DYNAMICS AND CONTROL OF TENSEGRITY SYSTEMS

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Abstract: Rather than the traditional vector differential equation, this paper introduces rigid body dynamics in a new form, as a matrix differential equation. For a system of n_b rigid bodies, the forces are characterized in terms of network theory, and the kinematics are characterized in terms of a directed graph of the connections of all members. The dynamics are characterized by a second order differential equation in a $3 \times 2n_b$ *configuration matrix*. The first contribution of the paper is the dynamic model of a broad class of systems of rigid bodies, characterized in a compact form, requiring no inversion of a variable mass matrix. The second contribution is the derivation of all equilibria as linear in the control variable. The third contribution is the derivation of a linear model of the system of rigid bodies. One significance of these equations is the exact characterization of the statics and dynamics of all class 1 tensegrity structures, where rigid bar lengths are constant and the string force densities are control variables. The form of the equations allow much easier integration of structure and control design since the control variables appear linearly. This is a significant help to the control design tasks.

Key words: tensegrity, rigid body dynamics.

1. INTRODUCTION

This paper introduces rigid body dynamics in a new form, as a matrix differential equation, rather than the traditional vector equation. This paper describes the dynamics and the static equilibria of a set of discontinuous rigid bodies, connected via a continuous set of strings to stabilize the system. In our theory, the “strings” are “springs” which can take compression or tension. However, in the special application of greatest interest, the “strings” can only take tension. All equilibria of such bar and string connections are described, and the dynamics of such systems are described in a new form, a second order differential equation of a $3 \times 2n_b$ matrix, called the *configuration matrix*. By parametrizing the configuration in terms of the components of vectors, the usual

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nonlinearities of angles, angular velocities and coordinate transformations are avoided. Indeed, there are no trigonometric functions in this formulation. We seek simplicity in the analytical form of the dynamics, for ease in designing control laws later. Among all the available equations for a system of rigid bodies, these equations produce the simplest form. Our model of dynamics is in a matrix form, opening new control research challenges to develop control theories for matrix models of the dynamics.

In the 60s and 70s, a variety of Newtonian and energy approaches (Hamilton and Lagrange) were introduced and traded for numerical efficiencies. NASA had great interest in building accurate deployable spacecraft simulations composed of a large number of rigid bodies in a topological tree. The typical form of these equations was vector-second order. For a large class of problems it is reasonable to assume that the rigid bodies are rod-shaped and have negligible inertia about their longitudinal axes. We will make this assumption.

1.1 Tensegrity Systems

Class k tensegrity systems are defined by the number (k) of rigid bodies that connect to each other (with frictionless ball joints) at a specific point (node). This paper entertains only class 1 tensegrity systems, so no rigid bodies are in contact, and the system is stabilized only by the presence of tensile members connecting the rigid bodies. In the steady state, such a system has only axially-loaded members, since the rigid bodies do not touch each other and the strings connected to the rigid bodies cannot apply torques at the site of the attachment. These features not only simplify the equations of motion, but the resulting models will be much more accurate than models of rigid bodies that are subject to bending moments. That is, the internal stresses in the rigid bodies have a specific direction.

Tensegrity systems have been around for over 50 years as an artform, with some architectural appeal, but analytical tools to design engineering structures from tensegrity concepts are still inadequate. The primary motivation for this paper is to provide a convenient analytical tool to describe both the statics and dynamics of class 1 tensegrity systems.

1.2 Notation

DEFINITION 1 *The set of vectors* $\underline{e}_i$, $i = 1, 2, 3$, *form a* dextral set, *if the dot products satisfy* $\underline{e}_i \cdot \underline{e}_j = \delta_{ij}$ *(where* δ_{ij} *is a Kronecker delta), and the cross products satisfy* $\underline{e}_i \times \underline{e}_j = \underline{e}_k$, *where the indices* i, j, k *form the cyclic permutations,* $i, j, k = 1, 2, 3$ *or* $2, 3, 1$, *or* $3, 1, 2$.

