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POST-FESTUM[1] AND HEURISTIC ANALOGIES

1. INTRODUCTION

Analogies have proven powerful tools in generating insight and understanding. However, analogies may also deeply mislead – scientists as well as learners. It appears that this two-sided nature of analogies relates to the following fundamental features of teaching and learning. Students make their own sense of material provided by the teacher as learning aids, and this is especially so with analogies. From the teacher's perspective, analogical relations between base and target domains of analogy properly rest on subject matter structure. They rely on propositionally based knowledge. Students however, interpret base and target domains in fundamentally different ways. Learning by analogy rests on visual perception. It traces a line of concrete visualisation and abstraction by transcending the concrete in a second step. To put it in a nutshell: a student's heuristic analogy is built on mental images rather than propositionally based knowledge (as opposed to the teacher's post-festum analogy). Hence, students seem determined to construct analogical relations other than the teacher's. Using analogies, then, is not simply a process of transferring certain structural features from the base to the target but a process of constructing the analogical relation the teacher aims at.

[1] Post-festum analogies are analogies that are constructed by the teacher from canonical knowledge and are used for the purpose of explaining scientific phenomena.

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2. A THEORY OF ANALOGICAL REASONING

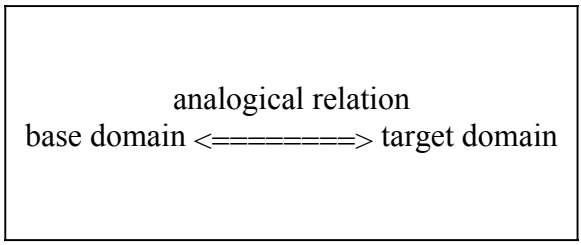

Figure 1. Analogy: Relation between base and target domains of analogy

A widely accepted definition determines analogy as a similarity between two domains commonly called base and target domain (Figure 1). The conclusion suggests itself that analogical reasoning is a matter of comparing their similarities. In fact, most theoretical frameworks for analogy and analogical reasoning are based on this assumption which will be questioned in this section. The predominant and most prominent of these frameworks are Gentner's "Structure-Mapping Theory" and Holyoak's pragmatic approach.

2.1 Common characteristics of theoretical frameworks for analogy

The process of analogical reasoning is usually divided into three distinct subprocesses called *access*, *mapping* and *generalisation* (Vosniadou & Ortony, 1989, p.7). From this theoretical viewpoint accessibility of analogy is governed by surface similarities but not by higher-order structures. The propositional structure of analogy is accessed through the mostly unstructured surface of the obviously similar but actually irrelevant. The mapping process is based on mental representations of both base and target. They are mentally represented in their propositional structure whereas the depth and complexity of the base's representation exceeds that of the target's. In principle, mapping is seen as a comparison of similarities between both representations. In Gentner's approach the focus is predominantly on carry over of propositional structures while Holyoak's (1985) pragmatic structure mapping approach also takes account of contextual factors. However, both approaches share the view that mental representations of propositional structures of base and target are crucial for analogical reasoning. The mode of drawing conclusions from base to target is essentially based on formal logic and the logical equivalence of features in both representations (Gentner, 1989, p.210). Moreover the learner making use of an analogy draws on means of assessing the heuristic power of conclusions drawn by analogy. Mapping is exclusively conceptualised as a transfer from base to target. In other words, the symmetrical nature of the analogical relation is not explicitly employed. Generalisation is a matter of abstraction targeting a new concept as the

learner builds a new concept by leaving aside the irrelevant surface similarities and the non-analogical structural features.

2.2 Post-festum and heuristic analogies: a kernel of a revised theoretical approach

Each time analogies are involved in the understanding of complex and abstract subject matter they potentially serve different objectives. Analogies may function as a means of illustrating abstract concepts and principles vividly. Their communicational intent may be to impart insight and knowledge. In this way, analogies become an important educational tool in class. If a teacher generates an analogy for these purposes the aim of analogy construction is just as well known to him as the concept the analogy is supposed to illustrate or the theory that it is based on. A student, however, who is confronted with a conceptual analogy finds himself or herself in a completely different situation. He or she is totally ignorant of the scientific concepts and principles at which the analogy aims. Therefore the teacher's use of analogy (e.g., generating an analogy in order to illustrate a concept well known to him or her) will presumably be different from the student's (e.g., searching for an analogy in order to approach an unknown phenomenon). Consistent distinction between the teacher's *post-festum analogy* and the learner's *heuristic analogy* builds the framework for our theoretical considerations. The two terms denote the different psychological processes the teacher and the student go through when making use of analogies. (Post festum [lat.]. Phrase denoting that something happens afterwards. In our case, generating an analogy after having constructed the conceptual framework at which an analogy aims.) This distinction is missing in present theories of analogical reasoning (Gentner & Medina, 1998). In the subsequent section we would like to outline a few arguments in favour of this dichotomy which touch on the epistemological, communicational and psychological dimensions of analogy.

