CHAPTER 4

MENSURATIONAL ASPECTS

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4.1. SAMPLE PLOTS

4.1.1 Plot size

In forest inventory problems, the size of the plots to be measured needs also to be decided in addition to selection of the number of plots. The larger the plots are, the more time-consuming and expensive it is to measure them, and obviously the number of sample plots that can be measured with a given budget is larger when the plots are small. On the other hand, the variation among plots in the population, S_y^2, diminishes as the plot size increases (Shiver and Borders 1995 p. 60).

Clusters of smaller sub-plots (or combined plots) have been used on many occasions instead of single plots, typically in large-area surveys such as national inventories (see Chapter 11). A cluster plot typically consists of small circular plots (or point-sampling plots, section 4.2) that form a geometrical figure such as a triangle or rectangle. There are two benefits entailed in the use of clusters (Loetsch et al. Vol II p. 345). First, the location and layout of a cluster of several small plots is faster and more accurate than the measuring of large single plots. In addition, the coefficient of variation is smaller than for single plots of the same total area.

The optimal plot size thus depends on both the measurement costs and the observed variation. This question has been studied by Nyyssönen (1966) and Nyyssönen et al. (1971), for instance. Gambill et al. (1985) presented a method for determining plot size that minimizes the total cruising time (i.e. costs) and provides a specified level of precision, while Scott et al. (1983) discussed a method for determining the optimal spacing of sub-plots in clusters and Scott (1993) one for determining the optimal cluster design.

It can be shown that the spatial pattern of forests has an effect on the optimal plot size. If the trees are located according to a Poisson distribution, the ratio

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of the variance to the population mean, σ^2/μ, for number of stems will assume the value 1, whereas it will be larger than 1 for a clustered population and smaller than 1 for a systematic population. The more clustered the population is, the larger the plot size should be in order to obtain a certain coefficient of variation in the number of trees per plot. Also, the smaller the plot, the faster the coefficient of variation $(CV = \sigma / \mu)$ increases with the variance/mean ratio (Fig. 4.1), although the latter ratio also depends on the sample plot size: i.e. σ^2/μ tends to increase as plot size increases (Loetsch et al. 1973 Vol II p. 332).

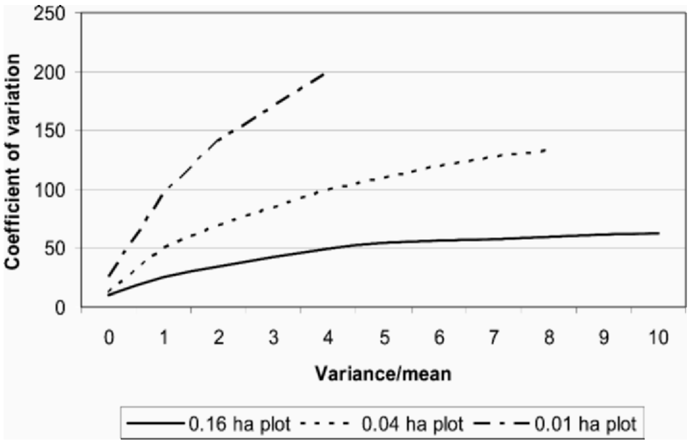

Figure 4.1 Illustration of the effect of spatial pattern (variance/mean ratio) and plot size on the coefficient of variation (modified from Loetsch et al. 1973).

The size of plot also determines the inclusion area for each tree. When circular plots of radius *r* are used, the inclusion area is a circle also of radius *r* centred on the tree. In other words, a certain tree will be tallied if the sample plot is located in its inclusion area. Thus the inclusion probability of a tree is its inclusion area divided by the total forest area (Schreuder et al. 1993 p. 114). The inclusion probabilities for rectangular plots are calculated in the same way, but now the inclusion area of a certain tree is a rectangle centred on it, having the same area and same orientation as the original sample plots (Ducey et al. 2004). These inclusion probabilities are needed for calculating edge corrections (section 4.4).

4.1.2 Plot shape

The usual plot shapes used in forest inventories are rectangular, square and circular. Rectangular plots are established by first defining one side and two corners, after which right angles are traced at these corners and the other two corners are located (Schreuder et al. 1993). The distance between the last two corners, and also the two diagonals if possible, should be measured in order to check the measurements, as a rectangular plot is fairly vulnerable to errors in determination of the right angles. If the angles at the first two corners are 5 degrees too wide, this will cause the plot area to be 8.3% too large (Loetsch et al. 1973 p. 317, Schreuder et al. 1993). A square plot can also be established working from the centre, by measuring the distance $a/\sqrt{2}$ to the corner along each diagonal. This approach is much less vulnerable to errors.

