Chapter 19

COOPERATIVE CODING AND ITS APPLICATION TO OFDM SYSTEMS

Jerry C. H. Lin
Polytechnic University
clin17@poly.edu

Andrej Stefanov
Polytechnic University
stefanov@poly.edu

Abstract: We study OFDM systems with cooperative coding over frequency selective Rayleigh fading channels. We derive the pairwise error probability for the block-fading OFDM channel model. We use the derived pairwise error probability to get an upper bound on the frame error probability for the coded cooperative OFDM system. This bound is then utilized in the study of the diversity and coding gains achievable through cooperative coding in OFDM systems for various inter-user channel qualities. We consider the design of cooperative convolutional codes based on the principle of overlays and provide simulation results for different cooperation scenarios. We observe significant gains over conventional non-cooperative OFDM systems.

Keywords: diversity methods, error–correction coding, fading channels.

1. Introduction

Information transfer through wireless networks involves simultaneous communication among multiple source–destination pairs. Wireless local area networks may operate in infrastructure mode or as ad-hoc networks. In the infrastructure mode the coordination of these multiple communications is done

Frank H. P. Fitzek and Marcos D. Katz (eds.), Cooperation in Wireless Networks: Principles and Applications, 593–605.

via the access point. The access point processes all the signals transmitted from the sources (uplink) and forwards them to their respective destinations (downlink). In the ad–hoc mode on the other hand there is no fixed infrastructure and the terminals utilize other terminals as relays to transfer information from the source to its destination. Motivated by the diversity effects and power efficiency of communicating via relaying, recent research efforts have focused on cooperation among the terminals in the network (user-cooperation), demonstrating the advantages of user-cooperation regardless of the mode of the network operation.

In a cooperative network, two or more terminals share their information and transmit jointly as a virtual antenna array. This enables them to obtain higher data rates and it leads to decreased sensitivity to channel variations [Sendonaris et al., 2003]. The terminals share information by tuning into each other's transmitted signals and by processing the information they overhear through the inter-user channel. The cooperation still leads to performance improvements over single user transmission, even though the inter-user channel may be faded and noisy. The fact, that in practice, the relaying terminal cannot receive and transmit at the same time was incorporated in [Laneman et al., 2004], where the authors considered different protocols to achieve diversity gains such as amplify and forward or decode and forward. From a coding perspective these protocols resemble repetition coding, and there are more effective ways of designing channel codes.

In [Stefanov and Erkip, 2004] we demonstrated that an overall block fading channel model is appropriate in the case of user-cooperation, since the cooperating terminals observe independently faded channels towards the destination. This resulted in a framework for the design of cooperative channel codes optimized for user-cooperation. As the next generation of wireless local area networks (WLAN's) and cellular systems will utilize Orthogonal Frequency Division Multiplexing (OFDM) [Nee and Prasad, 2000], it is necessary to consider the analysis and design of cooperative codes in the context of OFDM systems.

2. System Model

We consider an OFDM system with K subcarriers. Each code word spans P adjacent OFDM words, and each OFDM word consists of K symbols, transmitted simultaneously during one time slot. Each symbol is transmitted at a particular OFDM subcarrier. We assume that the fading is quasi-static during each OFDM word, but varies independently from one OFDM word to another.

At the receiver, the received signal can be expressed in the frequency domain as follows

$$y[p,k] = H[p,k]c[p,k] + z[p,k] \tag{19.1}$$

where $k = 0, \ldots, K-1, p = 1, \ldots, P$, and $H[p,k]$ is the complex channel frequency response at the kth subcarrier and at the pth time slot. $c[p,k]$ and $y[p,k]$ are the transmitted signal and the received signal, respectively, at the kth

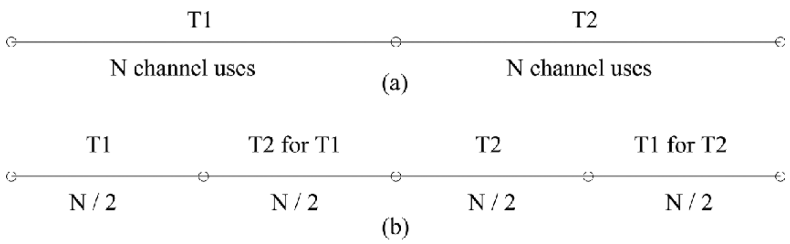

Figure 19.1. Time-division channel allocations: (a) orthogonal direct transmission and (b) orthogonal cooperative diversity transmission.

