MODELING THE EFFECTS OF STEPS ON BOUNDARY-LAYER TRANSITION

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Abstract: An experimental and computational study is conducted to develop a model for the effects of surface steps on transition to turbulence in boundary layers. The step effects are captured within the framework of a variable n-factor method, where the step results in a reduction in the TS-wave transition n-factor ΔN_{TS}. Data is presented for ΔN_{TS} for favorable and adverse pressure gradients. Backward-facing steps result in significantly larger reductions in the transition n-factor when compared to forward-facing steps. The results show that step effects can be accounted for by using a ΔN_{TS} for step heights up to 1.5 times the local boundary-layer displacement thickness.

Key words: Instability, Receptivity, Steps, Transition Prediction, Variable N-Factor

1. INTRODUCTION

In many flows of practical interest, the location of the transition to turbulence is influenced by the presence of surface imperfections in the form of steps, gaps, or protuberances. In aerodynamics some of the most common surface imperfections are forward- and backward-facing steps. New methods to improve transition predictions (based on better physical modeling) need to account for these typical surface imperfections.

Currently, the most useful tools for predicting transition are based on the so-called e^n method – originally devised by Smith & Gamberoni (1956) and Van Ingen (1956). In the e^n method, transition is assumed to occur when the amplification factor n reaches a critical value N, where N is established by

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correlation with experiments. For controlled experimental conditions, this provides an effective prediction method. However, when applied away from the correlation conditions the method does not yield consistent results. The primary shortcoming of the basic e^n method is that the receptivity (responsible for the initial amplitude A_0) and nonlinear-breakdown physics cannot be adequately accounted for in a single value of N.

To overcome the major shortcomings of the basic e^n method, variable n-factor methods have been proposed (Mack 1977; Crouch & Ng 2000). Here the value of N is given as a function of the external conditions, which influence the receptivity or locally change the growth rate. The methods are based on experimental correlation or a combination of correlation and theory. In this paper, we consider a new form of variable N-factor to account for the effects of steps on Tollmien-Schlichting (TS) wave transition.

The investigation of the effects of steps and protuberances on transition has a long history starting from the beginning of the last century (Prandtl 1914; Goldstein 1936; Dryden 1953, etc.). These early experiments led to the concept of a "critical obstacle height" necessary for affecting transition. The data form these experiments are the foundation for current empirical approaches to transition prediction (e.g. Fage 1943).

A more clear understanding of the physical mechanisms leading to transition came with the experiments of Klebanoff & Tidstrom (1972). They have shown that surface imperfections change the boundary-layer stability characteristics, leading to an accelerated transition. Aizin & Polyakov (1979) obtained another important result showing that surface imperfections can serve as effective sources for unstable Tollmien-Schlichting (TS) waves through the enhanced receptivity to free-stream acoustic disturbances. These TS waves were later shown to superimpose on the TS waves already existing in the smooth-surface boundary layer (Kosorygin 1985, Kosorygin & Polyakov 1985/1990), resulting in movement of the transition onset location.

Analytical and numerical studies show that some of the effects of the steps can be accounted for by modeling the details of the local flow perturbations in the calculation of amplification factors (Nayfeh 1992; Perraud & Seraudie 2000). Meanwhile, Boeing has initiated transition experiments aimed at incorporating the effects of steps into the critical N value. Gaster & Wang (2004) investigated the effects of rectangular backward- and forward-facing steps in a flat-plate zero pressure-gradient boundary layer. They obtained the changes in n-factor for steps of various heights. The present investigation aims at developing a prediction scheme to account for step effects on transition in boundary layers under favorable or adverse pressure gradients, characteristic of leading-edge regions in aerodynamic flows.

2. EXPERIMENTS

The experiments are conducted in the wind tunnel T-324 at the Institute of Theoretical and Applied Mechanics, Russian Academy of Sciences (Novosibirsk). This is a low-turbulence close-circuit wind tunnel with a *1m x 1m x 4m* test section and is well suited for receptivity, stability, and transition experiments. Two flat-plate models were used in the experiments. Each of these plates is manufactured from a *6-mm* aluminum alloy sheet and they are *0.996m* wide and *2.0m* long. The leading-edge shape is a semi-cylinder machined directly on the flat plate model with radius *2.25mm*. The position of the stagnation line and the pressure distribution in the vicinity of the leading edge are controlled by a trailing-edge flap during tests.