Another form of plot is a strip, i.e. a long, narrow rectangle. In strip sampling the measurer usually walks along the central line of the strip and checks its width now and then, e.g. with a pole (Loetsch et al. 1973 p. 318). It is also possible to walk along one side of the strip. Strips are not very commonly used nowadays, except for sampling rare populations (Chapter 8). This is due to the fact that the line-plot type of inventory includes far less measurements but is just as efficient (section 1.3).

The trees in plantation forests are often planted in rows and columns which are not exactly parallel, so that it may be difficult to establish a plot of exactly the specified size. It is therefore usually advisable to establish a plot with corners mid-way between the rows (Schreuder et al. 1993 p. 295), otherwise the plot estimates may be biased due to inaccuracies in the areas. Since plantations usually show periodic variation, systematic sampling may also be highly inefficient. If the plot centres always fall between two rows, for instance, the nearest rows will always be either just inside or just outside a plot (Shiver et Borders 1995 p. 60).

Circular plots are easy to establish when the radius is not very large, and they are also not very vulnerable to errors in plot area. The length of the perimeter will increase as the radius increases, however, and so will the number of trees on the edge of the plot. Thus circular plots with a large radius are not very efficient (Schreuder et al. 1993, Loetsch et al. 1973). In many cases combined circular plots can be established, i.e. plots that consist of several concentric circles, the smaller circles being used for smaller trees and the larger circles for larger trees.

It is assumed with all plot types that the terrain will be level and the plot will lie entirely within the stand. If these assumptions are not fulfilled, a slope correction or edge correction will be needed (section 4.4).

4.2 POINT SAMPLING

Point sampling (also known as angle-gauge sampling, Bitterlich sampling, plotless sampling or variable radius plot [VRP] sampling) is a sampling method that is unique in forest inventories. The principles were first introduced by Walter Bitterlich (1947, see also Bitterlich 1984). In point sampling the trees do not have an equal probability of being included in the sample, but instead the probability is proportional to the tree size, or more exactly to the basal area of the tree (PPS sampling). This was first noted by Grosenbaugh (1952).

Trees with a basal area exceeding a certain viewing angle α are selected for the sample (Fig. 4.2). The radius r at which the basal area of the tree just exceeds the critical angle defines the plot area for a tree of this size, and each tree is measured in a circular plot having an area proportional to its basal area, giving (Loetch et al. 1973 Vol II p. 348)

$$\frac{\frac{\pi}{4} d_i^2}{\pi r_i^2} = \frac{\frac{\pi}{4} d_j^2}{\pi r_j^2}, \tag{4.1}$$

where d_i (d_j) is diameter of tree i (j) and r_i (r_j) is the limiting radius for that diameter.

In angle-gauge sampling, the inclusion probabilities for each tree can be calculated as the inclusion area divided by the total area, as with circular plots. In this case, however, the inclusion area around each tree depends on its diameter, i.e. large trees have larger inclusion areas than small trees. The radius of this inclusion area is the limiting radius for trees of that size.

Each tree in a stand represents the same basal area, namely BAF m^2/ha, where BAF is the basal area factor. The estimator for any variable of interest is (see section 2.8 and Chapter 8)

$$\hat{Y} = \sum_{j=1}^{m} \sum_{i=1}^{N_k} \frac{y_i}{\pi_i}, \tag{4.2}$$

where the inclusion probability $\pi_i = g_{ji} / BAF$, and g_{ji} is the basal area of tree i at sampling point j, m is the number of sampling points, N_j is the number of trees at point j and BAF is the basal area factor of the angle gauge used. Typically, either 1 (m^2/ha) or 2 factors are used in Finland. If the variable of interest y_i is the basal area g_i, it is enough to count the trees filling the angle, so that the method provides a quick means of measuring the basal area. For other variables, such as the number of trees, the diameters of the trees also need to be measured.

Angle-gauge sampling can also be used in many other applications. In vertical point sampling, for example, the trees are selected in proportion to their squared height, i.e. the trees filling a vertical angle gauge are selected. This approach of estimating the mean squared tree height was proposed by Hirata (1955).