subcarrier and at the *p*th time slot. $z[p,k]$ is the circularly symmetric complex Gaussian noise with variance of $N_0/2$. The time domain channel response can be expressed as

$$h(\tau) = \sum_{l=1}^{L} \alpha(l)\delta(\tau - \frac{n_l}{K\triangle_f}) \tag{19.2}$$

where $\delta(\cdot)$ is the Dirac delta function, L denotes the number of non-zero taps and $\alpha(l)$ is the complex gain of the *l*th non-zero tap, whose delay is $\frac{n_l}{K\triangle_f}$, where n_l is an integer and $\triangle_f$ is the tone spacing of the OFDM system.

The channel frequency response between the transmit antenna and the receive antenna at the *p*th time slot and at the *k*th subcarrier is given by [Lu et al., 2002]

$$H[p,k] = H(pT, k\triangle_f) = \sum_{l=1}^{L} \alpha(l;pT)e^{-j2\pi k n_l/K} = h^H(p)w(k) \tag{19.3}$$

where $h(p) = [\alpha(1),\ldots,\alpha(L)]^H$ is the L-sized vector containing the time responses of all the non-zero taps and $w(k) = [e^{-j2\pi k n_1/K},\ldots,e^{-j2\pi k n_L/K}]^T$ contains the corresponding DFT coefficients.

We adopt a time-sharing cooperative scheme similar to that of [Laneman et al., 2004; Stefanov and Erkip, 2004; Hunter and Nosratinia, 2002], as illustrated in Figure 19.1. Terminal T_1 transmits the first half of its codeword to the destination and T_2. If T_2 is able to decode it correctly, it then transmits the second half of T_1's codeword to the destination. If T_2 fails to decode it correctly, it notifies T_1 and T_1 then transmits the rest of the codeword itself. In the next transmission, the role of T_1 and T_2 are interchanged.

3. Performance Analysis of Coded Cooperative OFDM Systems

In this section, we analyze the performance of coded cooperative OFDM systems. We first derive the Chernoff bound on the pairwise error probability of the block fading OFDM channel, resulting from cooperation. We then utilize the pairwise error probability in the analysis of the frame error probability of coded cooperative systems. In particular, we study the achievable diversity order for various inter-user channel qualities.

Pairwise Error Probability for Block Fading OFDM Systems

In the block fading OFDM channel model resulting from cooperation, each block may have a different received signal-to-noise ratio and different number of non-zero channel taps. The user T_i-destination channels have L_i non-zero taps, $i = 1, 2$, respectively. Assuming that perfect channel state information is available at the receiver and by applying the Chernoff bound, the pairwise error probability (PEP) of transmitting codeword $\boldsymbol{c}$, while another codeword $\boldsymbol{e}$ is decoded at the receiver, is upper bounded by

$$P(\boldsymbol{c} \to \boldsymbol{e}|H) \leq \exp\{-(d_1^2(\boldsymbol{c}, \boldsymbol{e})\gamma_1 + d_2^2(\boldsymbol{c}, \boldsymbol{e})\gamma_2)\} \tag{19.4}$$

where $\gamma_i = \frac{E_{s_i}}{N_0}$, $i = 1, 2$, denotes the signal-to-noise ratio for the first half and second half of the channel codeword, respectively. $d_i^2(\boldsymbol{c}, \boldsymbol{e})$, $i = 1, 2$, can be expressed as

$$d_i^2(\boldsymbol{c}, \boldsymbol{e}) = \sum_{k=0}^{K-1} \sum_{p=(i-1)P/2+1}^{iP/2} |H_i[p, k]\epsilon[p, k]|^2 = h_i^H D_i h_i \tag{19.5}$$

where, $\epsilon[p, k]_{1\times 1} = c[p, k] - e[p, k]$, and

$$D_{i_{L_i \times L_i}} = \sum_{k=0}^{K-1} \sum_{p=(i-1)P/2+1}^{iP/2} w_i(k)\epsilon[p, k]\epsilon^*[p, k]w_i^H(k) \quad i = 1, 2. \tag{19.6}$$