DEFINITION 2 *Let* $\underline{e}_i$, $i = 1, 2, 3$ *define a* dextral set *of unit vectors fixed in an inertial frame, and define the* vectrix $\underline{E}$ *by* $\underline{E} = [\ \underline{e}_1 \quad \underline{e}_2 \quad \underline{e}_3\ ]$.

The item we call $\underline{r}$ is a Gibbs vector. The items we call r^X and r^E are vectors in the linear vector spaces of linear algebra, where we use the notation, $r^X \in \mathbb{R}^3$ and $r^E \in \mathbb{R}^3$ to denote that the items r^X and r^E live in a real three-dimensional space. However, the items r^X and r^E tell us nothing unless we have previously specified the frames of reference $\underline{X}$ and $\underline{E}$ for these quantities. If we must assign a "dimension" to these quantities $\underline{X}$ and $\underline{E}$, then we must say they are 3×1 arrays, composed of the three elements $\underline{e}_i$, $i = 1, 2, 3$. However, these arrays contain quantities we call Gibbs vectors $\underline{e}_i$. So the 3×1 item $\underline{E}$ is not a *vector* in either the sense of Gibbs, nor in the sense of linear algebra. For these reasons Peter Hughes makes the logical choice to call the quantity $\underline{E}$ a *vectrix*.

Unlike many problems in aerospace, where multiple coordinate frames are utilized, this paper uses only one coordinate frame to describe all vectors. Since we always use the same frame of reference, the inertial frame, described by the vectrix $\underline{E}$, we will not complicate the notation of vectors with different superscripts, as would be required above to distinguish between components of a vector represented in different frames. Hence, we use the notation for the vector $\underline{n}_i$, as follows

$$\underline{n}_i = \underline{E} n_i^E, \qquad n_i^E = n_i, \tag{1}$$

where $n_i^T = [\, n_{i_1}, \quad n_{i_2}, \quad n_{i_3} \,]$ describes the components of the vector n_i in coordinates $\underline{E}$, where we have dropped the superscript E that would be used in the more complete and more general notation above (n_i^E), and we will write only n_i, hereafter, instead of n_i^E.

We generate a diagonal $n \times n$ matrix from an n-dimensional vector $v^T = [\, v_1 \quad v_2 \quad v_3 \quad v_4 \quad \dots \,]$, by denoting the *hat* operator by

$$\hat{v} = \mathrm{diag}\left[\, v_1 \quad v_2 \quad v_3 \quad v_4 \quad \dots \,\right]. \tag{2}$$

We generate a 3×3 matrix $\tilde{v}$ from the 3-dimensional vector $v^T = [\, v_1 \quad v_2 \quad v_3 \,]$ by the *tilde* operator as follows

$$\tilde{v} = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}. \tag{3}$$

We often use the fact that for any two n-dimensional vectors $v \in \mathbb{R}^n$, and $x \in \mathbb{R}^n$,

$$\hat{v} x = \hat{x} v. \tag{4}$$

2. DESCRIPTION OF A NETWORK OF BARS/STRINGS

We now show how to organize the equations for n_b rigid bars. We will show below how to describe all dynamics in the $\underline{E}$ frame, after the usual definition of dot and cross products. The 3×1 matrix b_i represents the components of vector $\underline{b}_i$ with respect to the fixed frame $\underline{E}$. That is,

$$\underline{b}_i = \sum_{j=1}^{3} \underline{e}_j b_{i_j} = \begin{bmatrix} \underline{e}_1 & \underline{e}_2 & \underline{e}_3 \end{bmatrix} \begin{bmatrix} b_{i_1} \\ b_{i_2} \\ b_{i_3} \end{bmatrix} = \underline{E} b_i. \tag{5}$$