Angle-gauge sampling requires certain assumptions to be fulfilled (Grosenbaugh 1958, Schreuder et al. 1993). These are fairly similar to the ones that apply to plot sampling, namely that

1. The trees are vertical and their cross-sections are circular.
2. The terrain is level (or else a slope correction is made).
3. The sample trees are visible from the point location (or from another point at same distance, or else their diameter and distance can otherwise be checked), and

4. The area from which the trees can be selected lies entirely within the stand (or else an edge correction is made)

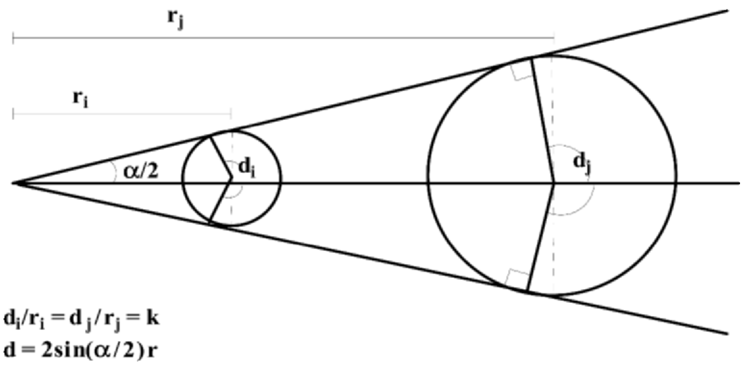

Figure 4.2 The principle of a relascope: k is a relascope constant, $k = 2\sin(\alpha/2)$*, α is the viewing angle and the basal area factor* $BAF = 10^4 \sin^2(\alpha/2)$ *(Loetsch et al. 1973 p.349).*

It has been seen in many studies (e.g. Laasasenaho and Päivinen 1986) that larger basal area factors give systematically larger basal areas. This is most probably due to the fact that a small basal area factor allows large trees to be included in the point sample from long distances. This means that not all the trees are necessarily visible, or that there is a possible inclusion area beyond the stand edge. It is obvious that many measurers will ignore edge corrections in practical work (section 4.4), or fail to check whether trees that are further away should be counted. Consequently, it is advisable to use factors giving about 6-10 trees per sample point on average, to avoid factors that would cause trees to be included from long distances.

4.3 COMPARISON OF FIXED-SIZED PLOTS AND POINTS

If fixed area sampling and variable radius sampling are compared in such a way that one plot is compared with one point, the result usually is that plot sampling is more efficient. This is because plots usually include more tallied trees than points. If measurement costs are accounted for, point sampling can be more efficient.

Matérn (1972), who compared the two sampling methods in a theoretical framework, concluded that with a given number of measured trees, the point sampling method is more efficient for determining the basal area or the volume of the stand. This result has been confirmed in other studies (Schreuder et al. 1987, Scott 1990). Although the number of stems is more efficiently measured with plots of a fixed size, Schreuder et al. (1987) found that the number of stems by diameter classes could also be measured more efficiently using variable-radius plots. It has also been stated that change (i.e. mortality, ingrowth) (Scott 1990) is more efficiently measured with fixed-radius plots. In any case, point sampling estimates for growth are often more complex (Chapter 5). In general, it can be concluded that variables associated with large diameter classes or correlated with current basal area are better estimated with point samples and variables associated with small diameter classes are more efficiently measured with fixed-area plots.

4.4 PLOTS LOCATED ON AN EDGE OR SLOPE

4.4.1 Edge corrections

In many cases a sample plot may happen to be located in a void in a stand, such as on a road, in a lake or beneath a power line. In these cases the surveyor may be tempted to move it to a wooded spot. If the stand area includes voids, plots located on those spots will, however, be needed in order to calculate the mean volume accurately (Shiver and Borders 1995). Only if power lines etc. are excluded from the stand area should one not place plots there. Correspondingly, if the distance between the plots should be 200 metres the distance across a power line, for instance, should not be counted in this (Shiver and Borders 1995).

A special approach is needed when plots are located near a stand edge. For example, if a circular plot is located so that the distance from its centre to the stand edge is less than its radius r, the total area of the plot inside the stand will be less than the nominal area, so that, if no corrections are made, fewer trees will be measured than should be and the approach will result in a biased volume, i.e. an underestimate (Schreuder et al. 1993). The basic reason for the bias, however, is that the varying inclusion probabilities of the trees are not accounted for. If a tree is so near to the stand edge that its inclusion area is partly outside the border, its inclusion probability will be smaller than it should be (see Gregoire 1982).