Note that $\epsilon[p, k]\epsilon^*[p, k]$ equals to 0 if the entries of codeword $\boldsymbol{c}$ and $\boldsymbol{e}$ corresponding to the kth subcarrier and the pth time slot are the same. Let $D_{(1)}$ denote the number of instances when $\epsilon[p, k]\epsilon^*[p, k] \neq 0, p = 1, \ldots, P/2, \forall k$; and let $D_{1_{eff}}$ denote the minimum $D_{(1)}$ over every possible pair of codewords [Lu et al., 2002; Schlegel and Costello, 1989]. Denoting $r_1 = rank(D_1)$, it follows that $\min_{c,e} r_1 \leq \min\{D_{1_{eff}}, L_1\}$. Similarly, $\min_{c,e} r_2 \leq \min\{D_{2_{eff}}, L_2\}$. We observe that D_1 and D_2 are non-negative definite Hermitian matrices. Hence, by an eigen-decomposition, we obtain

$$D_1 = V_1 \Lambda V_1^H \quad D_2 = V_2 \Phi V_2^H \tag{19.7}$$

where V_1 and V_2 are unitary matrices, while Λ and Φ are diagonal matrices with $\{\lambda_j\}_{j=1}^{r_1}$ and $\{\phi_j\}_{j=1}^{r_2}$ being positive eigenvalues of D_1 and D_2, respectively. All the L_1 elements, $\alpha_1(1), \ldots, \alpha_1(L_1)$, of $\{h_1\}$, and the L_2 elements,

$\alpha_2(1), \ldots, \alpha_2(L_2)$, of $\{h_2\}$, are assumed to be i.i.d. circularly symmetric complex Gaussian with zero means. Eq. (19.4) can be written as

$$P(\boldsymbol{c} \to \boldsymbol{e}|H) \leq \exp\left\{-\left(\gamma_1 \sum_{j=1}^{r_1} \lambda_j |\beta(j)|^2 + \gamma_2 \sum_{j=1}^{r_2} \phi_j |\kappa(j)|^2\right)\right\} \quad (19.8)$$

where $\beta(j) = [V_1^H h_1]_j$ and $\kappa(j) = [V_2^H h_2]_j$. Since V_1 and V_2 are unitary, $\beta(j)$ and $\kappa(j)$ are also i.i.d. circularly symmetric complex Gaussian with zero mean and their magnitudes $|\beta(j)|$ and $|\kappa(j)|$ are i.i.d. Rayleigh distributed. By averaging the conditional PEP over the Rayleigh distribution, the pairwise error probability is found to be

$$P(\boldsymbol{c} \to \boldsymbol{e}) \leq \left(\prod_{j=1}^{r_1} \lambda_j \prod_{j=1}^{r_2} \phi_j\right)^{-1} \gamma_1^{-r_1} \gamma_2^{-r_2} \quad (19.9)$$

where r_1 and r_2 are the diversity levels with maximum of L_1 and L_2, respectively. We observe that in the block fading model resulting from cooperation in an OFDM system, each block may have a different received signal-to-noise ratio and different number of nonzero channel taps. For the case when $\gamma_1 = \gamma_2 = \gamma$, the pairwise error probability expression simplifies to

$$P(\boldsymbol{c} \to \boldsymbol{e}) \leq \left(\prod_{j=1}^{r_1} \lambda_j \prod_{j=1}^{r_2} \phi_j\right)^{-1} \gamma^{-(r_1+r_2)}. \quad (19.10)$$

Note that the quasi-static fading case [Lu et al., 2002], can be readily obtained as a special case of the block fading OFDM model.