LEMMA 3 *Let for some chosen inertial reference frame $\underline{E}$,*

$$\underline{b}_i = \underline{E} b_i, \qquad \underline{f}_i = \underline{E} f_i, \qquad \underline{n}_i = \underline{E} n_i.$$

Then the cross product is given by

$$\underline{b}_i \times \underline{f}_{i+n_b} = (\underline{E} b_i) \times (\underline{E} f_{i+n_b}) = \underline{E} \tilde{b}_i f_{i+n_b}$$

and the dot product is given by

$$\underline{b}_i \cdot \underline{f}_{i+n_b} = (\underline{E} b_i) \cdot (\underline{E} f_{i+n_b}) = b_i^T \underline{E}^T \cdot \underline{E} f_{i+n_b} = b_i^T f_{i+n_b}$$

where the dot product $\underline{E}^T \cdot \underline{E} = I$ since $\underline{e}_i, i = 1, 2, 3$ form a dextral set of unit vectors.

Let a structural system be composed of n_b bars and n_s strings. The definitions below will later allow us to describe the connections between the rigid members and the strings.

DEFINITION 4 *A* node *(the i^{th} node $\underline{n}_i$) of a structural system is a point in space at which members of the structure are connected. The coordinates of this point in the $\underline{E}$ frame are $n_i \in \mathbb{R}^3$, as in (1).*

DEFINITION 5 *A* string *(the i^{th} string) is characterized by these properties:*

- *A massless structural member connecting two nodes.*
- *A vector connecting these two nodes is $\underline{s}_i$. The direction of $\underline{s}_i$ is arbitrarily assigned.*
- *The string provides a force to resist lengthening it beyond its rest-length, but provides no force to resist shortening the string below its rest-length.*
- *A* string *has no bending stiffness.*

DEFINITION 6 *A* bar *(the i^{th} bar of a n_b bar system) is characterized by these properties:*

- *A structural member connecting two nodes $\underline{n}_i$ and $\underline{n}_{i+n_b}$.*

- *The vector along the bar connecting nodes* $\underline{n}_i$ *and* $\underline{n}_{i+n_b}$ *is* $\underline{b}_i = \underline{n}_{i+n_b} - \underline{n}_i$, $i = 1, 2, \ldots n_b$.

- *The bar* $\underline{b}_i$ *has* length $\|b_i\| = \mathcal{L}_i = \sqrt{b_i^T b_i}$.

DEFINITION 7 *The vector* $\underline{r}_i$ *locates the mass center of bar* $\underline{b}_i$*, and* $\underline{r}_i = \underline{E} r_i$.

DEFINITION 8 *The vector* $\underline{t}_i$ *represents the force exerted on a node by string* $\underline{s}_i$*, where the direction of* $\underline{t}_i$ *is defined to be parallel to string vector* $\underline{s}_i$*. That is,* $\underline{t}_i = \gamma_i \underline{s}_i$ *and hence,* $t_i = \gamma_i s_i$ *for some positive scalar* γ_i.

DEFINITION 9 *The* force density γ_i *in string* s_i *is defined by* $\gamma_i = \frac{\|t_i\|}{\|s_i\|}$.

DEFINITION 10 $\underline{f}_i$ *represents the net sum of vector forces external to bar* $\underline{b}_i$ *terminating at node* $\underline{n}_i$*. The net sum of vector forces acting at the other end of bar* $\underline{b}_i$ *is* $\underline{f}_{i+n_b}$.

