CHAPTER 8

CAN STOCK-SPECIFIC SUSTAINABILITY CONSTRAINTS BE JUSTIFIED?

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Abstract. We show that the Suppes-Sen grading principle leads to stock-specific sustainability constraints in a class of resource models, provided that the resource is renewable or utility is derived directly from the resource stock. Decreasing the resource stock is not compatible with Suppes-Sen maximality, unless a smaller stock leads to higher natural growth.

1. INTRODUCTION

During the last decade sustainability has become one of the main issues in environmental economics and policy. Even though there exists a multitude of definitions of sustainability, they all boil down to the idea that living conditions on earth should not become worse in the course of time. Consequently, sustainability has in many instances been interpreted as a postulate to keep stocks of natural resources—as part of the whole vector of capital stocks—intact. This does not only conform to common sense but might also be justified from an economic perspective when the natural resource cannot be substituted by man-made capital. With respect to forests, this holds true from an instrumental as well as from a moral perspective. On the one hand, forests are indispensable as a source of biodiversity, and as sinks for carbon emissions—being important for climate protection—as well as a supplier of amenity value and raw material for the pulp and paper industry. On the other hand

people—feeling responsible for the preservation of the nature in itself—will also attribute to forests some kind of "existence value".

In economic theory, however, applying the usual discounting criteria to standard renewable resource models will in many cases have adverse effects on sustainability in a stock specific sense: Paths that are optimal w.r.t. to discounted utilitarianism will often lead to a deterioration of resource stocks—at least when the discount rate is sufficiently high.

Using different specific models Heal (1998, 2001) has analysed how such undesirable consequences can be avoided. In particular he has shown that two features are favorable for having non-decreasing resource stocks along optimal paths:

- Utility is not only derived from the flow of resource extraction, but also directly from the resource stock itself.

- Intertemporal paths are evaluated by means of social preferences—like undiscounted utilitarianism (in the form of overtaking) and Rawlsian maximin—that entail equal treatment of all generations. Such equal treatment corresponds to what is referred to as the Weak Anonymity condition in the social choice literature.

In this chapter we extend Heal's (2001) analysis by showing how stock-specific sustainability constraints can be obtained from rather weak ethical axioms. By combining Weak Anonymity with the uncontroversial Strong Pareto condition, the so-called Suppes-Sen grading principle is obtained (Sen, 1970; Suppes, 1966). We show that the Suppes-Sen grading principle leads to stock-specific sustainability constraints, provided that the resource is renewable or utility is derived directly from the resource stock. Under this provision, decreasing the resource stock contradicts Suppes-Sen maximality, unless a smaller stock leads to higher natural growth. Hence, there is an important class of models where extraction leading to a stock smaller than the one corresponding to *maximal sustainable yield* is incompatible with the Suppes-Sen grading principle. By starting from basic ethical axioms for intergenerational social preferences, the analysis of this chapter yields a new justification for stock-specific sustainability constraints in general, and—when applied to forests—for limits on deforestation in particular.

Within the framework of social choice theory, we have in Asheim, Buchholz, and Tungodden (2001) justified sustainability by means of the Suppes-Sen grading principle. In models that satisfy a certain productivity condition, which we refer to as "Immediate Productivity", the set of Suppes-Sen maximal utility paths is shown to equal the set of non-decreasing and efficient paths. This result cannot, however, be applied in the present setting, since none of these models considered in this chapter satisfies "Immediate Productivity". Nevertheless, it turns out that the Suppes-Sen grading principle leads to stock-specific sustainability constraints.

By deriving normative conclusions concerning resource management from *incomplete* social preferences like the Suppes-Sen grading principle, the motivation for this chapter is similar to the previous chapter by Mitra (2005). While Mitra

(2004) weakens a criterion satisfying both Strong Pareto and Weak Anonymity—namely undiscounted utilitarianism in the form of overtaking—by assuming that only paths coinciding beyond some finite point in time are comparable, we go a step further by analyzing social preferences satisfying nothing but Strong Pareto and Weak Anonymity.

We recapitulate in Section 2 the analysis of Asheim et al. (2001), introduce in Section 3 the class of models considered, and show in Section 4 under what conditions the Suppes-Sen grading principle leads to stock-specific sustainability constraints in these models. Since the models abstract from important features of real-world economies, the significance of these results is discussed in the concluding Section 5. Proofs are contained in Section 6.