Frame Error Probability Analysis

Without loss of generality, we study the cooperative coding performance gains from the perspective of node T_1. Similar results would also be obtained for node T_2. The frame error probability (FEP) can be obtained as

$$P_f^{coop} = (1 - P_f^{in}) P_f^{BF} + P_f^{in} P_f^{QS} \leq P_f^{BF} + P_f^{in} P_f^{QS} \quad (19.11)$$

where P_f^{in} denotes the FEP of the first half codeword over the inter-user channel, P_f^{BF} denotes the FEP over the block fading channel when the cooperation takes place, and P_f^{QS} denotes the frame error probability over the quasi–static fading T_1–destination channel which the destination observes if T_2 cannot decode T_1. Let γ_1 denote the received signal-to-noise ratio at the destination corresponding to the transmission from user T_1. Similarly, let γ_2 denote the received signal-to-noise ratio at the destination corresponding to the transmission from user T_2

and γ_{in} denote the received signal-to-noise ratio at user T_2 corresponding to the transmission from user T_1.

Utilizing the pairwise error probability for the block Rayleigh fading OFDM channel derived in the previous section and the union upper bound on the frame error probability, when node T_1 transmits in cooperation with node T_2, the upper bound on the frame error probability, P_f^{coop}, is

$$\begin{aligned} P_f^{coop} &\leq \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e} \neq \boldsymbol{c}} \frac{1}{\left(\prod_{b=1}^{2} \mu_b\right) \gamma_1^{r_1} \gamma_2^{r_2}} \right) \\ &+ \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e} \neq \boldsymbol{c}} \frac{1}{\left(\prod_{j=1}^{r_{in}} \delta_j\right) \gamma_{in}^{r_{in}}} \right) \\ &\left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e} \neq \boldsymbol{c}} \frac{1}{\left(\prod_{j=1}^{r} \xi_j\right) \gamma_1^{r}} \right) \end{aligned}$$

where r_b denotes the rank of the codeword difference matrices in the OFDM fading block b, $b = 1, 2$, and r denotes the rank of the codeword difference matrix between the two entire codewords of T_1. The μ_b's, $b = 1, 2$ are given by $\mu_1 = \prod_{i=1}^{r_1} \lambda_i$ and $\mu_2 = \prod_{i=1}^{r_2} \phi_i$, where the λ_i's and the ϕ_i's denote the nonzero eigenvalues of the product of the codeword difference matrix and its respective conjugate transpose for the OFDM fading block $b = 1$ and $b = 2$, respectively. The ξ_i's denote the nonzero eigenvalues of the product of the codeword difference matrix between the two entire codewords and its conjugate transpose. Similarly, the δ_i's denote the r_{in} nonzero eigenvalues of the product of the codeword difference matrix for the first half of the codeword and its conjugate transpose, utilized over the inter-user OFDM channel.

We consider the case when, $\gamma_1 \approx \gamma_2 \approx \gamma_{in} = \gamma$, that is all channels, including the inter–user channel, have similar quality. This assumption simplifies the diversity analysis and is quite reasonable at high signal-to-noise ratios in all channels. In this case, P_f^{coop}, can be approximately upper bounded by

$$\begin{aligned} P_f^{coop} &\leq \gamma^{-(L_1+L_2)} \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e} \neq \boldsymbol{c}} \frac{1}{\prod_{b=1}^{2} \mu_b} \right) \\ &+ \gamma^{-(L_1+L_{in})} \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e} \neq \boldsymbol{c}} \frac{1}{\prod_{j=1}^{L_{in}} \delta_j} \right) \left(\sum_{\boldsymbol{c}} \sum_{\boldsymbol{e} \neq \boldsymbol{e}} \frac{1}{\prod_{j=1}^{L_1} \xi_j} \right) \end{aligned}$$

Here we have assumed that all codeword difference matrices of interest are of full rank. Let $k = \min\{L_{in}, L_2\}$. At high signal-to-noise ratios, we have the following approximation

$$P_f^{coop} \approx \mathcal{K}_1 \, \gamma^{-(L_1+k)} \tag{19.12}$$

where the term $\mathcal{K}_1$ represents the coding parameters. This means that when all links have the same average quality, the diversity order achieved through cooperative coding depends on $k = \min\{L_2, L_{in}\}$, as indicated by the exponent of the signal-to-noise ratio. Note, that this diversity level is also indicated by the information theoretic cut-set bound [Cover and Thomas, 1991].