The flat plates have movable leading edges to create backward- and forward-facing steps (as well as gaps) over a wide range of heights. The accuracy of adjustment for the step height was about $10\mu m$. The model leading edges have different lengths (*127mm* and *450mm*) to enable the placement of the step in a favorable or an adverse pressure gradient. The polished working surfaces of the flat plates have slightly different natural undulations, which have been documented. Both plates contain a row of 24 *0.35mm* static pressure orifices parallel to the centerline, but shifted in the span direction. The surface pressure variation is created by means of wall contours, as shown in figure 1.

Detailed measurements of the basic flow are used to establish the flow conditions for the stability analysis. The measured pressure distribution is shown in figure 2, along with the spline fit used for the calculations. The minimum of the pressure distribution is located at a longitudinal distance *x=250mm* and the step can be placed at either *x=127mm* or *x=450mm*, in the

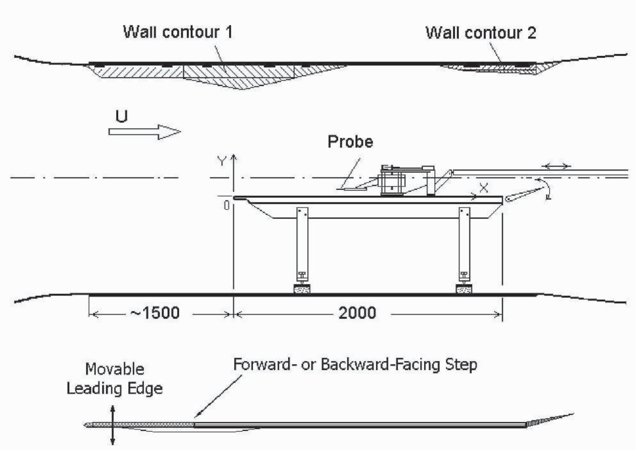

Figure 1. Experimental setup showing the flat-plate in the test section with wall contours (distances are in *mm*), and flat plate with movable leading edge

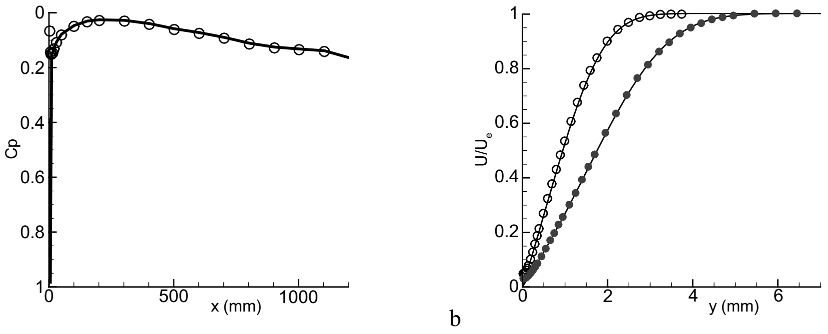

Figure 2. Basic-flow characteristics: (a) experimental pressure distribution with curve used for computations, (b) computational (line) and experimental (symbols) velocity profiles

favorable (FPG) or adverse (APG) pressure gradient, respectively. The free-stream velocity is varied from *18.3m/s* up to *27.5m/s*. All measurements are conducted under natural background disturbances, with an integral free-stream turbulence intensity of $\varepsilon = 0.028\%$ at $U=18.3m/s$, and $\varepsilon = 0.039\%$ at $U=27.5m/s$ (bandwidth $2 - 4\cdot 10^3$ Hz).

Figure 2 also shows velocity profiles measured at the streamwise locations $x=400mm$ (open symbols) and $x=900mm$ (closed symbols). The lines through the experimental points are boundary-layer calculations based on the measured *Cp* distribution. The experimental points were shifted in *y* using the calculated results to "find the wall." The calculated basic flow is in very good agreement with the measured flow conditions.

In addition to the mean-flow quantities, the transition location and velocity oscillations are also measured. The transition location is determined by means of a *1-mm* round Preston tube which is moved along the surface. The measured dynamic pressure can be interpreted in terms of the velocity profile slope close to wall (i.e. the skin friction). The dynamic pressure typically diminishes along the streamwise coordinate in a laminar boundary layer. At some distance downstream, the pressure (and skin friction) will rise as a result of the non-linear processes associated with transition. The transition location, x_T, is estimated based on the minimum of the dynamic pressure distribution, with an uncertainty in x_T of about *20mm*. The transition Reynolds number, $Re_T = U x_T / \nu$, is plotted in figure 3 as a function of the non-dimensional step height. An Increase in the step height results in a reduction in the extent of laminar flow. The results show a stronger reduction in Re_T for backward-facing steps.

