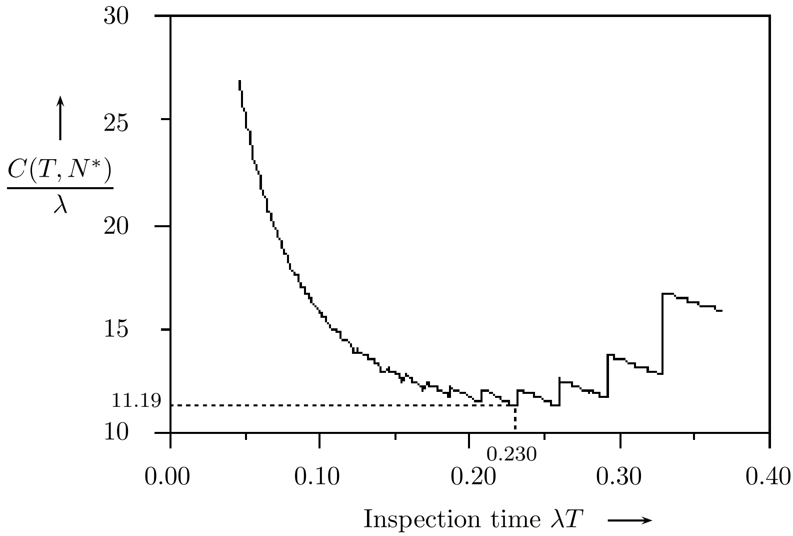

Fig. 8.3. Relation between λT and $C(T)/\lambda$ in the exponential case

Example 8.5. When the failure time of unit i has Weibull distribution $\{1 - \exp[-(\lambda_i t)^{1.5}]\}$ and $c_1 = 1$, $c_2 = 10$, $a = 0.1$, and $q = 0.8$, Figure 8.3 shows the relationship between λT and $C(T, N^*)/\lambda$, and that the optimum time is $\lambda T^* = 0.230$ and the resulting cost rate is $C(T^*, N^*)/\lambda = 11.19$. In this case, the optimum number is $N^* = 5$ and the total time is $\lambda(N^* T^* + t_0) = 1.34$. ∎

8.5 Intermittent Faults

Digital systems have two types of faults from the viewpoint of operational failures: permanent faults due to hardware failures or software errors, and intermittent faults due to transient failures [98, 99]. Intermittent faults are automatically detected by the error-correcting code and corrected by the error control [100, 101] or the restart [102, 103]. However, some faults occur repeatedly, and consequently, will be permanent faults. Some tests are applied to detect and isolate faults, but it would waste time and money to do more frequent tests.

Continuous and repetitive tests for a continuous Markov model with intermittent faults were considered in [48]. Redundant systems with independent modules were treated in [46]. Furthermore, they were extended for non-Markov models [98] and redundant systems with dependent modules [104].

This section applies the inspection policy to intermittent faults where the test is planned at periodic times kT $(k = 1, 2, \dots)$ to detect these faults (see Figure 8.4). We obtain the mean time to detect a fault and the expected

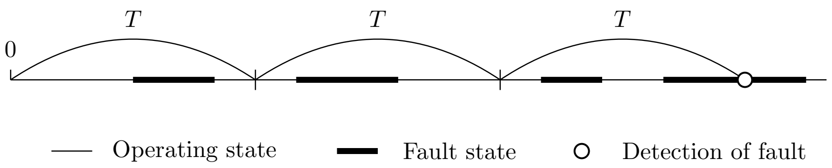

Fig. 8.4. Process of periodic inspection for intermittent faults

number of tests. In addition, we discuss optimum times T^* that minimize the expected cost until fault detection, and maximize the probability of detecting the first fault. An imperfect test model where faults are detected with probability p was treated in [50].

Suppose that faults occur intermittently; *i.e.,* a unit repeats the operating state (State 0) and fault state (State 1) alternately. The times of respective operating and fault states are independent and have identical exponential distributions $(1-\mathrm{e}^{-\lambda t})$ and $(1-\mathrm{e}^{-\theta t})$ with $\theta > \lambda$. The periodic test to detect faults is planned at times kT $(k = 1, 2, \dots)$. It is assumed that the faults of a unit are investigated only through test which is perfect; *i.e.,* faults are always detected by test when they occur and are isolated. The time required for test is negligible.