The problem has been known for a long time, and the first attempts to correct the bias were presented by Finney and Palca (1948). Their method is itself biased, however. One solution that is often attempted is to move the plot away from the stand edge to the inside of the forest (the "Move-to-r" approach). Circular plots located nearer to the edge than their radius r, for instance, are moved to a point at a distance r from the stand boundary. This means, however, that the trees within a distance r from the edge have a smaller inclusion probability than those further inside the stand. Furthermore, the inclusion probability of the trees in the zone to which the plots are moved increases. This will lead to biased estimates if the border zone is different from the interior forest (Schreuder et al. 1993 p. 299). If trees grow better near the boundary than inside the forest, for instance, the stand volume may be underestimated.

Another approach is to measure a sample plot on the edge so that only the portion inside the stand is actually measured. This means that the true area of the plot inside the stand needs to be defined, which may be a complex matter when using circular plots, for example. The area of each plot also has to be accounted for when calculating the mean volume of the stand, i.e. by attaching more weight to plots with a smaller area (see Beers 1966). This method also produces biased estimates (Schreuder et al. 1993), but correct estimates can be achieved if each tree in the plot is weighted separately according to the inverse of its inclusion area, (Beers 1966, Iles 2003 p. 627).

There are also other valid procedures for measuring plots at a stand edge. The most popular one is the "mirage method", as presented by Schmid-Haas (1969, see also Beers 1977, Schmid-Haas 1982, Gregoire 1982). When the radius *r* of a circular plot is larger than the distance *x* from the edge, a mirroring sample plot is located at a similar distance *x* from the edge on the other side and only the trees inside the original stand are measured on this mirroring plot (Fig. 4.3). This method exploits the concentric approach. With mirage method the folding of the plot works correctly, so that the inclusion area of each and every tree need not to be considered separately, and still the method provides unbiased estimates (Gregoire et al. 1982).

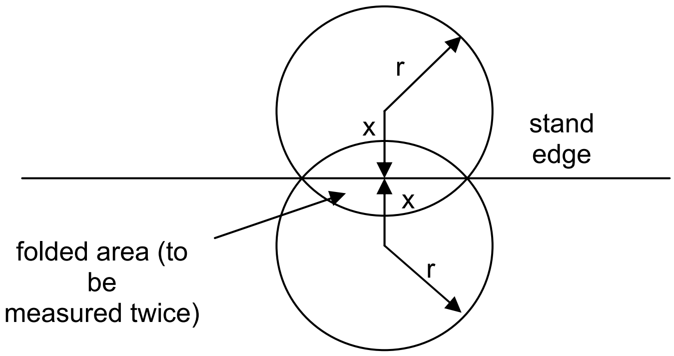

Figure 4.3 Illustration of the mirage method.

The good point about the mirage method is that it is also directly suited for point sampling. It is not without problems, however. It requires the assumption to be made that the border is almost linear (e.g. Iles 2003). Corners may encounter problems in the case of two crossing borders, for instance, where some trees need to be counted once, some twice and some three or four times (Fig. 4.4.). Erroneous use can also produce biased estimates: if the plot is not circular, the mirage plot may contain trees that were not in the original sample (e.g. Ducey et al. 2001). This possibility needs to be accounted for. In some cases it may also be difficult to define the mirage plot, as it may border onto a lake, a cliff or even the lawn of a house.

A new and very promising method for edge correction is the "walkthrough method" (Ducey et al. 2004). This is based on the inclusion areas for single trees. It requires measurement of the distance between the tree and the centre of the plot, after which a similar distance is measured on the other side of the tree (i.e. it is

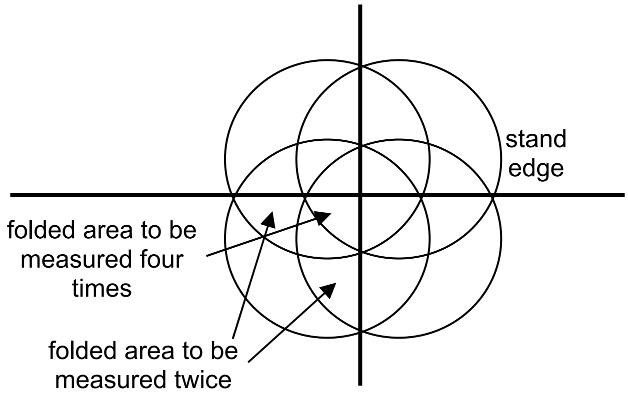

Figure 4.4 Illustration of the mirage method in a corner.