Velocity profile measurements are made using a single-wire probe built in-house with $5\mu m \times 1mm$ Pt-plate tungsten wire. The probe is connected to a DANTEC constant-temperature anemometer. Spectra of the velocity oscillations, measured just prior to transition onset, show a band of unstable

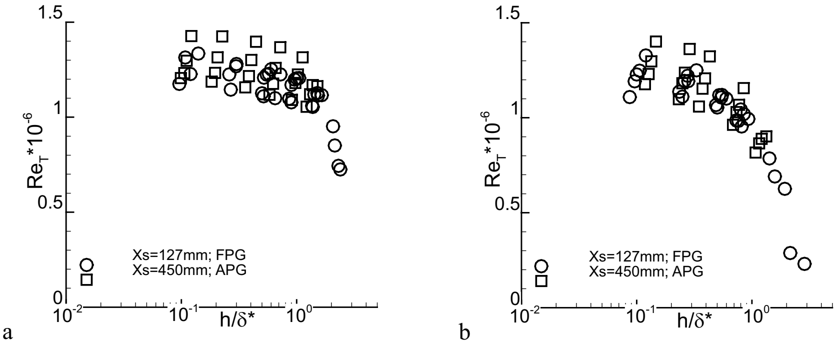

Figure 3. Transition Reynolds numbers as a function of non-dimensional step height, for: (a) forward-facing and (b) backward-facing steps, in favorable and adverse pressure gradients.

TS waves. The measurements also show the exponential growth characteristic of TS waves. Similar measurements in the presence of steps demonstrate that the same frequencies are responsible for transition, within some step-height range. However, the measured amplitudes of the leading frequencies are larger in presence of a step.

3. N-FACTOR RESULTS

Tollmien-Schlichting wave amplification n-factors are calculated based on quasi-parallel theory. The mean flow is obtained numerically by solving the compressible boundary layer equations for spanwise-uniform flow. The experimental pressure distributions are used to generate the edge velocities for the boundary layer. The instabilities are governed by the Orr-Sommerfeld equation, with imposed values for the real frequency ω and spanwise wavenumber β. The amplification n-factor is defined as:

$$n_{TS}(x) = \max_{\omega} \max_{\beta} \Big(\int_{x_0}^{x} \gamma(s;\omega,\beta)\, ds \Big), \qquad (1)$$

where γ is the spatial growth rate. These n-factors are calculated for the smooth-surface conditions, without accounting for any of the local effects due to the steps.

Figure 4 shows the amplification n-factors calculated for the test conditions of this study. The results show a slight destabilizing effect due to increasing the free-stream velocity from $U=18.3m/s$ to $U=27.5m/s$. This corresponds to an increase in the unit Reynolds number from $1.2\cdot10^{6}\ m^{-1}$ to $1.8\cdot10^{6}\ m^{-1}$. The change in unit Reynolds number also results in a change in

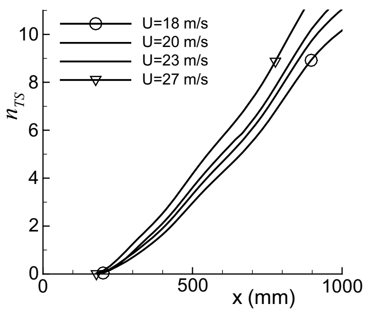

Figure 4 N-factor curves calculated for the experimental conditions, with different edge velocities: *U=18m/s, U=20m/s, U=23m/s, U=27m/s*

boundary-layer thickness at the $x=127mm$ step location from $\delta^*=0.52mm$ to $\delta^*=0.42mm$. The dominant frequencies calculated from the stability theory are in agreement with the dominant frequencies measured prior to transition.

The value of $n_{TS}(x_T)$ at the transition location, x_T, is designated N_{TS}. For each step height considered, the transition location is measured and the value of N_{TS} is determined. In the absence of any step, transition occurs at $x_T=1000mm$ for $U=18m/s$, and at $x_T=760mm$ for $U=27m/s$; the corresponding transition n-factors for these two cases are $N_{TS}=10$ and $N_{TS}=8.5$, respectively. This difference in n-factor is consistent with the difference in turbulence levels given in section 2, following the relationship of Mack (1977).