2.3 The epistemological dimension of analogy

The heuristic aim of analogy is a theoretical approach to a yet unknown phenomenon. The target is the ontic object that produces this phenomenon. Generally speaking analogy is the solution to the problem that Plato posed in his famous *paradox of the meno* and which Bereiter (1985) called the *learning paradox*. It is our thesis that analogy is a central way of leaping the epistemological gap between the already known and the yet unknown. In a "pretheoretical phase" a heuristic analogy allows the formation of hypotheses without having constructed a settled theory. The base is some sort of "proto-theory" that is available as a way to model the target. This holds as well phylogenetically for scientific enquiry as it does ontogenetically for individual learning. As a heuristic means, the analogy enables the scientist to plan experiments and to generate expectations with respect to their outcome by tentatively transferring theoretical aspects from the base to the target

domain. In a similar way, students are enabled to make theory laden observations when confronted with yet unknown phenomena. Currently employed theoretical approaches to analogical reasoning presuppose that the learner is able to evaluate the quality of his or her analogy based inferences. Gentner's structure mapping theory for example proposes a so called *systematicity principle* conveying analogical matches of high validity (Gentner, 1989). From our point of view these considerations pay only limited attention to the creative aspect of analogical reasoning. A heuristic analogy aims at the construction of analogical relations rather than the detection of objectively and antecedently existing correspondences. It is deeply utilitarian in the sense that it treats some unknown target phenomenon as if it were quite like the base in order to guide empirical action and to yield well-directed hypotheses predicting its outcomes. Or to put it in simple words: analogical transfer does not answer questions, it helps to ask questions. The answers to these questions are beyond analogy. In the case of scientists, they are solely given by experiment. The student relies on observations as well as on the teacher and classmates.

As a proto-theory the heuristic analogy can be seen as a way of overcoming the epistemological chasm between the familiar and the radically new knowledge. If an analogy has proven to be applicable and powerful, it contributes to a modelling of the target phenomenon or the construction of a target concept, either of them paving the way to a comparison of similarities between base and target. Following the above terminology this solely holds for post-festum analogies. If the target concept is not yet canonical or unknown to an individual learner, analogy is not a matter of comparison. Or to put it the other way round: similarity is not the starting point of a heuristic analogy, it is the structure at which the process of analogical reasoning aims. This disagrees with Gentner and Medina (1998) as well as Holyoak (1985) who consider analogical reasoning to be a process of comparing base and target in general and regardless of what the mode of analogy construction is.

It is commonly assumed that analogy is a form of induction (Holyoak & Koh, 1987; Seel & Dinter, 1991). Opposed to this, Bunge (1973) points out that analogies cannot be a source of inductive inferences. Induction is a method to test hypotheses. Heuristic analogies do not test hypotheses antecedently but allow them to develop. Analogical transfer is not amenable to any theoretical systematisation. There is no such thing as the logic of analogy. This in fact makes analogical transfer error prone at its heart. Bunge admits that analogies in fact may be powerful tools but he also claims that "a negative history of science, one recording failures rather than successes, might show that analogies are as often misleading as they are fruitful" (p.126). It is none the less obvious that in the history of science the successful and prolific use of analogies is prominent. Fourier's theory of heat conduction as well as Carnot's theory of the heat engine are based on an analogy to the flow of water. Fourier's theory then was adopted by Ohm when he developed his theory of electricity flow. Huygens developed his wave-theory of light by analogy to the already established wave-theory of sound (Mach, 1910). To treat the phenomenon of light as if it were quite like sound-waves is different from comparing sound- and light-waves. Only in the latter case can you rely on an empirically tested wave-theory of light, which allows you to estimate post festum the soundness of analogical transfer. In the case of heuristic analogies inferences lead to the

assumption that something might be the case. Inductive inferences show that something is actually reliable. The nature of heuristic analogical transfer is associative, as Mach (1920, pp.15-16) pointed out. Association though, is a creative process that extends beyond normativity.