The transition probabilities $P_{0j}(t)$ from state 0 to state j $(j = 0, 1)$ are, from Section 2.1,

$$P_{00}(t) = \frac{\theta}{\lambda+\theta} + \frac{\lambda}{\lambda+\theta}\mathrm{e}^{-(\lambda+\theta)t}, \qquad P_{01}(t) = \frac{\lambda}{\lambda+\theta}(1-\mathrm{e}^{-(\lambda+\theta)t}).$$

Using the above equations, we have the following reliability quantities. The expected number $M(T)$ of tests to detect a fault is

$$M(T) = \sum_{j=0}^{\infty}(j+1)[P_{00}(T)]^j P_{01}(T) = \frac{1}{P_{01}(T)}, \tag{8.51}$$

the mean time $l(T)$ to detect a fault is

$$l(T) = \sum_{j=0}^{\infty}(j+1)T[P_{00}(T)]^j P_{01}(T) = \frac{T}{P_{01}(T)}, \tag{8.52}$$

the probability $P_0(T)$ that the first occurrence of faults is detected at the first test is

$$P_0(T) = \int_0^T \mathrm{e}^{-\theta(T-t)}\lambda\mathrm{e}^{-\lambda t}\,\mathrm{d}t = \frac{\lambda}{\theta-\lambda}(\mathrm{e}^{-\lambda T} - \mathrm{e}^{-\theta T}), \tag{8.53}$$

the probability $P_1(T)$ that the first occurrence of faults is detected at some test is

$$P_1(T) = P_0(T) + \mathrm{e}^{-\lambda T}P_1(T),$$

i.e.,

$$P_1(T) = \frac{\lambda}{\theta - \lambda} \frac{\mathrm{e}^{-\lambda T} - \mathrm{e}^{-\theta T}}{1 - \mathrm{e}^{-\lambda T}}, \tag{8.54}$$

and the probability $Q_N(T)$ that some fault is detected until the Nth test is

$$Q_N(T) = 1 - [P_{00}(T)]^N. \tag{8.55}$$

Using the above quantities, we consider the following four optimum policies. The expected cost until fault detection is, from (8.51) and (8.52),

$$C(T) \equiv c_1 M(T) + c_2 l(T) = \frac{c_1 + c_2 T}{P_{01}(T)}, \tag{8.56}$$

where c_1 = cost of one test and c_2 = operational cost rate of a unit. We seek an optimum time T_1^* that minimizes $C(T)$. Differentiating $C(T)$ with respect to T and setting it equal to zero imply

$$\frac{1}{\lambda + \theta}(\mathrm{e}^{(\lambda+\theta)T} - 1) - T = \frac{c_1}{c_2}. \tag{8.57}$$

The left-hand side of (8.57) is strictly increasing from 0 to infinity. Thus, there exists a finite and unique T_1^* that satisfies (8.57).

We derive an optimum time T_2^* that maximizes the probability $P_0(T)$. From (8.53), it is evident that $\lim_{T\to 0} P_0(T) = 0$, and

$$\frac{\mathrm{d}P_0(T)}{\mathrm{d}T} = \frac{\lambda}{\theta - \lambda}(\theta \mathrm{e}^{-\theta T} - \lambda \mathrm{e}^{-\lambda T}).$$

Thus, by putting $\mathrm{d}P_0(T)/\mathrm{d}T = 0$ because $\theta > \lambda$, an optimum T_2^* is

$$T_2^* = \frac{\log\theta - \log\lambda}{\theta - \lambda}. \tag{8.58}$$

Furthermore, we derive a maximum time T_3^* that satisfies $P_1(T) \geq q_1$; *i.e.*, the probability that the first occurrence of faults is detected at some test is greater than a specified q_1 $(0 < q_1 < 1)$. It is evident that $\lim_{T\to 0} P_1(T) = 1$, $\lim_{T\to\infty} P_1(T) = 0$, and

$$\frac{\mathrm{d}P_1(T)}{\mathrm{d}T} = \frac{\lambda}{\theta - \lambda} \frac{\mathrm{e}^{-(\lambda+\theta)T}}{(1 - \mathrm{e}^{-\lambda T})^2}[\theta(\mathrm{e}^{\lambda T} - 1) - \lambda(\mathrm{e}^{\theta T} - 1)] < 0.$$

Thus, $P_1(T)$ is strictly decreasing from 1 to 0, and hence, there exists a finite and unique T_3^* that satisfies $P_1(T) = q_1$.