2. THE SUPPES-SEN GRADING PRINCIPLE AND SUSTAINABILITY

There is an infinite number of generations $t = 1, 2, \dots$. The utility level of generation t is given by u_t, which should be interpreted as the utility level of a representative member of this generation. Assume that the utilities need not be more than ordinally measurable and level comparable.

A binary relation R over paths $_1\mathbf{u} = (u_1, u_2, \dots)$ starting in period 1 expresses social preferences over different intergenerational utility paths. Any such binary relation R is throughout assumed to be reflexive and transitive on the infinite Cartesian product $\Re^\infty$ of the set of real numbers $\Re$, where $\infty = |\aleph|$ and $\aleph$ is the set of natural numbers. The social preferences R may be complete or incomplete, with I denoting the symmetric part, i.e. indifference, and P denoting the asymmetric part, i.e. (strict) preference.

In order to define sets of feasible paths, it suffices for the analysis of the present chapter to assume that the initial endowment of generation $t \geq 1$ is given by a stock x_t. A generation t acts by choosing a utility level u_t and a capital stock x_{t+1} which is bequeathed to the next generation $t+1$. For every t, the function F_t gives the maximum utility attainable for generation t if x_t is inherited and x_{t+1} is bequeathed; i.e., $u_t \leq F_t(x_t, x_{t+1})$ has to hold for any feasible *utility-bequest pair* (u_t, x_{t+1}) of generation t. Furthermore, it is assumed that the utility level of each generation cannot fall below 0. If $F_t(x_t, x_{t+1}) < 0$, then the bequest x_{t+1} is infeasible given the inheritance x_t. Hence, generation t's utility-bequest pair (u_t, x_{t+1}) is said to be *feasible* at t given x_t if $0 \leq u_t \leq F_t(x_t, x_{t+1})$. The sequence $_1\mathbf{F} = (F_1, F_2, \dots)$ characterizes the technology of the economy under consideration. Given the technology $_1\mathbf{F}$, a utility path $_t\mathbf{u} = (u_t, u_{t+1}, \dots)$ is feasible at t given x_t if there exists a path $_{t+1}\mathbf{x} = (x_{t+1}, x_{t+2}, \dots)$ such that, for all $s \geq t$, generation s's utility bequest pair (u_s, x_{s+1}) is feasible at s given x_s.

A utility path $_1\mathbf{v}$ weakly Pareto-dominates another utility path $_1\mathbf{u}$ if every generation is weakly better of in $_1\mathbf{v}$ than in $_1\mathbf{u}$ and some generation is strictly better off. A feasible path $_1\mathbf{v}$ is said to be efficient if there is no other feasible path that

weakly Pareto-dominates this path. A feasible path $_1\mathbf{v}$ is said to be R-*maximal*, if there exists no feasible path $_1\mathbf{u}$ such that $_1\mathbf{u}\,P\,_1\mathbf{v}$.

Within this framework, the justification for sustainability in Asheim et al. (2001) rests on one technological assumption and two conditions on the social preferences.

First, we in Asheim et al. (2001) impose the following domain restriction on the technological framework.

Assumption 1 (*Immediate Productivity of* $_1\mathbf{F}$). If $_t\mathbf{u} = (u_t, u_{t+1},...)$ is feasible at t given x_t with $u_t > u_{t+1}$, then $(u_{t+1}, u_t, u_{t+2},...)$ is feasible and inefficient at t given x_t.

This assumption means that if a generation has higher utility than the next, then its excess utility can be transferred at negative cost to its successor. It thus generalizes positive net capital productivity to a setting where utilities need not be more than ordinally measurable and level comparable.

Second, we in Asheim et al. (2001) impose the following two conditions on the social preferences R (with I and P as symmetric and asymmetric parts).

Condition 1 (*Strong Pareto*). *For any* $_1\mathbf{u}$ *and* $_1\mathbf{v}$, *if* $v_t \geq u_t$ *for all* t *and* $v_s > u_s$ *for some* s, *then* $_1\mathbf{v}\,P\,_1\mathbf{u}$.

Condition 2 (*Weak Anonymity*). *For any* $_1\mathbf{u}$ *and* $_1\mathbf{v}$, *if for some finite permutation* π, $v_{\pi(t)} = u_t$ *for all* t, *then* $_1\mathbf{v}\,I\,_1\mathbf{u}$.