assumed that the measurer walks through the tree in a constant direction for the same distance). If the point achieved in this way lies within the area, the tree is measured once, otherwise (i.e. the point is reached over the boundary) it is measured twice (Fig. 4.5). This method is simple to apply in the field and does not require linear borders. There are still problems involved, however. There may be cases in narrow areas where both a sample point that lies within the inclusion area of the tree and its walkthrough point are outside the area (Iles 2003). In such a case these areas are neither counted in the original sample nor compensated for by the walkthrough method.

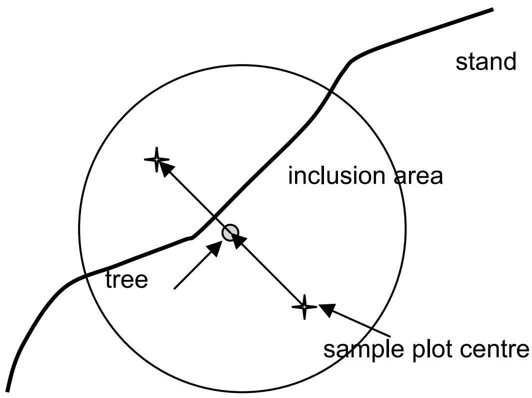

Figure 4.5 Illustration of the walkthrough method. The tree in the figure is counted twice, as the walkthrough point lies outside the boundary.

There are also many other methods for tackling this problem. Those interested could consider including sample points that lie outside the areas

(Masuyama 1954), or the related "toss-back" method of Iles (2001, 2003) and Flewelling and Iles (2004). Other unbiased approaches are to use the enlarged tree circle method (Barrett 1964, Schreuder et al. 1993), Grosenbaugh's method (1958) or the tree concentric method (Gregoire and Scott 1990).

Edge correction problems also need to be accounted for when using clusters of sub-plots (Scott and Bechtold 1995, Hahn et al. 1995), although the edge effect becomes smaller the larger is the area to be surveyed.

4.4.2 Slope corrections

If plots are located on a slope and the distance is measured along the slope, the plot as projected to the horizontal will actually be an ellipse with too small an area. This will obviously cause bias in the estimates if it is not accounted for. The slope can be accounted for exactly if the distance of each tree from the centre of the plot is measured horizontally. This also applies to point sampling. Measuring the exact horizontal distances may be tedious, however, if there are many such plots, and difficult if the slope is steep.

Another possibility is to enlarge the radius of the circular plot by multiplying it by $\sqrt{1/\cos\beta}$, where β is the maximum slope angle (Bryan 1956). The plot as projected to the horizontal will then be a circle with the correct area. In the case of a rectangular plot, the sides perpendicular to slope will remain unaffected but the sides parallel to slope need to be extended by $1/\cos\beta$ (Loetsch et al. 1973 Vol II p. 324). If the plot is not oriented parallel or perpendicular to the gradient of the slope, the corrections will obviously be more complicated.

A correction for the slope can be made in point sampling by dividing the estimate for the basal area by the cosine of the maximum angle of the slope at the sampling point (Schreuder et al. 1993 p. 119). The problem with this method, however, is the varying sampling intensity on different slopes (Del Hodge 1965). Furthermore, the correction only applies to total basal area, since it means varying the basal area factor for individual trees (Loetsch et al. 1973 Vol II p. 354).

Del Hodge (1965) presented a method in which the angle gauge was adjusted for the maximum slope so that the inclusion areas of the trees were correct. Another possibility is to adjust the angle gauge separately for each tree (Bruce 1955). This can be done fairly conveniently with a prism. There also exist instruments that make such corrections automatically, e.g. the Spiegel relascope (Shiver and Borders 1995 p. 91). This last method is the only one in which the inclusion areas for the trees are circular.

All in all, it is fairly easy to make a slope or edge correction. The most problematic cases are ones where both types of correction are needed (Ducey et al. 2001). When the inclusion areas are ellipses, for instance, the mirage plot may contain trees even if the original plot does not, i.e. the mirrored area does not entirely overlap with the original plot.

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