From these definitions, define matrices, $F \in \mathbb{R}^{3\times 2n_b}$, $N \in \mathbb{R}^{3\times 2n_b}$, $T \in \mathbb{R}^{3\times n_s}$, $S \in \mathbb{R}^{3\times n_s}$, $B \in \mathbb{R}^{3\times n_b}$, $\Gamma \in \mathbb{R}^{n_s\times n_s}$, as follows

$$F = \begin{bmatrix} F_1 & F_2 \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & \ldots & f_{n_b} & | & f_{n_b+1} & \ldots & f_{2n_b} \end{bmatrix} \quad (6)$$

$$N = \begin{bmatrix} N_1 & N_2 \end{bmatrix} = \begin{bmatrix} n_1 & n_2 & \ldots & n_{n_b} & | & n_{n_b+1} & \ldots & n_{2n_b} \end{bmatrix} \quad (7)$$

$$T = \begin{bmatrix} t_1 & t_2 & \ldots & t_{n_s} \end{bmatrix} \quad (8)$$

$$S = \begin{bmatrix} s_1 & s_2 & \ldots & s_{n_s} \end{bmatrix} \quad (9)$$

$$B = \begin{bmatrix} b_1 & b_2 & \ldots & b_{n_b} \end{bmatrix} \quad (10)$$

$$R = \begin{bmatrix} r_1 & r_2 & \ldots & r_{n_b} \end{bmatrix} \quad (11)$$

$$\hat{\gamma} = \Gamma = \text{diag}\begin{bmatrix} \gamma_1 & \ldots & \gamma_{n_s} \end{bmatrix}, \quad (12)$$

where $\hat{\gamma}$ represents the diagonalizing operation on the vector $\gamma \in \mathbb{R}^{n_s}$. It follows from (7), (10), and Definition 6 that

$$B = N_2 - N_1 = N \begin{bmatrix} -I \\ I \end{bmatrix}, \quad (13)$$

and the locations of the mass centers of all bars are described by

$$R = N_1 + \frac{1}{2} B. \quad (14)$$

It follows from Definition 9 and (8), (9) that

$$T = S\Gamma. \quad (15)$$

2.1 Angular Momentum

LEMMA 11 *Assume that the mass of the bar is uniformly distributed only along its length, and that its length is much longer than its diameter. Then the angular momentum of the bar b_i about the center of mass of bar b_i, expressed in the $\underline{E}$ frame, is*

$$h_i = \frac{m_i}{12}\tilde{b}_i\dot{b}_i \tag{16}$$

3. DYNAMICS OF A RIGID BAR

For a single bar, with bar vector $\underline{b}$, at nodes $\underline{n}_1$ and $\underline{n}_2$, these forces are applied $\underline{f}_1$ and $\underline{f}_2$.

LEMMA 12 *The translation of the mass center of bar $\underline{b}$, located at position $\underline{r}$ obeys*

$$m\ddot{\underline{r}} = \underline{f}_1 + \underline{f}_2 \tag{17}$$

or, in the $\underline{E}$ frame of reference,

$$m\ddot{r} = f_1 + f_2 \tag{18}$$

LEMMA 13 *The rotation of bar b_i about it mass center obeys*

$$\frac{m}{12}\tilde{b}\ddot{b} = \frac{1}{2}\tilde{b}(f_2 - f_1). \tag{19}$$

3.1 Constrained Dynamics

We now wish to develop the dynamics constrained for constant bar lengths. We add a non-working constraint torque τ to get

$$\frac{m}{12}\tilde{b}\ddot{b} = \frac{1}{2}\tilde{b}(f_2 - f_1) + \tau \tag{20}$$

$$\kappa = b^T b - \mathcal{L}^2 = 0, \tag{21}$$

where the added constraint is $\kappa = 0$, and τ is the non-working torque associated with this constraint. The torque τ does no work in the presence of any feasible perturbation of the generalized coordinate b. Hence, $\tau^T \delta b = 0$. The constraint must also hold in the presence of a feasible perturbation. Hence, $d\kappa = (\frac{\partial \kappa}{\partial b})^T \delta b = 0$. Thus,

$$\begin{bmatrix} \tau^T \\ (\frac{\partial \kappa}{\partial b})^T \end{bmatrix} \delta b = 0, \tag{22}$$