The term 'permutation', as used in Condition 2, signifies a bijective mapping of $\{1, 2, \ ...\}$ onto itself, is finite whenever there is a T such that $\pi(t) = t$ for any $t > T$. While Strong Pareto (sometimes referred to as 'Efficiency') ensures that the social preferences are sensitive to utility increases of any one generation, Weak Anonymity (also called 'Equity') can be considered a basic fairness norm as it ensures that everyone counts the same in social evaluation. In the intergenerational context the Weak Anonymity condition implies that it is not justifiable to discriminate against some generation only because it appears at a later stage on the time axis. It thereby rules out discounted utilitarianism.

Define sustainability in the following standard way (cf. the discussion in Pezzey and Toman, 2002, Section 3.1).

Definition 1 (*Sustainability*). Generation t with inheritance x_t is said to behave in a sustainable manner if it chooses a feasible utility-bequest pair (u_t, x_{t+1}) so that the constant utility path $(u_t, u_t,...)$ is feasible at $t+1$ given x_{t+1}. The utility path $_1\mathbf{u} = (u_1, u_2,...)$ is called sustainable given x_1 if there exists $_2\mathbf{x} = (x_2, x_3,...)$ such that every generation behaves in a sustainable manner along $(_1\mathbf{x}, _1\mathbf{u}) = (x_1, (u_1, x_2),$ $(u_2, x_3),...)$.

Hence, a generation behaves in sustainable manner if its utility level can also potentially be shared by all future generations. While any feasible non-decreasing

path is sustainable, it is not in conflict with sustainability that some generation makes a large sacrifice to the benefit of future generations, leading to its own utility being lower than that of its predecessor.

Our justification for sustainability can now be stated.

Proposition 1 (Asheim et al. 2001). *If the social preferences R satisfy Strong Pareto and Weak Anonymity, and the technology satisfies Immediate Productivity, then only sustainable utility paths are R -maximal.*

As noted in the introduction, this result is not applicable to the models that we consider in this chapter since the assumption of Immediate Productivity will not be satisfied. Instead, we will directly consider the conditions of Strong Pareto and Weak Anonymity, which jointly generate the *Suppes-Sen grading principle*.

Definition 2 (*Suppes-Sen grading principle*). The Suppes-Sen grading principle R^S deems two paths to be indifferent if one is obtained from the other through a finite permutation, and one utility path to be preferred to another if a finite permutation of the former weakly Pareto-dominates the other.

Strong Pareto and Weak Anonymity generate the Suppes-Sen grading principle R^S in the following sense: It holds that

- $_1\mathbf{v}\, I^s{}_1\mathbf{u}$ implies $_1\mathbf{v}\, I\,_1\mathbf{u}$ and

- $_1\mathbf{v}\, P^s{}_1\mathbf{u}$ implies $_1\mathbf{v}\, P\,_1\mathbf{u}$,

if and only if the social preferences R satisfy Strong Pareto and Weak Anonymity.

3. A CLASS OF MODELS

Consider a class of models, where consumption is derived from resource extraction, where the resource may be renewable, and where, following Krautkraemer (1985), utility may be derived directly from the resource stock. In the framework of Section 2, we have that F_t is independent of time t and given by:

$$F(x_t, x_{t+1})\begin{cases} = u(x_t + g(x_t) - x_{t+1}, x_t)\ \text{if}\ \ x_t \geq 0\ \ \text{and}\ \ x_t + g(x_t) \geq x_{t+1} \geq 0, \\ \qquad\qquad\ < 0\ \text{otherwise,} \end{cases}$$

indicating that feasibility at time t requires that $x_t \geq 0$ and $x_t+g(x_t) \geq x_{t+1} \geq 0$, that $c_t = x_t+g(x_t)-x_{t+1}$ is the consumption at time t , and that x_t is the resource stock at time t .