2.4 The communicational dimension of analogy

Especially in educational contexts, analogies also serve a communicative function. This implies that analogies can be treated as signs in a semiotic sense. Gentner (1983) as well as Holyoak (1985) implicitly presuppose the idea of analogies as signs. They both share a representational conception of signs (Wilbers, 1999). According to a representational semiotics, analogies work as a means of communication in class because they represent certain ideas, concepts, notions or models. Tracing back to the work of Morris (1955), representational semiotics regards signs as three-dimensional objects, which have a semantic, a pragmatic and a syntactic dimension. Basically the theoretical approaches of Gentner and Holyoak emphasise different facets of analogies as signs. Gentner stresses the syntactic aspect of analogy whereas Holyoak highlights its pragmatics. Within a representational framework communication is regarded as the transmission of information passed on to the other by means of signs which function as representations. Applied to the use of analogies in class this means: the teacher "wraps" a certain idea or concept into a conceptual analogy and thereby encodes information. By unwrapping or decoding the information the students receive and thus learn the ideas and concepts at which the analogy aims. From this point of view analogies are "containers" for the purpose of transporting ideas. This of course meets the information processing model of cognitive psychology held by Gentner as well as Holyoak. From a constructivist perspective a model of communication like this is problematic. Signs do not contain information. They do not work on the basis of packing, sending and unpacking conceptions and ideas.

From the viewpoint of an instrumental semiotics (Wittgenstein, 1969; Keller, 1995) the primary function of signs is to be interpreted. Analogies work as a means of communication in class because they can be interpreted by students. Thus communication is an inferential process. Basically two different kinds of inferences are involved in communicational processes (Keller, 1995, pp.113-114). If we are confronted with radically new material (word, phrase, metaphor, analogy, etc.) we tend to draw associative conclusions. If we face well known material, which is so to speak, "lexical" to us we draw rule-based conclusions. In class, analogies are an instrument of teaching and learning scientific concepts. If students make use of heuristic analogies, they have not yet formed the concepts that shall be learned via analogy. Conceptual analogies are a learning aid that they can make use of by making associations. Via associative inferences they successively form analogical relations which are conceptually generalised in the course of the heuristic use of analogy. Beyond the heuristic analogy analogical inferences gain a new quality. The user of an analogy is now able to make rule-based inferences from an abstract concept the analogy represents to commonalities between base and target. The

analogy is not a heuristic tool anymore and has turned into a post-festum analogy that serves communicational and explicative purposes rather than heuristic ones. From now on propositionally based knowledge that has emerged from the heuristic use of analogy guides analogy use. From a heuristic perspective analogical inferences are associations, whereas from a post-festum perspective they are rule-based inferences. In recent years Gentner and Medina (1998) have highlighted the significance of rule-based processing for analogical reasoning. It appears that in Gentner's approach the post-festum-perspective is somehow dominant. Further hints to this thesis can be found in the subsequent section.

2.5 The psychological dimension of analogy

Currently employed theoretical approaches emphasise the role of propositionally based knowledge in analogical reasoning. The above considerations suggest that propositionally based knowledge is solely employed in the construction of post-festum analogies. The rule-based inferences of the post-festum analogy operate on propositional structures. In accordance with various other authors we expect non-propositional knowledge based on visual imagery to be of the essence in the heuristic use of analogies. Zeitoun (1984) underlines the significance of visual imagery for analogical reasoning and mental images, so mental models seem to be of the essence. Clement (1993) takes regard of abstract imagery that he calls *intuitive schemata*. Sfard (1994) shares the view that *image schemata* are crucial for analogy use while Lakoff and Johnson (1980) and Johnson (1987) arrive at equivalent terms with respect to metaphors using the notion of *embodied schemata*. Taking a cognitive point of view, analogy hinges on pictorial rather than on propositional elements of cognition, such as intuitive schemata, mental images, mental models, etc.

