# A finite element formulation based on the theory of a Cosserat point – Extension to Ogden material

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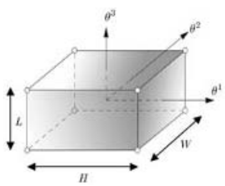
**ABSTRACT**

The theory of Cosserat points is the basis of a finite element formulation for a solid three-dimensional continuum, which was presented by [1]. Previous investigations [2] have revealed, that this formulation is free of showing undesired locking or hourglassing-phenomena. It additionally shows excellent behaviour for large deformations in any type of incompressible material and for sensitive structures such as plates or shells. Within the theory of Cosserat points, the position vectors $\mathbf{X}$ and $\mathbf{x}$ of an 8-node-brick element are described through director vectors $\mathbf{D}_i$ and $\mathbf{d}_i$.




$$\mathbf{X} = \sum_{i=0}^{7} N^i(\theta^1,\theta^2,\theta^3)\mathbf{D}_i \quad , \quad \mathbf{x} = \sum_{i=0}^{7} N^i(\theta^1,\theta^2,\theta^3)\mathbf{d}_i$$
$$N^0 = 1, N^1 = \theta^1, N^2 = \theta^2, N^3 = \theta^3, N^4 = \theta^1\theta^2,$$
$$N^5 = \theta^1\theta^3, N^6 = \theta^2\theta^3, N^7 = \theta^1\theta^2\theta^3$$

The special choice of shape functions $N^i$ allows to split the deformation as well as resulting stresses into homogeneous and inhomogeneous parts respectively. The stresses due to the inhomogeneous part of the deformation are obtained by incorporating analytical solutions to the deformation modes bending, torsion and higher-order hourglassing for a rectangular parallelepiped shaped reference element, see [1] and [2].

This work shows approaches on how to overcome the difficulty of initially distorted element geometries that differ strongly from the shape of a rectangular parallelepiped. The formulation initially was restricted to a Neo-Hookean material. This work will present the extension to a general elastic Ogden material as well as to metal plasticity for large deformations with isotropic hardening. It will also give insight to the properties of the Cosserat point element and its behaviour for rubber-like materials.

## References

[1] Nadler, B.; Rubin, M.B. (2003), A new 3-D finite element for nonlinear elasticity using the theory of a Cosserat point, *Solids & Structures*, 40, 4585-4614.

[2] Loehnert, S.; Boerner, E.F.I.; Rubin, M.B.; Wriggers, P. (2005), Response of a nonlinear elastic general Cosserat brick element in simulations typically exhibiting locking and hourglassing, *Computational Mechanics*, Vol. 36:266-288.