Next, suppose that the testing times T_i $(i = 1, 2, 3)$ are determined from the above results. The probability that a fault is detected until the Nth test is greater than q_2 $(0 < q_2 < 1)$ is $Q_N(T) \geq q_2$. Thus, a minimum number N^* that satisfies $[P_{00}(T_i^*)]^N \leq 1 - q_2$ is

Table 8.6. Optimum time T_1^* to minimize $C(T)$ and maximum time T_3^* to satisfy $P_1(T) \geq q_1$

θ/λ	T_1^*					T_3^*				
	c_1/c_2					q_1 (%)				
	1	5	10	50	100	50	60	70	80	90
1.2	0.80	1.39	1.70	2.50	2.87	1.29	0.96	0.68	0.43	0.20
1.5	0.85	1.49	1.82	2.70	3.10	1.33	0.99	0.69	0.44	0.20
2.0	0.90	1.60	1.97	2.93	3.37	1.38	1.02	0.71	0.44	0.21
5.0	1.03	1.86	2.30	3.49	4.03	1.49	1.07	0.74	0.45	0.21
10.0	1.09	1.97	2.45	3.73	4.32	1.54	1.10	0.75	0.46	0.21
50.0	1.14	2.07	2.59	3.95	4.59	1.58	1.12	0.76	0.46	0.21

Table 8.7. Optimum time T_2^* to maximize $P_0(T)$ and minimum number N^* such that $Q_N(T_2^*) \geq q_2$

θ/λ	T_2^*	N^*				
		q_2 (%)				
		50	60	70	80	90
1.2	1.09	2	2	3	4	5
1.5	1.22	2	2	3	4	6
2.0	1.39	3	3	4	5	7
5.0	2.01	5	6	8	10	14
10.0	2.56	8	11	14	19	26
50.0	4.00	36	48	62	83	119

$$N^* = \left[\frac{\log(1-q_2)}{\log P_{00}(T_i^*)}\right] + 1 \tag{8.59}$$

where $[x]$ denotes the greatest integer contained in x.

Example 8.6. Suppose that $\theta/\lambda = 1.2$, 1.5, 2.0, 5.0, 10.0, 50.0; *i.e.,* all times are relative to the mean fault time $1/\theta$. Table 8.6 presents the optimum time T_1^* that minimizes the expected cost $C(T)$ in (8.56) for $c_1/c_2 = 1$, 5, 10, 50, 100, and the maximum time T_3^* that satisfies $P_1(T) \geq q_1$ for $q_1 = 50$, 60, 70, 80, 90 (%). Table 8.7 shows the optimum time T_2^* that maximizes $P_0(T)$ and minimum number N^* that satisfies $Q_N(T_2^*) \geq q_2$.

For example, when $\theta/\lambda = 10$ and $c_1/c_2 = 10$, the optimum time is $T_1^* = 2.45$. In particular, when $1/\lambda = 24$ hours and $1/\theta = 2.4$ hours, the test should be done at about every 6 ($\doteqdot 2.45 \times 2.4$) hours. To maximize the probability of detecting the first fault at the first test, $T_2^* = 2.01$ for $\theta/\lambda = 5.0$. If the same test in this case is repeated ten times, a fault is detected with more than 80% probability from Table 8.7. Furthermore, if the test is done at $T_3^* = 0.45$, the probability of detecting the first fault is more than 80% from Table 8.6.

We have adopted the testing time T_1^* in cost, and T_2^* and T_3^* in probabilities of detecting the first occurrence of faults. In particular, the result of $T_2^* =$

$(\log\theta - \log\lambda)/(\theta-\lambda)$ is quite simple. If λ and θ vary a little, we can compute T_2^* easily and should make the next test at time T_2^*. These testing strategies could be applied to real digital systems by suitable modifications. ▮

8.6 Inspection for a Finite Interval

Most units would be operating for a finite interval. Practically, the working time of units is finite in actual fields. Very few papers treated with replacements for a finite time span. The optimum sequential policy [1] and the asymptotic costs [105, 106] of age replacement for a finite interval were obtained.

This section summarizes inspection policies for an operating unit for a finite interval $(0, S]$ $(0 < S < \infty)$ in which its failure is detected only by inspection. Generally, it would be more difficult to compute optimum inspection policies in a finite case than those in an infinite one. We consider three inspection models of periodic and sequential inspections in Section 8.1, and asymptotic inspection in Section 8.2.

In periodic inspection, an interval S is divided equally into N parts and a unit is checked at periodic times kT $(k = 1, 2, \dots, N)$ where $NT \equiv S$. When the failure time is exponential, we first compute a checking time in an infinite case, and using the partition method, we derive an optimum policy that shows how to compute an optimum number N^* of checks in a finite case.

