RELIABILITY-BASED TOPOLOGY OPTIMIZATION (RBTO)

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Abstract: Recently, topology optimization has been the subject of a significant amount of academic and industrial research due to its simplicity in implementation and impact on product design. In this paper, reliability-based topology optimization (RBTO) is introduced for structures, electromagnetics, heat transfer, and coupled systems with consideration of uncertainties in topology optimization. Continuum design sensitivity using the adjoint variable method, the density method, and optimization algorithm (SLP and MMA) are used for topology optimization. PMA is mainly used for reliability computations. Examples are given to validate the proposed method.

Keywords: Reliability, topology optimization, uncertainty, multi-physics.

1. INTRODUCTION

Conventional structural optimization techniques such as sizing or shape/configuration optimizations are aimed at the improvement of current designs. Conversely, topology optimization focuses on obtaining an initial conceptual design and does not require a sophisticated initial design. As such, any geometry within the boundaries for conditions and loads is sufficient for commencing an initial topology optimization. Starting from basic structural systems, topology optimization is now of an age where it extends throughout various physical systems including electromagnetic, acoustic, thermal, and even to coupled-physics systems.

Reliability-based design optimization (RBDO) together with topology optimization has received high attention from optimization societies [1]. The primary goal of probabilistic optimization is to consider the variations of per-

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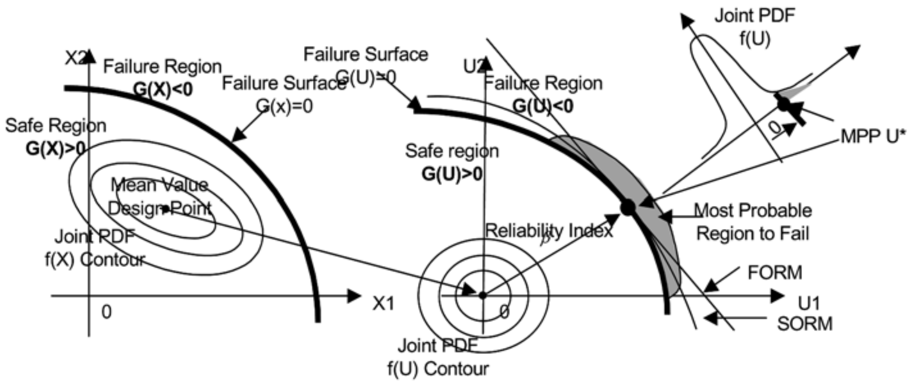


Figure 1. Concept of reliability-based design optimization.

formances caused by uncertainties. In deterministic optimization, these uncertainties are not considered and therefore the optimum design can be unreliable with respect to failures. On the other hand, in probabilistic optimization, minimizing the cost and bringing the probabilistic constraints on target should be done simultaneously.

In this research, the RBDO concept is applied to Topology Optimization and Reliability-Based Topology Optimization (RBTO) [2] is introduced. RBTO determines the optimal topology that satisfies the given probability in consideration of the variances of the uncertainties. RBTO is then applied to various multi-physics systems and numerical examples are presented.

2. RELIABILITY-BASED TOPOLOGY OPTIMIZATION

2.1 Concept of Reliability-Based Design Optimization

In general, engineering design problems are based on the control of the elements of a system such that they satisfy various criteria for performance, safety, serviceability, and durability under various demands. For example, a structure should be designed so that its strength or resistance is greater than the effects of the applied loads. However, there are numerous sources of uncertainty in system parameters. The goal of RBDO is to incorporate this uncertainty information into the actual design problems. The main idea for RBDO and its corresponding evaluations are shown in Figure 1.

RBDO has the same cost function as deterministic optimization, but it has probabilistic constraints used in the consideration of the probability of the satisfaction/failure potential of the constraints.

The general form of RBDO is described as follows:

$$\begin{aligned} &\text{Minimize} && f(X) \\ &\text{subject to} && P_f(X) = P[G(x) < 0] \leq P_{ft} \\ &&& x_i^L \leq x_i \leq x_i^U, \quad i = 1, \ldots, n, \end{aligned} \tag{1}$$

where P_{ft} is the target probability of failure. In RBDO, a limit state function, G, is formulated, and the system fails when $G < 0$.

2.2 General Formulation of Reliability-Based Topology Optimization (RBTO)

Conventional topology optimization is a deterministic method, so the RBDO concept was applied to topology optimization, resulting in the development of Reliability-Based Topology Optimization (RBTO) [2]. RBTO can be interpreted as a problem which finds an optimum topology under probabilistic constraints such that it becomes reliable for these uncertainties.