requiring that the matrix coefficient of δb must have deficient rank. Thus, $\tau = (\frac{\partial \kappa}{\partial b})\zeta$, for some ζ (called a Lagrange multiplier). Furthermore, $\frac{\partial \kappa}{\partial b} = 2b$. Hence, the constrained dynamic system obeys,

$$\frac{m}{12}\tilde{b}\ddot{b} = \frac{1}{2}\tilde{b}(f_2 - f_1) + b\zeta \tag{23}$$

$$\kappa = b^T b - \mathcal{L}^2 = 0. \tag{24}$$

where we have absorbed some constants into the scalar ζ. Note that the constraint holds over time, hence $\kappa = \dot{\kappa} = \ddot{\kappa} = 0$. Differentiating the constraint yields,

$$\dot{b}^T b + b^T \dot{b} = 0 = 2b^T \dot{b}. \tag{25}$$

Differentiating (25) yields

$$\dot{b}^T \dot{b} + b^T \ddot{b} = 0,$$

or,

$$b^T \ddot{b} = -\dot{b}^T \dot{b}. \tag{26}$$

The conclusion thus far is that constant length rigid bar rotations obey, for some scalar ζ,

$$\begin{bmatrix} \tilde{b} \\ b^T \end{bmatrix} \ddot{b} = \begin{bmatrix} \tilde{b}(f_2 - f_1)\frac{6}{m} \\ -\dot{b}^T \dot{b} \end{bmatrix} + \begin{bmatrix} b \\ 0 \end{bmatrix} \zeta. \tag{27}$$

The following identity will be useful, and is proved by substitution.

LEMMA 14 *For any skew-symmetric matrix $\tilde{b}$, the following is true.*

$$\tilde{b}^2 = bb^T - b^T bI.$$

The properties of the Moore–Penrose inverse are well known, and are used to obtain the following.

LEMMA 15 *The unique Moore-Penrose inverse of* $\begin{bmatrix} \tilde{b} \\ b^T \end{bmatrix}$ *is given by*

$$\begin{bmatrix} \tilde{b} \\ b^T \end{bmatrix}^+ = \begin{bmatrix} -\tilde{b} & b \end{bmatrix} \mathcal{L}^{-2}$$

LEMMA 16 *The solution of (27) for $\ddot{b}$ has the unique solution*

$$\ddot{b} = \frac{6}{m}(f_2 - f_1) - b\left(\frac{\dot{b}^T \dot{b}}{\mathcal{L}^2} + \frac{6}{m\mathcal{L}^2} b^T (f_2 - f_1)\right) \tag{28}$$

The results of this section applies for any number of bars. The next section will write the matrix construction for the general case.

3.2 An n_b-Bar System

It follows clearly that these equations apply to any number of bars, so that the following is true, where θ_i is the i^{th} element of the vector $\theta \in \mathbb{R}^{n_b}$.

THEOREM 17 *Consider an n_b-bar system with constant length bar vectors b_i, $i = 1, 2, \dots, n_b$, and matrices defined by,*

$$R = N_1 + \frac{1}{2}B \tag{29}$$

$$B = \begin{bmatrix} b_1 & b_2 \dots & b_{n_b} \end{bmatrix} = N_2 - N_1, \quad N = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \tag{30}$$

$$N_1 = \begin{bmatrix} n_1 & n_2 \dots & n_{n_b} \end{bmatrix}, \quad N_2 = \begin{bmatrix} n_{n_b+1} & \dots & n_{2n_b} \end{bmatrix} \tag{31}$$

$$F = \begin{bmatrix} F_1 & F_2 \end{bmatrix}, \qquad F_1 = \begin{bmatrix} f_1 \dots & f_{n_b} \end{bmatrix} \tag{32}$$