Assume throughout that $u : \mathfrak{R}_+^2 \to \mathfrak{R}$ is a continuously differentiable and quasi-concave utility function that assigns utility $u(c,x)$ to any non-negative consumption-amenity pair and satisfies:

$$u(0,0) = 0, \quad u_c > 0 \text{ if } c > 0, \quad \text{and} \quad u_x \geq 0 \text{ if } x > 0$$

Moreover, assume throughout that $g : [\,0, \bar{x}\,] \to \Re_+$ is a continuously differentiable natural growth function that assigns non-negative natural growth to any stock in $[0, \bar{x}]$ and satisfies:

$$g(0) = 0 \quad \text{and} \quad g(\bar{x}) = 0.$$

Four different models are obtained by considering combinations of the following four assumptions.

Assumption 2 (*No resource amenities*). $\forall c \geq 0, \forall x > 0, u_x = 0$.

Assumption 3 (*Positive resource amenities*). $\forall c \geq 0, \forall x > 0, u_x > 0$.

Assumption 4 (*No natural growth*). $\forall x \in [0, \bar{x}], g(x) = 0$.

Assumption 5 (*Positive natural growth*). The natural growth function is continuously differentiable and strictly concave and satisfies $\forall x \in (0, \bar{x}), 0 < g(x) \leq \bar{x} - x$.

The restriction of Assumption 5, namely that $g(x) \leq \bar{x} - x$, means that the stock cannot grow beyond its natural biological equilibrium and is satisfied if g' is bounded below and the period length is small enough. We follow Heal (2001) by representing the renewable resource by means of a biomass model, realizing that such modelling is only in special cases adequate for forest management.

Since Assumptions 2 and 3 are mutually exclusive, and so are Assumptions 4 and 5, the following four models are obtained.

Model 1 (Cake-eating) satisfies Assumptions 2 and 4.

Model 2 (Renewable resource) satisfies Assumptions 2 and 5.

Model 3 (Non-renewable resource yielding amenities) satisfies Assumptions 3 and 4.

Model 4 (Renewable resource yielding amenities) satisfies Assumptions 3 and 5.

These are the models that Heal (2001) investigates. In additional to considering the applicability of the Chichilnisky (1996) criterion, he applies discounted utilitarianism, undiscounted utilitarianism (in the form of overtaking), and Rawlsian maximin as social preferences over different intergenerational utility paths. Undiscounted utilitarianism and lexicographic versions of Rawlsian maximin satisfy both Conditions 1 (Strong Pareto) and 2 (Weak Anonymity), while discounted utilitarianism satisfies Strong Pareto, but not Weak Anonymity.

It does not come as a surprise that in Model 1 there is no way to have sustainability as a optimal solution, independently of the social preferences used. In Models 2–4, however, all social preferences considered by Heal (1998, 2001) may lead to optimal solutions in which stock specific sustainability constraints are obtained; i.e., in which part of the resource stock is forever kept intact. In the case of discounted utilitarianism, this result holds at least when the discount rate is sufficiently low (and marginal utility of consumption is bounded away from infinity). Therefore, Heal considers that there is no inherent conflict between 'optimality' and 'sustainability'.

Instead of applying specific forms of intergenerational social preferences as Heal does, we here investigate the implications in these four models of imposing the Suppes-Sen grading principle (i.e., the conditions of Strong Pareto and Weak Anonymity), leading to consequences that are shared by undiscounted utilitarianism and Rawlsian maximin, but not necessarily by discounted utilitarianism.

4. APPLYING THE SUPPES-SEN GRADING PRINCIPLE

Proposition 1 entails that the Suppes-Sen grading principle leads to sustainable paths in technologies satisfying the assumption of Immediate Productivity. This result cannot be applied to Models 1–4 since they do not satisfy this technological assumption.

Proposition 2. *Assumption 1 (Immediate Productivity) is not satisfied by Models 1–4.*

The proofs of this and the other results of this section are contained in Section 6.

Moreover, the direct application of the Suppes-Sen grading principle does not yield any restriction on the depletion policy in Model 1, except that the resource stock must be exhausted as time goes to infinity, so that the path is efficient. Hence, the following result is obtained.

Proposition 3. *Consider Model 1 and social preference given by the Suppes-Sen grading principle R^S. A utility path is R^S-maximal if and only if it is efficient.*

Hence, in Model 1 and for any social preferences R satisfying Conditions 1 (Strong Pareto) and 2 (Weak Anonymity), a utility path is R-maximal only if it is efficient.

However, the direct application of the Suppes-Sen grading principle yields a restriction on the depletion policy in Models 2-4, leading to the following stock-specific sustainability constraint.