In sequential inspection, we show how to compute optimum checking times. Such computations might be troublesome, because we have to solve some simultaneous equations, however, they would be easier than those of Algorithm 1 in Section 8.1 as recent personal computers have developed greatly.

In asymptotic inspection, we introduce an inspection intensity and show how to compute approximate checking times by a simpler method than that of the sequential one. Finally, we give numerical examples and show that the asymptotic inspection has a good approximation to the sequential one.

(1) Periodic Inspection

Suppose that a unit has to be operating for a finite interval $(0, S]$ and fails according to a general distribution $F(t)$ with a density function $f(t)$. To detect failures, a unit is checked at periodic times kT $(k = 1, 2, \dots, N)$. Then, from (8.1), the total expected cost until failure detection or time S is

$$C(N) = \sum_{k=0}^{N-1} \int_{kT}^{(k+1)T} \{c_1(k+1) + c_2[(k+1)T - t]\}\,\mathrm{d}F(t) + c_1 N\overline{F}(NT) + c_3$$
$$= \left(c_1 + \frac{c_2 S}{N}\right)\sum_{k=0}^{N-1}\overline{F}\left(\frac{kS}{N}\right) - c_2\int_0^S \overline{F}(t)\,\mathrm{d}t + c_3 \quad (N = 1, 2, \dots). \quad (8.60)$$

Table 8.8. Approximate time $\widetilde{T}$, optimum number N^*, and time $T^* = S/N^*$, and expected cost $\widetilde{C}(N^*)$ for $S = 100$, 50 and $c_1/c_2 = 2$, 5, 10 when $\lambda = 0.01$

S	c_1/c_2	$\widetilde{T}$	N^*	T^*	$\widetilde{C}(N^*)/c_2$
	2	19.355	5	20.0	76.72
100	5	30.040	3	33.3	85.48
	10	41.622	2	50.0	96.39
	2	19.355	3	16.7	47.85
50	5	30.040	2	25.0	53.36
	10	41.622	1	50.0	60.00

It is evident that $\lim_{N\to\infty} C(N) = \infty$ and

$$C(1) = c_1 + c_2 \int_0^S F(t)\,\mathrm{d}t + c_3.$$

Thus, there exists a finite number N^* $(1 \le N^* < \infty)$ that minimizes $C(N)$.

In particular, assume that the failure time is exponential; *i.e.,* $F(t) = 1 - \mathrm{e}^{-\lambda t}$. Then, the expected cost $C(N)$ in (8.60) can be rewritten as

$$C(N) = \left(c_1 + \frac{c_2 S}{N}\right)\frac{1-\mathrm{e}^{-\lambda S}}{1-\mathrm{e}^{-\lambda S/N}} - \frac{c_2}{\lambda}(1-\mathrm{e}^{-\lambda S}) + c_3 \quad (N = 1, 2, \dots). \tag{8.61}$$

To find an optimum number N^* that minimizes $C(N)$, we put $T = S/N$. Then, Equation (8.61) becomes

$$C(T) = (c_1 + c_2 T)\frac{1-\mathrm{e}^{-\lambda S}}{1-\mathrm{e}^{-\lambda T}} - \frac{c_2}{\lambda}(1-\mathrm{e}^{-\lambda S}) + c_3. \tag{8.62}$$

Differentiating $C(T)$ with respect to T and setting it equal to zero, we have

$$\mathrm{e}^{\lambda T} - (1 + \lambda T) = \frac{\lambda c_1}{c_2} \tag{8.63}$$

which agrees with (8.5). Thus, there exists a finite and unique $\widetilde{T}$ $(0 < \widetilde{T} < \infty)$ that satisfies (8.63).

Therefore, we show the following partition method.

(i) If $\widetilde{T} < S$ then we put $[S/\widetilde{T}] \equiv N$ and calculate $C(N)$ and $C(N+1)$ from (8.61), where $[x]$ denotes the greatest integer contained in x. If $C(N) \le C(N+1)$ then $N^* = N$, and conversely, if $C(N) > C(N+1)$ then $N^* = N + 1$.
(ii) If $\widetilde{T} \ge S$ then $N^* = 1$.

Note that $\widetilde{T}$ gives the optimum checking time for an infinite time span in an exponential case.