$$F_2 = \begin{bmatrix} f_{n_b+1} \dots & f_{2n_b} \end{bmatrix} \tag{33}$$

$$\mathcal{Q} = \begin{bmatrix} B & R \end{bmatrix} \tag{34}$$

$$K_0 = \begin{bmatrix} I \\ 0 \end{bmatrix} \hat{\theta} \begin{bmatrix} I & 0 \end{bmatrix} \tag{35}$$

$$\theta_i = b_i^T (f_{n_b+i} - f_i)/2\mathcal{L}_i^2 + m_i \|\dot{b}_i\|^2/12\mathcal{L}_i^2 \tag{36}$$

$$\Phi = \begin{bmatrix} -\frac{1}{2}I & I \\ \frac{1}{2}I & I \end{bmatrix} \tag{37}$$

$$M = \text{diag}\begin{bmatrix} \dots & m_i & \dots \end{bmatrix} \tag{38}$$

$$\mathcal{M} = \begin{bmatrix} \frac{1}{12}M & 0 \\ 0 & M \end{bmatrix}. \tag{39}$$

Then the rigid body dynamics are given by

$$\ddot{\mathcal{Q}}\mathcal{M} + \mathcal{Q}K_0 = F\Phi, \qquad \mathcal{Q}\Phi^T = N, \tag{40}$$

where the coordinate transformation from coordinates N to coordinates $\mathcal{Q}$ is provided by the invertible matrix Φ^T.

For a given square matrix J, define $\lfloor J \rfloor = \text{diag}\begin{bmatrix} \dots & J_{ii} & \dots \end{bmatrix}$. Then, it may be shown that,

COROLLARY 18

$$\hat{\theta} = \frac{1}{2}\hat{\mathcal{L}}^{-2}\lfloor B^T(F_2 - F_1) + \frac{1}{6}\dot{B}^T\dot{B}M \rfloor \tag{41}$$

$$= \frac{1}{2}\hat{\mathcal{L}}^{-2}\lfloor \begin{bmatrix} I & 0 \end{bmatrix} \mathcal{Q}^T(F_2 - F_1) + \frac{1}{6}\begin{bmatrix} I & 0 \end{bmatrix} \dot{\mathcal{Q}}^T M \rfloor \tag{42}$$

$$\mathcal{L} = \begin{bmatrix} L_1 & L_2 & \cdots & L_{n_b} \end{bmatrix}^T. \tag{43}$$

4. CHARACTERIZING BAR/STRING CONNECTIONS

DEFINITION 19 *Define the "string connectivity matrix" C by*

$$C_{ij} = \begin{cases} 1 & \textit{if string vector } s_i \textit{ terminates on node } n_j. \\ -1 & \textit{if string vector } s_i \textit{ eminates from node } n_j. \\ 0 & \textit{if string vector } s_i \textit{ does not connect with node } n_j. \end{cases} \tag{44}$$

$$C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \quad C_1 \in \mathbb{R}^{n_s \times n_b}, \quad C_2 \in \mathbb{R}^{n_s \times n_b}$$

DEFINITION 20 *Define the "disturbance connectivity matrix" D by*

$$D_{ij} = \begin{cases} 1 & \textit{if disturbance vector } w_i \textit{ connects to node } n_j. \\ 0 & \textit{if disturbance vector } w_i \textit{ does not connect with node } n_j. \end{cases} \tag{45}$$

$$D = \begin{bmatrix} D_1 & D_2 \end{bmatrix}, \quad D_1 \in \mathbb{R}^{n_w \times n_b}, \quad D_2 \in \mathbb{R}^{n_w \times n_b}$$

For n_w disturbance vectors applied at nodes selected by the matrix D,

$$W = \begin{bmatrix} w_1 & w_2 \cdots & w_{n_w} \end{bmatrix} \tag{46}$$

$$D = \begin{bmatrix} D_1 & D_2 \end{bmatrix}, \quad D_1 \in \mathbb{R}^{n_w \times n_b}, \quad D_2 \in \mathbb{R}^{n_w \times n_b}. \tag{47}$$