In the presence of a step, the transition n-factor is reduced due to the forward movement of the transition location. The reduction in transition n-factor can be modeled with a variable n-factor relationship

$$N_{TS} = N_{TS0} - \Delta N_{TS}(h/\delta^*), \tag{2}$$

where the function ΔN_{TS} accounts for the local change in the stability characteristics at the step. The value of the smooth-surface transition n-factor N_{TS0} can account for the free-stream turbulence level as suggested by Mack (1977). Delta-n-factors are calculated for each of the step-heights considered, and are plotted against the step height in figure 5. The step heights are normalized by the local displacement thickness at the step location. Results are presented for both favorable and adverse pressure gradients. In general, the adverse-pressure-gradient ΔN_{TS} results show a better collapse, when compared to the favorable-pressure-gradient results. In both pressure gradients, the backward-facing steps have a much bigger impact on the transition n-factors. For forward-facing steps, the reduction in n-factor is generally larger for adverse pressure gradients. The ΔN_{TS} results have also been plotted against the trip-height Reynolds number, $Re_h = U{\cdot}h/\nu$ (not shown). This yields distinct curves for the favorable-pressure-gradient

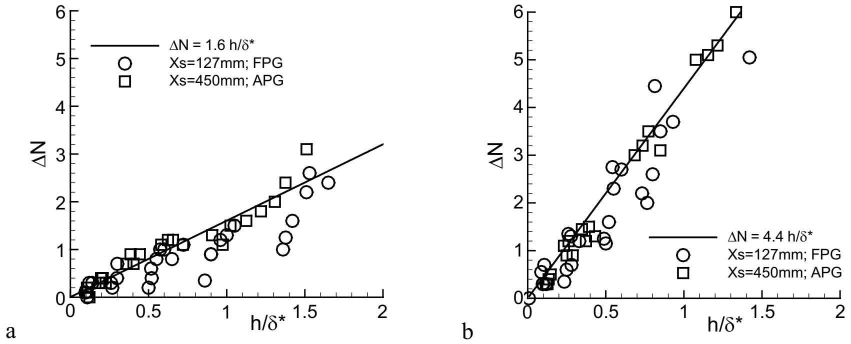

Figure 5. ΔN-factor results for: (a) forward-facing steps, and (b) backward-facing steps in favorable and adverse pressure gradients

and adverse-pressure-gradient data since it only accounts for the unit Reynolds number, and not the actual boundary-layer thickness.

The lines drawn through the calculated n-factors of figure 5 are based on a fit to the adverse-pressure-gradient data. These lines provide a rough upper bound for the favorable-pressure-gradient data. These results lead to the TS-wave n-factor expressions:

$$\text{FFS}: \quad N_{TS} = N_{TS0} - 1.6\, h/\delta^*, \tag{3}$$

$$\text{BFS}: \quad N_{TS} = N_{TS0} - 4.4\, h/\delta^*, \tag{4}$$

for forward- and backward-facing steps, respectively. For forward-facing steps, a step height equal to the displacement thickness results in a reduction in the TS-wave transition n-factor of 1.6. The delta n-factors for backward-facing steps are much larger. A backward-facing step at a height of the boundary-layer displacement thickness results in a reduction of 4.4 in the transition n-factor – almost three times the reduction of a forward-facing step of the same height. These expressions for N_{TS} capture the effects of steps on transition for step heights up to 1.5 times the boundary-layer displacement thickness.

4. CONCLUSIONS

Detailed measurements are made to quantify the effects of forward- and backward-facing steps on transition. The transition results from the amplification of TS waves, both for the smooth surface and in the presence of steps. Measurements and boundary-layer calculations are used to establish a basic flow. The calculated boundary-layer profiles are shown to be in very good agreement with the experiments. Linear-stability theory is used to determine the amplification n-factors for the TS waves.

The effects of the steps are modeled by a variable transition n-factor. The amplification n-factors are calculated for a smooth surface, but the critical value that signifies transition is given as a function of the step height. The transition n-factor varies linearly with the step height, nondimensionalized with the boundary-layer displacement thickness at the step location. Backward-facing steps result in n-factor reductions that are almost three times as large as forward-facing steps of the same height. The variable n-factor relationships are well correlated with the experiments for steps heights up to 1.5 times the local boundary-layer displacement thickness. This provides a very useful method to account for surface steps in practical applications.

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