Proposition 4. *Consider Models 2–4 and social preferences given by the Suppes-Sen grading principle R^S. If the initial stock x_1 satisfies $g'(x_1 + g(x_1)) \geq 0$, then a utility path is R^S-maximal only if $c_1 \leq g(x_1)$ (so that $x_1 \leq x_2$) and $c_1 \leq c_2$ (so that $u_1 \leq u_2$).*

In the case of Models 2 and 4, and for a "small" period length (so that the maximal per period growth $g(x_1)$ is "small" compared to x_1), the condition that $g'(x_1 + g(x_1)) \geq 0$ can be identified with the condition that x_1 does not exceed the stock size corresponding to the maximal sustainable yield (MSY); i.e., the stock size maximizing $g(x)$ over all $x \in [0, \overline{x}]$. Hence, Proposition 4 states, unless the stock exceeds the MSY size so that a smaller stock leads to higher natural growth, further depletion of the stock is incompatible with any social preferences R satisfying Conditions 1 (Strong Pareto) and 2 (Weak Anonymity).

In order to show that the results of Propositions 3 and 4 are not empty, we must establish that there exist R^S-maximal utility paths in the case of Models 2–4. By the following result, such existence poses no problem.

Proposition 5. *Consider Models 2–4 and social preferences given by the Suppes-Sen grading principle R^S. For any initial stock x_1, there exists a R^S-maximal utility path.*

Hence, imposing that the social preferences R satisfy Strong Pareto and Weak Anonymity does not rule out the existence of R-maximal utility paths.

While we through Proposition 4 provide conditions that are necessary for R^S-maximal paths in Models 2–4, and through the proof of Proposition 5 give a condition that is sufficient for R^S-maximality in these models, we do not have available conditions that are both sufficient and necessary and thus characterize the set of R^S-maximal paths in these settings.

5. THE SIGNIFICANCE OF THE RESULTS

Although the results of the previous section indicate that the seemingly weak and uncontroversial axioms of Strong Pareto and Weak Anonymity entail that a resource stock should not be further reduced if smaller than the size corresponding to MSY, one must keep in mind that the models abstract from factors that are important in the real world.

- The models of Section 3 do not have any production activities other than resource extraction. If production also depends on reproducible capital and the produced output can be split between consumption and accumulation of reproducible capital, then along any Pareto-efficient path there can be no profitable arbitrage possibilities between the two kinds of capital goods, i.e., in any period holding a stock of the natural resource must be as profitable as holding a stock of the reproducible capital. As along a Suppes-Sen maximal utility path the rate of productivity of reproducible capital may well be positive, it therefore follows that for, e.g., a renewable resource that does not yield amenities (cf. Model 2), the marginal rate of growth of the resource stock has to be positive, too. This will reduce the resource stock strictly below its msy size.

- In the real world, natural capital consists of many different types of resources. Since the simple models of Section 3 include only one resource, the results obtained in these models say nothing about how sustainability constraints should be imposed if there are multiple resources. Even though it is quite possible that models with multiple resources would imply sustainability constraints for some or all of these resources, this will naturally depend on how such models are formulated.

- Finally, real world resource stocks are geographically distributed. Since the simple models of Section 3 has no geographical dimension, the results obtained in these models say nothing about how sustainability constraints should be applied to a setting where resource stocks are geographically distributed. Even though it is quite possible that models where resources are geographically distributed would imply sustainability constraints in some or all of the regions, this will also depend on how such models are formulated.

Still, the models suggest that calls for resource conservation and sustainability based on ethical intuition may be provided with a more solid normative underpinning through basic axioms like Strong Pareto and Weak Anonymity.

6. PROOFS

Proof of Proposition 2. We must show that Assumption 1 is not satisfied in Models 1–4.

Model 1: Assume $_t\mathbf{u} = (u_t, u_{t+1}, ...)$ is feasible at t given x_t with $u_t > u_{t+1}$. Then $(u_{t+1}, u_t, u_{t+2}, ...)$ is feasible at t given x_t, but is not inefficient, unless $_t\mathbf{u}$ inefficient.

Model 2: Assume $_t\mathbf{u} = (u_t, u_{t+1}, ...)$ is feasible at t given x_t with $u_t > u_{t+1}$ and $g'(x_t) < 0$ and $g'(x_{t+1}) < 0$. Then $(u_{t+1}, u_t, u_{t+2}, ...)$ is not even feasible at t given x_t, unless $_t\mathbf{u}$ inefficient.