Example 8.7. Table 8.8 presents the approximate checking time $\widetilde{T}$, the optimum checking number N^*, and time $T^* = S/N^*$, and the expected cost

$\widetilde{C}(N^*) \equiv C(N^*) + (c_2/\lambda)(1 - \mathrm{e}^{-\lambda S}) - c_3$ for $S = 100$, 50 and $c_1/c_2 = 2$, 5, 10 when $\lambda = 0.01$. If S is large then it would be sufficient to compute approximate checking times $\widetilde{T}$. ∎

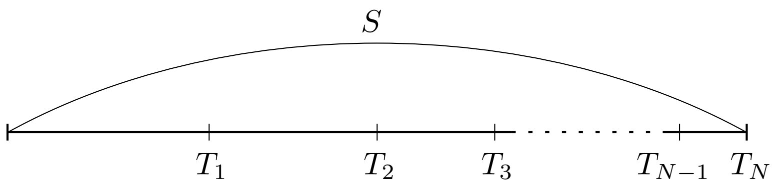

Fig. 8.5. Process of sequential inspection in a finite interval

(2) Sequential Inspection

An operating unit is checked at successive times $0 < T_1 < T_2 < \cdots < T_N$, where $T_0 \equiv 0$ and $T_N \equiv S$ (see Figure 8.5). In a similar way to that of obtaining (8.60), the total expected cost until failure detection or time S is

$$\mathbf{C}(N) = \sum_{k=0}^{N-1} \int_{T_k}^{T_{k+1}} [c_1(k+1) + c_2(T_{k+1} - t)]\,\mathrm{d}F(t) + c_1 N\overline{F}(T_N) + c_3 \qquad (N = 1, 2, \dots). \tag{8.64}$$

Putting that $\partial \mathbf{C}(N)/\partial T_k = 0$, which is a necessary condition for minimizing $\mathbf{C}(N)$, we have

$$T_{k+1} - T_k = \frac{F(T_k) - F(T_{k-1})}{f(T_k)} - \frac{c_1}{c_2} \qquad (k = 1, 2, \dots, N-1) \tag{8.65}$$

and the resulting minimum expected cost is

$$\widetilde{\mathbf{C}}(N) \equiv \mathbf{C}(N) + c_2 \int_0^S \overline{F}(t)\,\mathrm{d}t - c_3 = \sum_{k=0}^{N-1} [c_1 + c_2(T_{k+1} - T_k)]\overline{F}(T_k) \qquad (N = 1, 2, \dots). \tag{8.66}$$

For example, when $N = 3$, the checking times T_1 and T_2 are given by the solutions of equations

$$S - T_2 = \frac{F(T_2) - F(T_1)}{f(T_2)} - \frac{c_1}{c_2}$$

$$T_2 - T_1 = \frac{F(T_1)}{f(T_1)} - \frac{c_1}{c_2}$$

Table 8.9. Checking time T_k and expected cost $\widetilde{\mathbf{C}}(N)$ for $N = 1, 2, \ldots, 8$ when $S = 100$, $c_1/c_2 = 2$, and $F(t) = 1 - \mathrm{e}^{-\lambda t^2}$

N	1	2	3	4	5	6	7	8
T_1	100	64.14	50.9	44.1	40.3	38.1	36.8	36.3
T_2		100	77.1	66.0	60.0	56.2	54.3	53.3
T_3			100	84.0	75.4	70.5	67.8	66.6
T_4				100	88.6	82.3	78.9	77.3
T_5					100	91.1	87.9	85.9
T_6						100	94.9	92.5
T_7							100	97.2
T_8								100
$\widetilde{\mathbf{C}}(N)/c_2$	102.00	93.55	91.52	91.16	91.47	92.11	92.91	93.79

and the expected cost is

$$\widetilde{\mathbf{C}}(3) = c_1 + c_2 T_1 + [c_1 + c_2(T_2 - T_1)]\overline{F}(T_1) + [c_1 + c_2(S - T_2)]\overline{F}(T_2).$$

From the above discussion, we compute T_k $(k = 1, 2, \ldots, N-1)$ which satisfies (8.65), and substituting them into (8.66), we obtain the expected cost $\mathbf{C}(N)$. Next, comparing $\mathbf{C}(N)$ for all $N \geq 1$, we can get the optimum checking number N^* and times T_k^* $(k = 1, 2, \ldots, N^*)$.