The general form of an RBTO problem is described as the following:

Find the design variable vector such that:

$$\begin{aligned} &\text{Min/Max} && f(\eta_i) \\ &\text{Subject to} && P_s(X) = P[G(\eta_i, X_j) \geq 0] \geq P_t \\ &&& 0 \leq \eta_i \leq 1 \\ &&& i = 1, \ldots, ndv \ \text{ and } \ j = 1, 2, \ldots, \text{no. of uncertain variables,} \end{aligned} \tag{2}$$

where X_j is the jth uncertain variable, P_s is the system probability of success, P_t is the target probability of success, and G is the limit state function (performance function). Design variables are density functions, η_i, in each finite element.

Applying the Performance Measure Approach (PMA) to Equation (2) yields:

$$\begin{aligned} &\text{Min/Max} && f(\eta_i) \\ &\text{Subject to} && G^*(\eta_i, X_j) \geq 0 \\ &&& \text{when } \beta_s = \beta_t \text{ for each evaluation} \\ &&& 0 \leq \eta_i \leq 1 \\ &&& i = 1, \ldots, ndv \ \text{ and } \ j = 1, 2, \ldots, \text{no. of uncertain variables,} \end{aligned} \tag{3}$$

where β_s is the system reliability index for success, and β_t is the target reliability index for success.

In an RBTO problem, the performance (or limit-state) function should be defined, and each sensitivity analysis should be performed with respect to each uncertain variable.

2.3 RBTO for Structural Systems

Displacement is considered the limit-state function for a static problem. Young's modulus, thickness, and loading are considered uncertain variables.

The limit state function, G, is defined by:

$$\begin{aligned} G &= -\psi + \psi_{\max} \geq 0, \\ \psi &= z(\hat{x}) = \int_{\Omega} \hat{\delta}(x-\hat{x}) z(x) d\Omega, \\ X &= [X_1, X_2, X_3]^T = [E, t, F]^T, \end{aligned} \tag{4}$$

where ψ is the displacement at an isolated point $\hat{x}$ and $\hat{\delta}$ is the Dirac-Delta function, and X_j is the jth uncertain variable. The limit-state, Equation (4), implies that if the displacement, ψ, is larger than the limit value, $\psi_{\max}$, the system fails.

2.4 RBTO for Electromagnetic System

In RBTO for electromagnetic systems, in order to estimate the failure probability, the magnetic energy is considered the limit-state function for static problems. Design variables are density functions, η_i, in each finite element and the permeability, applied current density, and coercive force are uncertain variables [4–7].

The limit-state function G is defined by:

$$\begin{aligned} G &= \psi - \psi_{\min} \geq 0, \\ \psi &= \int_{\Omega} g(A, u) d\Omega = \frac{1}{2} \int_{\Omega} B \cdot H d\Omega, \\ X &= [X_1, X_2, X_3]^T = [\mu, J_s, H_c]^T, \end{aligned} \tag{5}$$

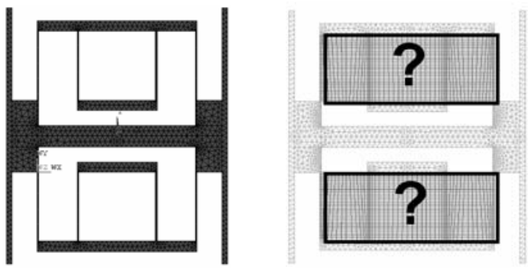
where ψ is the magnetic energy. The limit-state implies that if the magnetic energy, ψ, is smaller than the limit value, $\psi_{\min}$, the system fails.

2.5 RBTO for Thermal System

For thermal systems, the temperature of the kth nodal point is chosen as the performance response, and the convection coefficient, h, is assumed to be an uncertain variable.

The limit-state function becomes:

$$G = \psi - \psi_{\min} \geq 0,$$



(a) Conventional Type (b) Design Domain

Figure 2. Basis model for topology optimization.

$$\begin{aligned} \psi &= \mathbf{u}^T\mathbf{T}, \\ X &= [X_1]^T = [h]^T, \end{aligned} \tag{6}$$

where **u** is the vector which has 1 in the kth position and 0 elsewhere.

2.6 RBTO for Electric-Thermal-Structural Coupled System

For coupled systems, the objective function is to maximize the displacement of a target node. Since the example system is an actuator, the maximum displacement leads to a well-working device. The performance function is the total input current, I_m, and it should be less than a limit value. The electric conductivity, σ, is assumed to be an uncertain variable.