Good inter-user channel. Next, we focus on the case when the inter–user channel is very good, *i.e.*, it has a very high signal-to-noise ratio. This could represent the scenario when the two partners are located very close to each other. This means that P_f^{in} is small and we simply have $P_f^{coop} \approx P_f^{BF}$. Hence,

$$P_f^{coop} \approx \mathcal{K}_2 \, \gamma^{-(L_1+L_2)} \tag{19.13}$$

where $\mathcal{K}_2$ represents the coding parameters. We observe that when the inter-user channel quality is very good, the inter-user channel does not represent a bottleneck and a diversity level of $(L_1 + L_2)$ is achieved.

Poor inter-user channel. Finally, when the inter-user channel quality is poor, the inter-user channel signal-to-noise ratio, γ_{in}, will be lower than the signal-to-noise ratio of the user-destination channel. We can assume that $\gamma_{in}^{r_{in}} \leq C_{in}$, for all signal-to-noise ratios of interest. Hence, P_f^{coop} is upper bounded by the term $P_f^{in} P_f^{QS}$, yielding

$$P_f^{coop} \approx \frac{1}{C_{in}} \cdot \frac{\gamma^{-L_1}}{\min_{\boldsymbol{c},\boldsymbol{e}}\{(\prod_{j=1}^{L_{in}} \delta_j \prod_{j=1}^{L_1} \xi_j)\}} \tag{19.14}$$

where $\min_{\boldsymbol{c},\boldsymbol{e}}\{(\prod_{j=1}^{L_{in}} \delta_j \prod_{j=1}^{L_1} \xi_j)\}$ represents the dominant term in the union bound at high signal-to-noise ratios. In this case, the diversity level is only L_1. This is the same diversity level achieved by T_1 when there is no cooperation. However, there is still some coding gain, as indicated by the eigenvalue product, compared to the conventional OFDM system.

4. Simulation Results

In this section we provide numerical examples illustrating the performance of cooperative convolutional codes in OFDM systems. We assume that the OFDM system has $K = 128$ subcarriers. We consider the constraint length 7 convolutional code (133,171,117,165). This convolutional code belongs to the family of convolutional codes designed on the principle of overlays [Stefanov

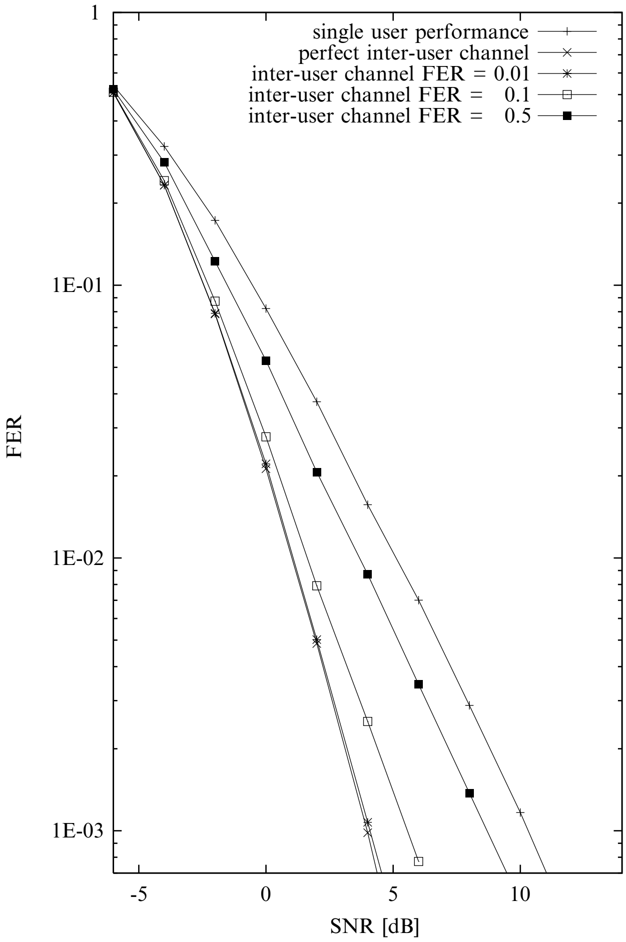

Figure 19.2. Single user performance vs. two user cooperation, for different inter-user channel qualities.

and Erkip, 2004; Gamal et al., 2003]. The coded bits are suitably multiplexed in order to achieve the maximum degree of diversity available in the cooperative communication scenario. We consider BPSK modulation. We assume maximum likelihood detection [Lin and Costello, 1983] and perfect channel state information at all the respective receivers. The frame size is 256 bits and each codeword spans $P = 2$ OFDM words. Both user-destination channels are assumed to have two taps, namely, $L_1 = L_2 = 2$.