From a developmental perspective the way analogies are constructed either depends on general cognitive abilities (and thus age) or expertise in the field in question. According to Vosniadou (1989) it is not the mechanisms of analogical transfer that develops but the conceptual structures on which they operate. She thereby opposes Gentner (1989, p.223) who assumes "a developmental shift from attributional focus to relational focus in production, choice and rating of analogy interpretations" (*relational shift*) from childhood to adulthood. Our theoretical considerations propounded in this chapter, support Vosniadou's thesis. The mode of analogy production changes if the concept the analogy represents has already been formed. This basically holds for experts as well as for novices. It applies phylogenetically to the development of conceptual systems and models in scientific inquiry and ontogenetically to individual learning processes in science classes. The consistent distinction of *post-festum* and *heuristic analogies* expresses this on a conceptual level. With regard to:

1. epistemology, students' heuristic analogies are not a matter of comparison.
2. semiotics, they are not a product emerging from the transmission of information from the teacher to the students.
3. psychology, they are not a process based on propositional knowledge.

We already hinted at the fact that Gentner's structure-mapping approach presumes various features and mechanisms of analogical reasoning in general that strongly relate to our notion of post-festum analogies. Superordinate concepts and conceptual systems make the source and target examples which can be compared to each other. Relevance and appropriateness of analogical transfer may be assessed on the background of conceptual generalisations. Propositional knowledge and rule-based inferences appear to be the very essence of post-festum analogies. But in order to understand the use of analogies in science learning, heuristic analogies should be taken into consideration.

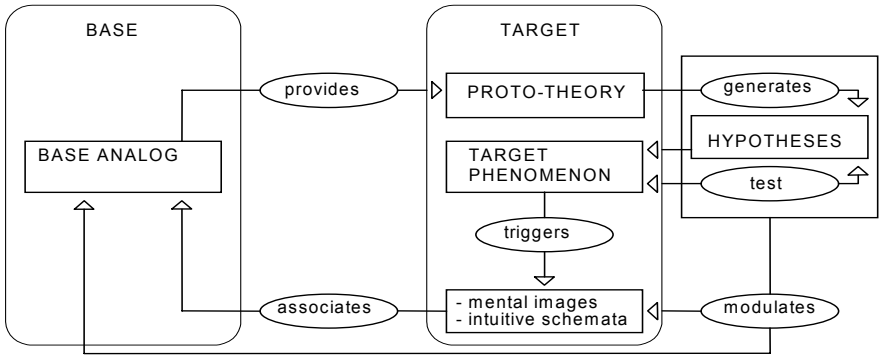

Figure 2. The heuristic analogy: a model of analogical reasoning

Both Gentner's structure mapping and Holyoak's pragmatic approach consider propositionally based knowledge as a starting point for analogy use. As an alternative our revised model of analogical reasoning (Figure 2) claims that intuitive schemata and mental models spontaneously generated by the students when first confronted with the target phenomenon are of the essence in analogy use. They lead to a preliminary associative link between target and base. The subsequent process of analogy construction is guided by these spontaneously generated associations. Or to put it the other way round: the analogy is a means of constructing (propositionally based) hypotheses on the basis of (image like) mental models and intuitive schemata triggered by the target phenomenon. This process of analogy construction which serves a heuristic exploration of the target draws on a better known base analog which provides some proto-theory for the yet unexplored target. This implies that analogies are more of a tool to bring about hypotheses instead of supporting them. The support of hypotheses is a matter of empirical testing and beyond analogy use (Bunge, 1973).

3. A STUDY IN THE DOMAIN OF LIMITED PREDICTABILITY OF CHAOTIC SYSTEMS

The above views of differentiating heuristic and post-festum analogies emerged from the analysis of students' learning processes when they investigated a chaotic pendulum. The major results are summarised in the following to illustrate the significance of the model presented in Figure 2.

A variant of the *teaching experiment* proposed by Steffe and D'Ambrosio (1996) was employed. These teaching experiments draw on Piagetian critical interviews where the interview situation is deliberately turned into a teaching situation from time to time. Instead of interviews with single students a small group setting of four students was employed. Twelve groups of four students (German Grammar school; average age 16) were "interviewed" in two sessions of about 120 minutes each. All sessions were video-taped and transcribed.