THEOREM 21 *Let any connection of rigid bars and elastic strings be described by the string connectivity matrix C and the disturbance connectivity matrix D in Definitions (44-45), and let the arbitrary convention be established that vectors entering a node have a positive sign. Then, the sum of all forces entering the nodes of a class 1 tensegrity structure may be computed by,*

$$F = -(TC + WD), \tag{48}$$

and the string vectors s_i, $i = 1, \dots, n_s$ are linearly related to the nodal vectors n_j, $j = 1, \dots, n_{2n_b}$ by

$$S = NC^T.$$

LEMMA 22

$$F = -Q\Phi^T C^T \Gamma C - WD, \qquad \Phi^T = \begin{bmatrix} -\frac{1}{2}I & \frac{1}{2}I \\ I & I \end{bmatrix}. \tag{49}$$

THEOREM 23 *The dynamics of all Class 1 tensegrity systems with rigid, fixed length bars are described by*

$$\ddot{Q}\mathcal{M} + Q\mathcal{K} + WD\Phi = 0, \tag{50}$$

$$\mathcal{Q} = \begin{bmatrix} B & R \end{bmatrix}, \tag{51}$$

$$\mathcal{M} = \begin{bmatrix} \frac{1}{12}M & 0 \\ 0 & M \end{bmatrix}, \tag{52}$$

$$\mathcal{K} = \begin{bmatrix} \hat{\theta} & 0 \\ 0 & 0 \end{bmatrix} + \Phi^T C^T \Gamma C \Phi, \tag{53}$$

$$\Phi^T = \begin{bmatrix} -\frac{1}{2}I & \frac{1}{2}I \\ I & I \end{bmatrix} \tag{54}$$

$$\begin{aligned} \hat{\theta} \;&=\; \frac{1}{12}\hat{\mathcal{L}}^{-2}\Bigg\lfloor 6\begin{bmatrix} I & 0 \end{bmatrix}\mathcal{Q}^T(\mathcal{Q}\Phi^T C^T \Gamma C + WD)\begin{bmatrix} I \\ -I \end{bmatrix} \\ &\quad + \begin{bmatrix} I & 0 \end{bmatrix}\dot{\mathcal{Q}}^T\dot{\mathcal{Q}}\begin{bmatrix} I \\ 0 \end{bmatrix} M\Bigg\rfloor. \end{aligned} \tag{55}$$

From (55), the i^{th} element of the diagonal matrix $\hat{\theta}$ is given by

$$\theta_i = \frac{1}{2}\mathcal{L}_i^{-2} b_i^T(\mathcal{Q}\Phi^T C^T \hat{C}_{\Delta_i}\gamma + WD_{\Delta_i}) + \frac{m_i}{12\mathcal{L}_i^2}||\dot{b}_i||^2, \tag{56}$$

where the following definitions characterize the i^{th} columns of the matrices $C_1 - C_2$ and $D_1 - D_2$.

$$C_{\Delta_i} = i^{th} col(C_1 - C_2) \tag{57}$$

$$C_{+_i} = i^{th} col(C_1 + C_2) \tag{58}$$

$$D_{\Delta_i} = i^{th} col(D_1 - D_2) \tag{59}$$

$$D_{+_i} = i^{th} col(D_1 + D_2) \tag{60}$$

5. CONCLUSIONS

The dynamics of a system of n_b rigid bodies, connected by tensile elements, has been derived from a network point of view. The resulting equations are in matrix form, rather than the traditional vector form. The nonlinear equations are given in the form of a second order differential equation of a $3 \times 2n_b$ *configuration matrix*. These equations contain no trigonometric nonlinearities, and require no inversion of a mass matrix containing configuration variables.

These equations open the door a bit wider for feedback control design and structure design as integrated activities, since all freedom in the desired equilibria can be utilized in the control problem. The result should be more efficient controlled structures. This efficiency will be demonstrated in a future paper.