Figure 19.2 illustrates the frame error rate (FER) performance comparison between the non-cooperative case and the cooperative case for different inter-user channel qualities. Both user-destination channels have similar quality. We observe that when the inter-user channel quality is very good, we achieve full diversity. The gain over the single user performance is about 6.5 dB at a FER of 10^{-3}. We note that even in the case when the inter-user channel FER is 0.5, we still obtain about 2 dB improvement at a FER of 10^{-3} as compared to the non-cooperative case.

Next we consider the scenario when one of the users has much better channel to the destination than the other partner. Figure 19.3 illustrates the FER performance for both users in this asymmetric scenario. We assume that user T_1 has better channel quality to the destination, *i.e.*, its signal-to-noise ratio is fixed at 10.3 dB, resulting in a FER of 10^{-3}. We observe the performance of both users as we vary the signal-to-noise ratio of user T_2. The inter-user channel FER is 10^{-1}. From Figure 19.3 it can be observed that both users benefit from cooperation. User T_1 achieves the FER of 10^{-3} when the signal-to-noise ratio of user T_2 is

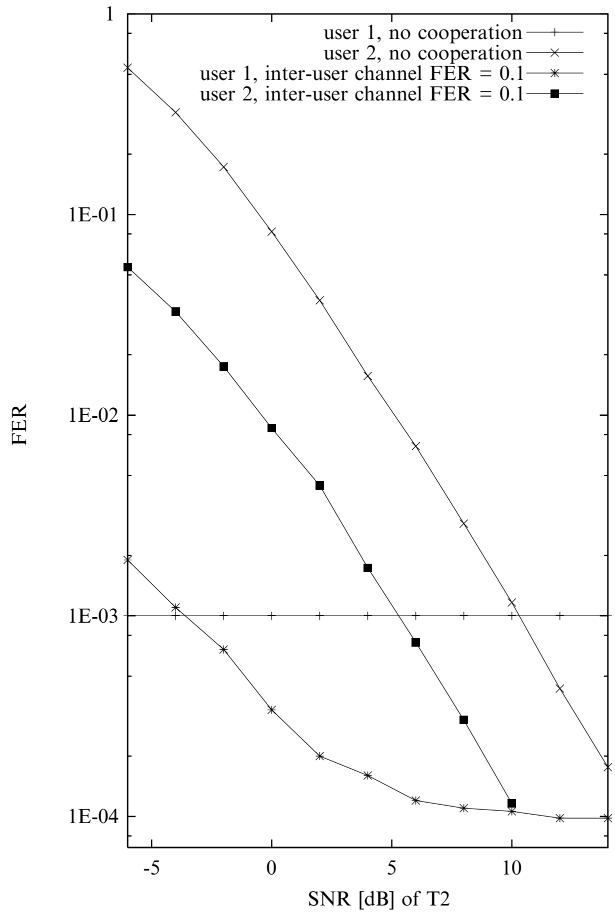

Figure 19.3. Single user performance versus two user cooperation, for two users with different channel qualities.

about -4 dB. At higher signal-to-noise ratios, its performance is even better than in the non-cooperative case. User T_2 also has significant gains, as it improves its performance by about 5 dB with respect to the non-cooperative case.