Model 3: Consider the following explicit counterexample. The utility function

$$u(c, x) = \begin{cases} 2c + x & \text{if } c \leq 8 \\ \frac{63}{4} + (c - \frac{127}{16})^{\frac{1}{2}} + x & \text{if } c > 8 \end{cases}$$

is continuously differentiable and satisfies Assumption 3. Let $x_1 = 20$, $x_2 = 14$, and $x_3 = 6$, and let $_3\mathbf{u} = (u_3, u_4, ...)$ be efficient at time 3 given $x_3 = 6$. We have that $c_1 = x_1 - x_2 = 6$ and $c_2 = x_2 - x_3 = 8$, so that $u_1 = 2*6 + 20 = 32$ and $u_2 = 2*8 + 14 = 30$. Decreasing utility at time 1 to u_2 entails decreasing consumption at time 1 to $\tilde{c}_1 = 5$ so that $v_1 = 2*5 + 20 = 30 = u_2$ and $\tilde{x}_2 = 15$. Since $_3\mathbf{u} = (u_3, u_4, ...)$ is efficient at time 3 given $x_3 = 6$, we can only increase con-

sumption at time 2 to $\tilde{c}_1 = \tilde{x}_2 - x_3 = 15 - 6 = 9$ to keep the remaining utility path unchanged. However,

$$v_2 = \tfrac{63}{4} + (9 - \tfrac{127}{16})^{\frac{1}{2}} + 15 = \tfrac{123}{4} + (\tfrac{17}{16})^{\frac{1}{2}} < 32 = u_1 .$$

Hence, the utility path $(u_2, u_1, u_3 \ldots)$ is not feasible at time 1 given x_1 .

Model 4: The result follows by combining the features of the proofs in the case of Models 2 and 3. □

Proof of Proposition 3. *Only if.* Assume that $_1\mathbf{u} = (u_1, u_2, \ldots)$ is not efficient. Then it follows, since R^S satisfies Strong Pareto, that there exists $_1\mathbf{v} = (v_1, v_2, \ldots)$ such that $_1\mathbf{v} P^S {}_1\mathbf{u}$.

If. Write $u(c)$ since, by Assumption 2, u does not depend on x . Assume that $_1\mathbf{u} = (u_1, u_2, \ldots) = (u(c_1), u(c_2), \ldots)$ is efficient, i.e.,

$$\sum_{t=1}^{\infty} c_t = s_1 .$$

Then any finite permutation of $_1\mathbf{u}$ also satisfies

$$\sum_{t=1}^{\infty} c_{\pi(t)} = s_1$$

and is thus efficient. Hence, there is no $_1\mathbf{v} = (v_1, v_2, \ldots)$ such that $_1\mathbf{v} P^S {}_1\mathbf{u}$. □

The following result is helpful for the proof of Proposition 4.

Lemma 1. *Consider Models 2–4. Let the feasible consumption path $_1\mathbf{c} = (c_1, c_2, \ldots)$ be given with $_1\mathbf{x} = (x_1, x_2, \ldots)$ as the accompanying path of resource stocks. If there exists some time t such that $g'(x_t + g(x_t)) \geq 0$ and $c_t > c_{t+1}$, then there exists a feasible consumption path $_1\tilde{\mathbf{c}} = (\tilde{c}_1, \tilde{c}_2, \ldots)$ satisfying*

$$\tilde{c}_s \begin{cases} = c_s & \text{for} \quad s = 1, \ldots, t-1, t+2, \ldots \\ = c_{s+1} & \text{for} \quad s = t \\ \geq c_{s-1} & \text{for} \quad s = t+1, \end{cases}$$

where the latter inequality can be made strict if Assumption 5 is satisfied. The accompanying path of resource stocks $_1\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, \ldots)$ satisfies

$$\tilde{x}_s \begin{cases} = x_s & \text{for} \quad s = 1, \ldots, t, t+2, \ldots \\ > x_s & \text{for} \quad s = t+1. \end{cases}$$