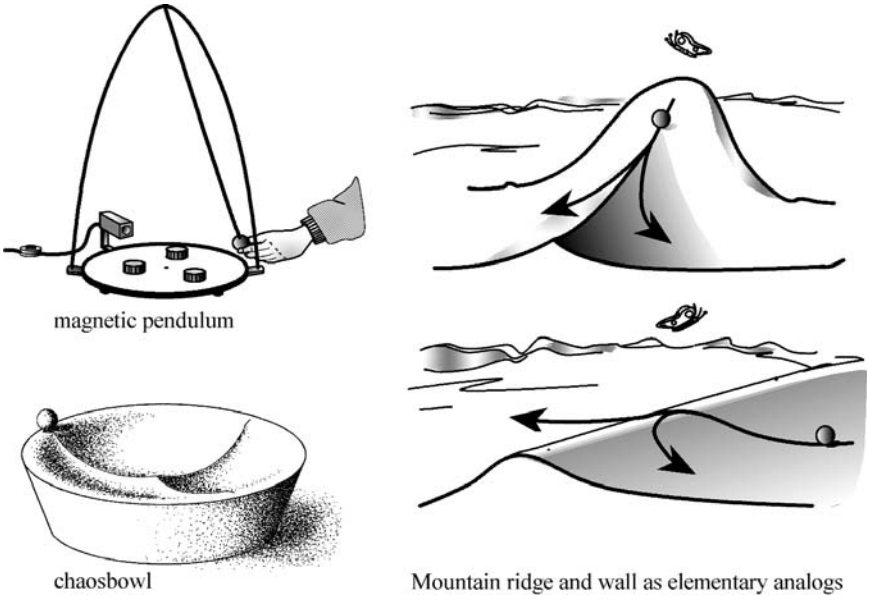

Figure 3. The chaotic pendulum and analogies to explain its behaviour

The focus of the sessions was on investigating a magnetic pendulum as shown in Figure 3. Even if the pendulum is repeatedly released from the same starting position the trajectories diverge and one cannot possibly predict the magnet over which the bob will come to a rest. Briefly put, its chaotic behaviour is caused by zones of unstable equilibrium (the Y-like figure between the magnets where the magnetic forces to the right and left balance out) that are passed by the pendulum bob several times. Such sensitive zones are typical of chaotic systems. The analogical relations investigated by students represent the unstable equilibrium in

various artefacts such as a chaos bowl or dice. Students are given much time in quasi-open-inquiry sessions to construct their understanding of the magnetic pendulum. At a certain point of time the two elementary analogies shown in Figure 3 (ridge and wall situation) are presented as drawings. A computer simulation program which allows students to investigate the behaviour of the pendulum in an ideal world is also employed. It offers the opportunity to inquire into its motion under identical starting conditions and without noise disturbing its motion.

The corpus of data comprises video-transcriptions and all artefacts produced by the students during class. There also were homework assignments between the two sessions. Standard methods of qualitative content analysis were used (Guba & Lincoln, 1989). A particular focus was on constructing students' intuitive schemata about the zones of unstable equilibrium. Another focus was on their ways of talking (i.e., their dictions) about chaotic motion. These schemata and dictions were interpreted as hints to mental images the students held with respect to the dynamic of chaos.

4. INTERPRETATIONS

The study confirms that analogies *can* be powerful tools in guiding students towards an understanding of the principle of limited predictability. The theoretical and methodological setting of the study allowed us to construct a fine-grained picture of analogy use, that is, a micro-level description of the role of analogies in learning about chaos theory. In accordance with our theoretical considerations we focused our data analysis on the role of intuitive schemata and mental images for analogy construction and the heuristic use of analogies. Indeed, the intuitive schemata students hold of the notion of unstable equilibrium and the mental images of chaotic motion seem to play a crucial role in analogy use. They both directed the ways the students in our study made use of presented conceptual analogies (e.g., Figure 3).

Space limitations, however, prevent the presentation of references from our data which substantiate the claims made in the subsequent section. Further details and abundantly discussed examples from our data are presented in Wilbers (1999). Key features of the intuitive schemata and dictions derived from the study are outlined below. These are then used to suggest processes of analogical reasoning.

4.1 Intuitive schemata of unstable equilibrium

Data analysis led to the following types of students' intuitive schemata of unstable equilibrium. Every schema includes a particular conception for explaining limited predictability.

- *Zones of decision.* When the system encounters a sensitive zone it has to decide where to go so to speak. Its decisions are random and hence non-predictable.

- *Neutral zones.* There are no forces acting on the ball: Hence small disturbances, predominantly those occurring when the ball is in a neutral zone, determine its

future path. As it is impossible to foretell the disturbances the future path is unpredictable.