In the previous discussion, we considered the performance of the coded cooperative OFDM system for various inter-user channel qualities. Next, we will consider the case where the signal-to-noise ratio in the inter-user channel varies in proportion with the signal-to-noise ratio in the user-destination channel. Figure 19.4 represents the scenario when the inter-user channel has $L_{in} = 2$ taps. We consider two cases for the signal-to-noise ratio in the inter-user channel. In the first case the signal-to-noise ratio in the inter-user channel is approximately the same with the signal-to-noise ratios in the user-destination channels, *i.e.*, $\gamma_{in} \approx \gamma$. This could represent the scenario when all three nodes are at a similar distance from one another. We observe that the in this scenario

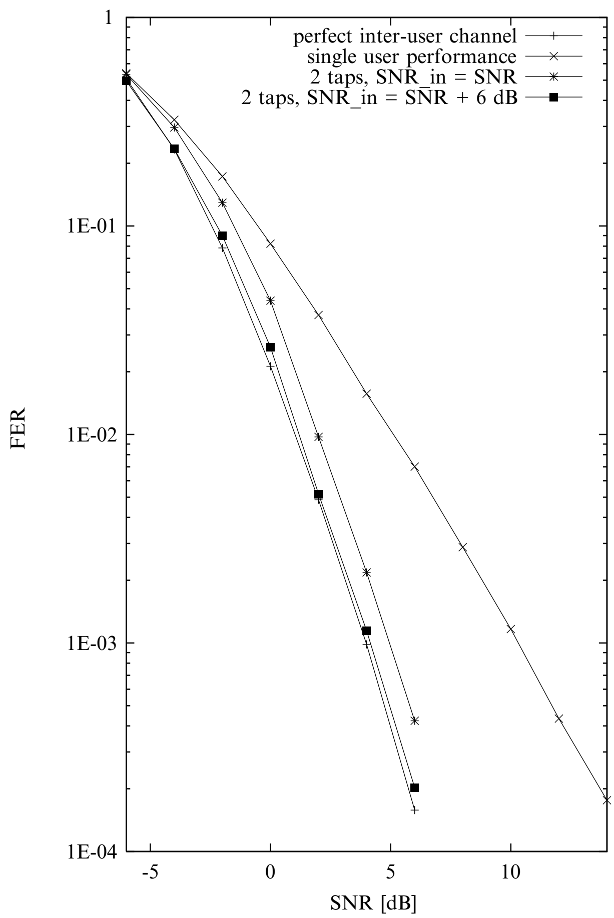

Figure 19.4. Cooperative coding example, the inter-user channel has two taps, $L_{in} = 2$.

the coded cooperative OFDM system achieves a FER of 10^{-3} at about 5.5 dB. This is only about 1.5 dB away from the performance with a perfect inter-user channel. It also results in a gain of almost 5 dB compared to the non-cooperative case. In the second scenario, we consider the case when the signal-to-noise ratio in the inter-user channel is approximately 6 dB better than the signal-to-noise ratio in the user–destination channels, *i.e.*, $\gamma_{in} \approx \gamma + 6$ dB. This could represent the scenario when the two cooperating nodes are closer to each-other than to the destination. We observe that in this case the coded cooperative OFDM system essentially achieves the same performance that it would have with a perfect inter-user channel.

In Figure 19.5, we consider the case when the signal-to-noise ratio in the inter-user channel is approximately the same with the signal-to-noise ratio in the