Then, the limit state function becomes:

$$\begin{aligned} \psi &= I_m, \\ G &= -\psi + \psi_{\max} \geq 0, \\ X &= [X_1]^T = [\sigma]^T. \end{aligned} \tag{7}$$

3. NUMERICAL EXAMPLES

3.1 Structural System: Double-Folded-Spring for MEMS

In this system [8], the spring is the most important part for the overall system performance (k_θ and k_x). Therefore, the spring is selected as the design domain for topology optimization. The conventional model is given in Figure 2(a) and the design domain from the conventional model is shown in Figure 2(b).

Three topology optimization problems are performed: (1) Deterministic Topology Optimization (DTO); (2) RBTO with $\beta = 1.0$; and (3) RBTO with $\beta = 1.5$. The number of design variables is 3400. The system should be

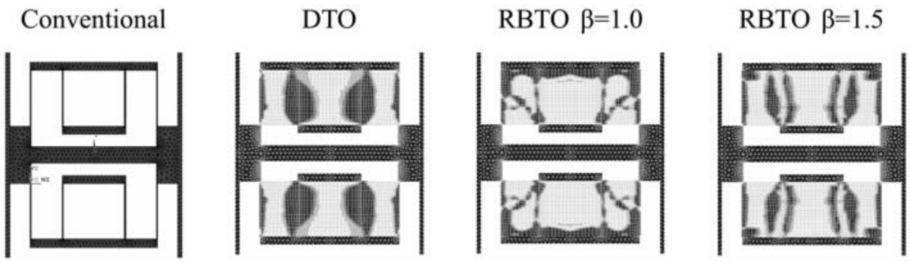


Figure 3. Summary of topology designs.

Table 1. Comparison of topology designs.

	Conventional	DTO	RBTO $\beta = 1.0$	RBTO $\beta = 1.5$
UX	0.02209	0.1473	0.03601	0.02270
UY	8.428E-06	7.974E-06	7.323E-06	6.951E-06
k_x	226.3	33.95	138.85	220.3
k_θ	4.878E+09	5.157E+09	5.615E+09	5.910E+09

symmetric, and a symmetric condition is internally imposed. The uncertain variables are Young's modulus, thickness, and loading. The uncertain variables have a 5% standard deviation of the mean values and they have normal distributions.

In the following equations, UX is an x-directional displacement of the target node and is inversely proportional to k_x. This property is associated with the actuation of the device. Additionally, UY is a y-directional displacement of the target node and is inversely proportional to k_θ. This property is associated with the stability of the device. The target value of the constraint is selected from the analysis result of the conventional model (Figure 2(a)).

RBTO using PMA

$$
\begin{aligned}
\text{Max} \quad & UX, && \text{(Under Force Set 1: Translation)} \\
\text{s.t.} \quad & UY < 8.0E-6, && \text{(Under Force Set 2: Torsion)} \\
& \text{when } \beta = \beta_t(1.0, 1.5) \\
& X = [E, t, F], \sigma_i = 0.05 \times \mu_i, \quad i = 1, 2, 3.
\end{aligned} \tag{8}
$$

Figure 3 and Table 1 summarize the obtained results.

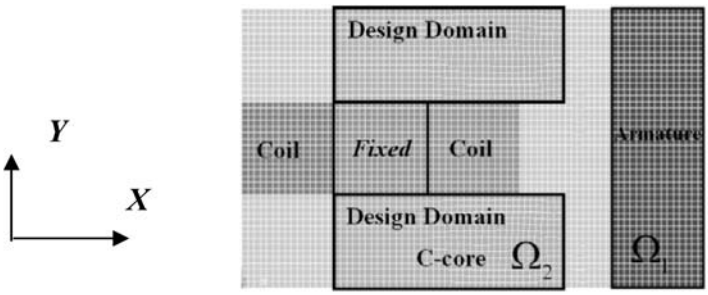


Figure 4. C-core actuator.

3.2 Electromagnetic System: C-Core Model

A numerical example [5] is a C-core model, as shown in Figure 4. The C-core actuator has three parts; an armature, a core, and a coil. The width of both the armature and the core is 20 mm. The length of the core and the blade are 60 mm and 50 mm, respectively. The relative permeability is 1000 at both the core and armature. The current density of the coil is 2.0 [A/mm^2].

The limit-state function is the magnetic energy and should be larger than 140 [J/m], which is the target magnetic energy (ψ_t) in this example. The permeability and current density are the uncertain variables. The permeability and current density have 10% and 5% variance of the initial values, respectively, and they are assumed to be normal random variables.