- *Dividing lines.* Many students are of the conviction that the lines of unstable equilibrium (forming the Y-like lines in the case of the magnetic pendulum) are basically borderlines for the fields of the single magnets, where the one field ends and the other begins. It is interesting that chaotic behaviour does not need any explanation for the students holding this view. It is presupposed in order to explain the random sequence of target magnets. Accordingly the analogies provided are not used in the intended way (for an explanation of chaotic motion).

- *Zones of topple over.* In such a zone, the direction of an object's further motion is random. Chance alone is seen as a generating "force" for future behaviour.

- *Sensitive zones.* This intended view includes that small changes of starting conditions and small disturbances result in small changes of the path the pendulum bob as it passes the zones of unstable equilibrium. If one compares two subsequent paths they more and more deviate when the sensitive zones are passed. At one of the passages it happens that they totally deviate. Only a small number of students in our, study was able to proceed this far in their understanding.

4.2 Dictions according to the dynamic of systems

Second, we analysed students ways of talking about a system's dynamic. We identified the following patterns of diction:

- *Static.* Explanations include the starting point and the target magnet but not any discussion of the trajectory of the moving pendulum bob.

- *Animistic.* Especially in the beginning a remarkable number of students use animistic dictions. However, they do not appear to hamper understanding but merely serve as first heuristics.

- *Dynamic - local.* Among the dictions that include arguments concerning the motion of the pendulum bob there are several students focusing on the behaviour at certain points, e.g., in zones of unstable equilibrium. Local arguments are either animistic or include force arguments (which are often not in complete accordance with the scientific concept).

- *Dynamic - global.* This diction is the intended. Two or more trajectories are compared. Main emphasis is given to the significance of local changes for the development of trajectories.

4.3 Processes of analogical reasoning

The particular intuitive schemata and dictions deeply influence the processes of analogical reasoning we observed. With regard to our attempts towards a revised theory of analogical reasoning we would like to emphasise the following findings:

- Gentner's theory holds that access to analogies as learning aids is predominantly facilitated by surface features. In contrast, a number of cases in our studies show that access is also possible via deep structure similarities between base and target.

- Gentner also emphasises the key role of propositional representations of base and target in the mapping process. There is much evidence in our data that such mapping often does not come about even if students are familiar with the base and should understand it in the appropriate manner. The essential key to engagement in a mapping process in Gentner's sense appears to be students' mental models and dictions of base and target. If students exhibit different frames of mental models regarding the unstable equilibrium and employ different dictions describing a system's motion for base and target, transfer does not take place.

- The mapping process from the perspective of Gentner's and Holyoak's theory of analogical reasoning is exclusively a process from base to target. In the present study and previous studies students usually make use of the symmetrical nature of the analogy relation. In other words, they often switch the roles of base and target, i.e., view the base from the perspective of the target and vice versa.

- Approaches of analogy use in instruction usually hold that intimate familiarity with the base is essential (Duit, 1991). In accordance with findings by Corkill and Fager (1992) there are several cases in our data of students who are familiar with the base and nonetheless do not engage in a mapping process. As mentioned, this occurs if students view base and target within different mental models and dictions and hence do not see the potential explanatory power of the base with regard to the target. But this may also happen if base and target are seen from the perspective of the same (or at least similar) mental models and dictions. In this case students may be pleased with the similarities between base and target they constructed and hence do not feel the need for further search of similarities.

5. DISCUSSION

The model of heuristic analogies (Figure 2) is grounded on theoretical considerations as outlined in this chapter as well as on pilot studies. It has provided a viable framework for the analysis of the present study which is based on 12 teaching experiments with 48 students in the domain of understanding chaotic behavior. Moreover it is supported by the findings of previous learning process studies in the same domain (Duit, Komorek, & Wilbers, 1997; Duit, Komorek & Wilbers, 2001). Nonetheless it is preliminary. Further research is necessary to investigate its validity for analogy based learning processes in general. Its applicability beyond the field of learning about chaos theory has not yet been tested. Moreover the data corpus it has been applied to so far is very limited. If further studies support the model, significant changes of instructional strategies of analogy use like Glynn's (1989) "Teaching-with-Analogies model" (TWA) or the FAR-Guide by Treagust, Harrison and Venville (1996) are necessary. Both TWA as well as FAR are instructional models that presuppose structure mapping as a learning model. Basically, they suggest that all teachers need to do is analyse an analogy in class merely on a propositional level from base to target in succession. The significance of non-propositional knowledge is not taken into account nor is the symmetric nature of analogy considered important.

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