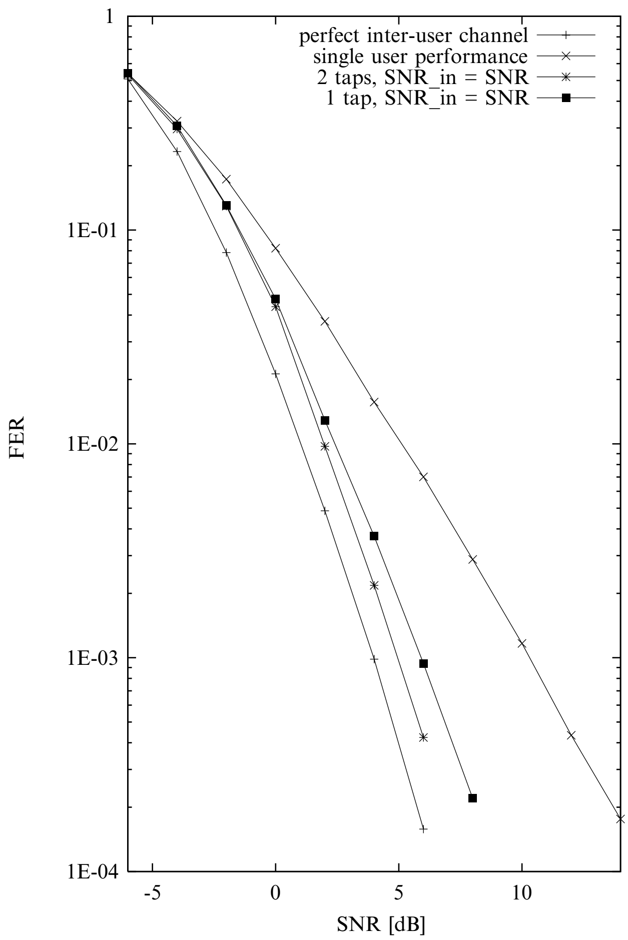

Figure 19.5. Cooperative coding example, ($\gamma_{in} \approx \gamma$), the inter-user channel has either one or two taps, $L_{in} = 1$ or $L_{in} = 2$.

user-destination channels, *i.e.*, $\gamma_{in} \approx \gamma$. Again, this could represent the scenario when all three nodes are at a similar distance from one another. We focus on the case when the inter-user channel may or may not be frequency selective, *i.e.*, $L_{in} = 2$ or $L_{in} = 1$. In either case, the (133,171) convolutional code used in the inter-user channel achieves the best performance over that channel regardless whether it has frequency selectivity or not. As expected, we observe that the overall coded cooperative OFDM system achieves a better performance when there is frequency selectivity in the inter-user channel, as this leads to better performance of the inter-user channel code and allows cooperation to take place more often. Nonetheless, even in the case when there is no frequency selectivity in the inter-user channel, which represents the worst case, we observe that the coded cooperative system achieves a FER of 10^{-3} at about 6 dB, which is less than 2 dB away from the performance with a perfect inter-user channel. It also results in a gain of over 4 dB compared to the non-cooperative case.

5. Conclusions

We considered cooperative coding and its application to OFDM systems. We derived the Chernoff bound on the pairwise error probability for the block fading OFDM model and subsequently used it in the analysis of the frame error probability of the coded cooperative OFDM system. The performance analysis indicated that cooperative coding can provide increased diversity and coding gains over conventional OFDM systems. We also provided examples of convolutional codes based on the principle of overlays that could exploit the cooperative gains. We illustrated that the codes perform well for a variety of cooperation scenarios and inter-user channel qualities.

References

Cover, T. M. and Thomas, J. A. (1991). *Elements of Information Theory*. John Wiley and Sons, Inc.

Gamal, H. E., Hammons, A. R., and Stefanov, A. (2003). Space–time overlays for convolutionally coded systems. *IEEE Trans. Commun.*, 51:1601–1612.

Hunter, T. and Nosratinia, A. (2002). Cooperation diversity through coding. In *Proc. of IEEE ISIT*, Lausanne, Switzerland.

Laneman, J. N., Tse, D. N. C., and Wornell, G. W. (2004). Cooperative diversity in wireless networks: Efficient protocols and outage behavior. *IEEE Trans. Inform. Theory*, 50(12):3062–3080.

Lin, S. and Costello, D. J. (1983). *Error Control Coding: Fundamentals and Applications*. Prentice–Hall.

Lu, Ben, Wang, X., and Narayanan, K. R. (2002). LDPC–based space–time coded OFDM systems over correlated fading channels: Performance analysis and receiver design. *IEEE Trans. Commun.*, 50:74–88.

Nee, R. V. and Prasad, R. (2000). *OFDM for wireless multimedia communications*. Artech House Publishers.

Schlegel, C. and Costello, D. J. (1989). Bandwidth efficient coding for fading channels: Code construction and performance analysis. *IEEE J. Select. Areas Commun.*, pages 1356–1368.

Sendonaris, A., Erkip, E., and Aazhang, B. (2003). User cooperation diversity–part I: System description. *IEEE Trans. Commun.*, pages 1927–1938.

Stefanov, A. and Erkip, E. (2004). Cooperative coding for wireless networks. *IEEE Trans. Commun.*, 52(9):1470–1476.