

## Numerical Integration of the Nonlinear Dynamics of Elastoplastic Solids

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### ABSTRACT

Classical numerical schemes for the numerical integration of the equations of solid and structural dynamics in time show serious limitations when applied to the finite deformation range. These include the classical Newmark and HHT type schemes. The unconditional stability property of some of these schemes is lost in the nonlinear range, with the simulations showing an unbounded growth of energy. Furthermore, the law of conservation of angular momentum is, in general, not inherited by the schemes in this nonlinear range either. To handle these difficulties, the formulation of the so-called energy-momentum schemes has received a great deal of attention lately. As illustrated in [1], these difficulties are also present in the physically dissipative problem of finite strain plasticity. The schemes do not lead to dissipative discrete dynamical systems.

We present in this contribution a new class of integration schemes for multiplicative finite strain plasticity that exhibit the non-negative energy dissipation characteristic of these physical systems and that preserve the conservation laws of linear and angular momenta in time. The exact physical dissipation is obtained, hence recovering previously developed energy-momentum schemes with the exact energy conservation for elastic problems in elastic steps of the elastoplastic simulations. Furthermore, the schemes allow the consideration of extensions showing a controllable high-frequency (numerical) energy dissipation to handle the numerical schemes of typical problems of interest. The new schemes rely on an alternative integration of the plastic evolution equations, or return mapping algorithm, that define the discrete in time evolution of the plastic internal variables and the exact enforcement of the yield constraint on the stresses driving the motion. We built on the developments presented in [1] and incorporate an additional set of new considerations for the volumetric contributions, thus arriving to algorithms that also preserve exactly the isochoric character of the plastic flow in some common models of finite strain plasticity (e.g.  $J_2$ -flow theory based on von Mises yield criteria). Moreover, we discuss the implementation of the new time-stepping schemes in the context of new formulations of assumed strain and mixed finite elements for the handling of the well-known volumetric locking in these cases, while obtaining the aforementioned conservation and dissipation properties in time. Several numerical simulations are presented illustrating the performance of the new schemes.

### References

- [1] F. Armero, Energy-Dissipative Momentum-Conserving Time-Stepping Algorithms for Finite Strain Multiplicative Plasticity, *Computer Methods in Applied Mechanics and Engineering*, in press.