If the PMA is used for RBTO, the RBTO problem is written as:

$$\begin{aligned} &\text{Minimize} && \text{Total Volume} \\ &\text{Subject to} && G = \psi - \psi_t \geq 0 \text{ when } \beta_s = 3, \quad \psi_t = 140, \end{aligned} \tag{9}$$

where ψ_t is the target magnetic energy.

The optimum results of several topology problems are shown in Figure 5 and Table 2. DTOSV is the Deterministic Topology Optimization with the Same Volume as the RBTO result. Since RBTO requires a reliability analysis, it needs a greater computational time than DTO. Empirically, RBTO needs about three times more computations than DTO. However, as shown in Table 2, RBTO gave a more reliable solution than DTOSV, while both methods used the same volume. Moreover, RBTO can satisfy the target reliability exactly as requested.

3.3 Thermal System: 2-D Cooling Fin

A 2-D cooling fin [9] is examined as a verification purpose. A 20 × 50 mm^2 plate (Figure 6) is considered to have heat conditions such that $k = 0.2$. $h_f = 0.005$, $T_b = 25$°C, $T^S = 300$°C. Knowing that a larger temperature difference

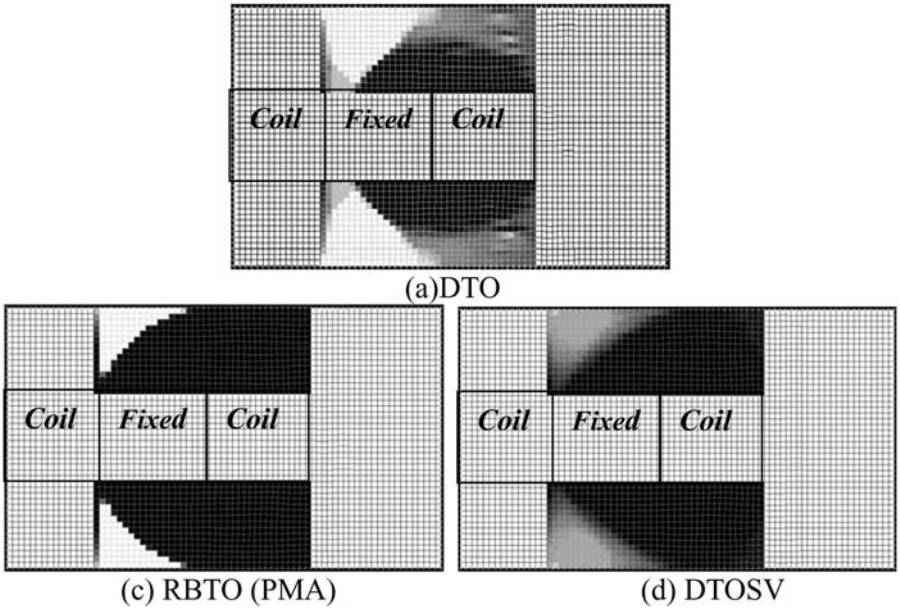


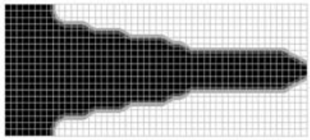
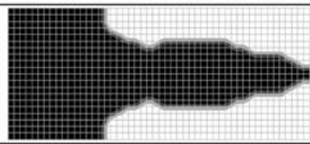
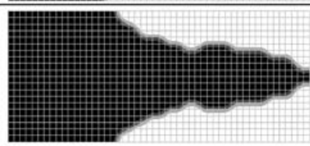
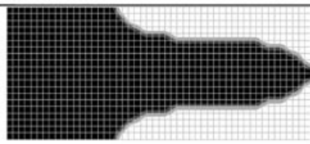
Figure 5. Topology results for electomagnetic systems.

Table 2. Comparison between DTO and RBTO.

	Objective (volume)	Energy at mean value	Force (F_x) [Nm]	Reliability
DTO	59.03	139.96	–6048.5	–0.070274
RBTO with 3 uncertainties	86.40	140.81	–6110.6	3.00241
DTOSV (with the same volume of RBTO result)	86.40	140.73	–6102.9	2.40125

Figure 6. A simple heat transfer system illustrating energy balance.

Table 3. Optimal 2-D fin designs with consideration of different reliability indices.