Example 8.8. Table 8.9 gives the checking time T_k $(k = 1, 2, \ldots, N)$ and the expected cost $\widetilde{\mathbf{C}}(N)$ for $S = 100$ and $c_1/c_2 = 2$ when $F(t) = 1 - \exp(-\lambda t^2)$. In this case, we set that the mean failure time is equal to S; *i.e.*,

$$\int_0^\infty \mathrm{e}^{-\lambda t^2}\,\mathrm{d}t = \frac{1}{2}\sqrt{\frac{\pi}{\lambda}} = S.$$

Comparing $\widetilde{\mathbf{C}}(N)$ for $N = 1, 2, \ldots, 8$, the expected cost is minimum at $N = 4$. That is, the optimum checking number is $N^* = 4$ and optimum checking times are 44.1, 66.0, 84.0, 100. ▮

(3) Asymptotic Inspection

Suppose that $n(t)$ is an inspection intensity defined in **(3)** of Section 8.2. Then, from (8.21) and (8.64), the approximate total expected cost is

$$\mathbf{C}(n(t)) = \int_0^S \left[c_1 \int_0^t n(u)\,\mathrm{d}u + \frac{c_2}{2n(t)}\right] \mathrm{d}F(t) + c_1\overline{F}(S)\int_0^S n(t)\,\mathrm{d}t + c_3. \tag{8.67}$$

Differentiating $\mathbf{C}(n(t))$ with $n(t)$ and setting it equal to zero, we have (8.22).

We compute approximate checking times $\widetilde{T}_k$ $(k = 1, 2, \ldots, N-1)$ and checking number $\widetilde{N}$, using (8.22). First, we put that

$$\int_0^S \sqrt{\frac{c_2 h(t)}{2c_1}}\, \mathrm{d}t \equiv X$$

and $[X] \equiv N$, where $[x]$ is defined in policy (i) in **(1)**. Then, we obtain A_N $(0 < A_N \le 1)$ such that

$$A_N \int_0^S \sqrt{\frac{c_2 h(t)}{2c_1}}\, \mathrm{d}t = N$$

and define an inspection intensity as

$$\widetilde{n}(t) = A_N \sqrt{\frac{c_2 h(t)}{2c_1}}. \tag{8.68}$$

Using (8.68), we compute checking times T_k that satisfy

$$\int_0^{T_k} \widetilde{n}(t)\, \mathrm{d}t = k \qquad (k = 1, 2, \ldots, N), \tag{8.69}$$

where $T_0 = 0$ and $T_N = S$. Then, the total expected cost is given in (8.66).

Next, we put N by $N+1$ and do a similar computation. At last, we compare $\mathbf{C}(N)$ and $\mathbf{C}(N+1)$, and choose the small one as the total expected cost $\mathbf{C}(\widetilde{N})$ and the corresponding checking times $\widetilde{T}_k$ $(k = 1, 2, \ldots, \widetilde{N})$ as an asymptotic inspection policy.

Example 8.9. Consider a numerical example when the parameters are the same as those of Example 8.8. Then, because $\lambda = \pi/4 \times 10^4$, $n(t) = \sqrt{\lambda t/2}$, $[X] = N = 4$, and $A_N = (12/100)/\sqrt{\pi/200}$, we have that $\widetilde{n}(t) = 6\sqrt{t}/10^3$. Thus, from (8.69), checking times are

$$\int_0^{T_k} \frac{6}{1000}\sqrt{t}\, \mathrm{d}t = \frac{1}{250} T_k^{3/2} = k \qquad (k = 1, 2, 3).$$

Also, when $N = 5$, $A_N = (15/100)/\sqrt{\pi/200}$, and $\widetilde{n}(t) = 3\sqrt{t}/4 \times 10^2$. In this case, checking times are

$$\int_0^{T_k} \frac{3}{400}\sqrt{t}\, \mathrm{d}t = \frac{1}{200} T_k^{3/2} = k \qquad (k = 1, 2, 3, 4).$$

Table 8.10 shows the checking times and the resulting costs for $N = 4$ and 5. Because $\widetilde{\mathbf{C}}(4) < \widetilde{\mathbf{C}}(5)$, the approximate checking number is $\widetilde{N} = 4$ and its checking times $\widetilde{T}_k$ are 39.7, 63.0, 82.5, 100. These checking times are a little smaller than those in Table 8.9, however, they are closely approximate to the optimum ones. ∎

Table 8.10. Checking time $\widetilde{T}_k$ and expected cost $\widetilde{C}(N)$ for $N = 4, 5$ when $S = 100$, $c_1/c_2 = 2$, and $F(t) = 1 - e^{-\lambda t^2}$

N	4	5
1	39.7	34.2
2	63.0	54.3
3	82.5	71.1
4	100.0	86.2
5		100.0
$\widetilde{\mathbf{C}}(N)/c_2$	91.22	91.58

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