DTO		Vol = 52.88 (%) β = 0.0 Mean Temp = 30.64
RBTO 1		Vol = 60.77 (%) β = 1.0 Mean Temp = 33.18
RBTO 2		Vol = 66.03 (%) β = 2.0 Mean Temp = 35.80
RBTO 3		Vol = 63.38 (%) β = 3.0 Mean Temp = 38.37

ensures a higher heat transfer rate by convection, an optimization problem is suggested that has a high temperature in the middle of the right side on the end.

$$\begin{aligned} &\text{Minimize} && \text{Total Volume} \\ &\text{Subject to} && \Pr(T_p > 30.6) \le \Pr(\beta_{tgt}). \end{aligned} \tag{10}$$

The convection coefficient, h, is assumed to be an uncertain variable which has 20% standard deviation of the mean value. Three different reliability indices (1.0, 2.0, 3.0) are used to test influences on the optimal results. PMA is used for RBTO in this example. As shown in Table 3, as the reliability index gets higher, the more volume is used, and the higher the mean temperature achieved. This result coincides with the fact that the rectangular fin (full material) has the highest temperature.

3.4 Electric-Thermal-Structural Coupled System: Electro-thermal Actuator

A switch with a bi-stable actuator for RF and optical applications is a useful device. The switch is electro-thermally actuated and it exhibits a clear bi-stable performance. The micro-switch mainly consists of a bi-stable actu-

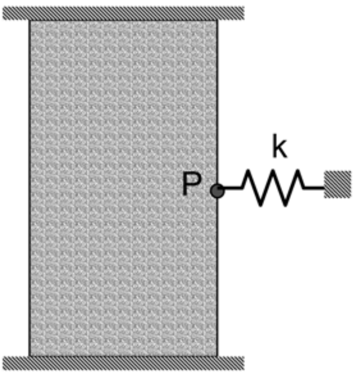


Figure 7. Initial design domain of the actuator for topology optimization.

ator enabling two separate transmission lines to connect physically, and a thermoelastic micro-actuator for making the bi-stable micro- actuator latch-up.

For this device, topology optimization is performed on the cascaded bent-beam area to obtain a more efficient structure in terms of displacement and power. The initial design domain is chosen as a simple rectangle, and then meshed into 30 $times60$ finite elements. The leaf spring is replaced with a spring element [10].

Constraint on the total input current is considered to limit the overall power usage.

For the electro-thermal actuator RBTO is applied as:

$$\begin{aligned} &\text{Maximize} && \text{Displacement at } P(\delta_p) \\ &\text{Subject to} && \Pr(I_{\text{in}} < 50mA) \leq \Pr(\beta_t), \end{aligned} \tag{11}$$

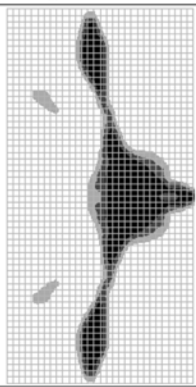
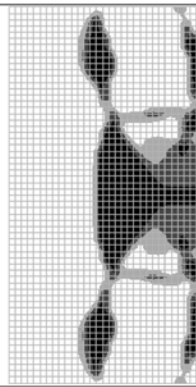
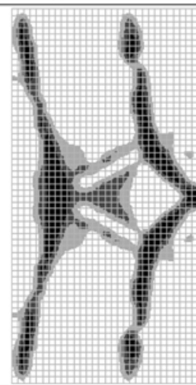
where I_{in} is the total input current.

Since an electric analysis works as a constraint, the uncertain variable is selected from one of the electric properties, i.e., the electric conductivity, σ. It is assumed that σ has a 10% standard deviation of the mean value. PMA is used for RBTO. The test proceeds with two different target reliability indices, i.e., $\beta_t = 1.0, 2.0$. As shown in Table 4, the objective function decreases when the target reliability index increases. The resulting input current decreases when the higher target reliability index is imposed, which explains that the limit of the constraint function (input current) is affected by the imposition of the reliability analysis.

4. CONCLUSION

In this research, the RBDO concept is applied to Topology Optimization and Reliability-Based Topology Optimization (RBTO) is introduced. RBTO de-

Table 4. Electro-thermal actuator design using RBTO.

DTO	RBTO1	RBTO2
-	$\beta_{tgt} = 1.0$	$\beta_{tgt} = 2.0$
$\delta_P = 72.4\,\mu m$ $I_{in} = 50.0\,mA$	$\delta_P = 69.2\,\mu m$ $I_{in} = 45.4\,mA$	$\delta_P = 63.6\,\mu m$ $I_{in} = 41.5\,mA$
		

termines an optimal topology that satisfies the given probability in consideration of the variances of the uncertainties. RBTO is then applied to multi-physics systems and each formulation is derived.

Structural displacement, magneto-static energy, thermal temperature, and electro-thermal actuator problems are solved using the RBTO methodology. RBTO is mainly compared to the deterministic Topology Optimization with the same volume of RBTO and the effectiveness of RBTO is shown.

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