Proof. The path $_1\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, ...)$ coincides with $_1\mathbf{x}$ up to and including time t. For time $t+1$, it follows that $x_{t+1} = x_t + g(x_t) - c_t$, while

$$\tilde{x}_{t+1} = x_t + g(x_t) - \tilde{c}_t = x_t + g(x_t) - c_{t+1} > x_t + g(x_t) - c_t = x_{t+1}$$

since $\tilde{c}_t = c_{t+1} < c_t$. As $g'(x_t + g(x_t)) \geq 0$,

i. $g(x_{t+1}) < g(\tilde{x}_{t+1})$ if Assumption 5 is satisfied, since $g' > 0$ between $x_{t+1} = x_t + g(x_t) - c_t$ and $\tilde{x}_{t+1} = x_t + g(x_t) - c_{t+1}$ by the strict concavity of g.

ii. $g(x_{t+1}) = g(\tilde{x}_{t+1})$ if Assumption 4 is satisfied, since $g' = 0$ between $x_{t+1} = x_t + g(x_t) - c_t$ and $\tilde{x}_{t+1} = x_t + g(x_t) - c_{t+1}$.

Let $\tilde{c}_{t+1} = c_t + g(\tilde{x}_{t+1}) - g(x_{t+1})$, so that $\tilde{c}_{t+1} \geq c_t$, with strict inequality if Assumption 5 is satisfied. It follows that the resource stock at time $t+2$, $\tilde{x}_{t+2}$, in the alternative path equals the resource stock at time $t+2$, x_{t+2}, in the original path:

$$\begin{aligned}\tilde{x}_{t+2} &= x_t + g(x_t) - \tilde{c}_t + g(\tilde{x}_{t+1}) - \tilde{c}_{t+1} \\ &= x_t + g(x_t) - c_{t+1} + g(\tilde{x}_{t+1}) - (c_t + g(\tilde{x}_{t+1}) - g(x_{t+1})) \\ &= x_t + g(x_t) - c_t + g(x_{t+1}) - c_{t+1} = x_{t+2}.\end{aligned}$$

Hence, it is feasible to keep consumption unchanged from time $t+1$ on. □

Proof of Proposition 4. Assume that the initial stock x_1 satisfies $g'(x_1 + g(x_1)) \geq 0$, but $c_1 > g(x_1)$ or $c_1 > c_2$. We must show that $u(c_1, x_1)$ cannot constitute the initial period of a R^S-maximal utility path.

Model 2: If $c_1 > c_2$, then clearly there exists $t \geq 1$ so that $g'(x_t + g(x_t)) \geq 0$ and $c_t > c_{t+1}$. If $c_1 > g(x_1)$ so that $x_1 > x_2$, then $(x_1, x_2, ...)$ would be decreasing at an increasing pace as long as $(c_1, c_2, ...)$ is non-decreasing. Hence, there exists $t \geq 1$ so that $g'(x_t + g(x_t)) \geq 0$ and $c_t > c_{t+1}$ also in this case. Since Model 2 satisfies Assumption 5, it follows from Lemma 1 that there exists a utility path $_1\mathbf{v} = (v_1, v_2, ...) = (u(\tilde{c}_1), u(\tilde{c}_2), ...)$ that Pareto-dominates and thus, by Strong Pareto, is preferred to

$$(u(c_1), ..., u(c_{t-1}), u(c_{t+1}), u(c_t), u(c_{t+2}), ...),$$

which, by Weak Anonymity, is equally good as

$$(u(c_1), ..., u(c_{t-1}), u(c_t), u(c_{t+1}), u(c_{t+2}), ...),$$

(where we write $u(c)$ since, by Assumption 2, u does not depend on x). By transitivity, the latter utility path is not R^S-maximal given x_1.

Models 3–4: The proof by contradiction consists of two cases.

CASE 1: *There exists $t \geq 1$ so that $g'(x_t + g(x_t)) \geq 0$, $c_t > c_{t+1}$, and $g(x_t) \geq c_{t+1}$.*
By Lemma 1, there exists a feasible consumption path $_1\tilde{\mathbf{c}} = (\tilde{c}_1, \tilde{c}_2, ...)$ derived from
$_1\mathbf{c} = (c_1, c_2, ...)$ by permuting c_t and c_{t+1}, with an accompanying path of resource
stocks $_1\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, ...)$ that coincides with $_1\mathbf{x} = (x_1, x_2, ...)$, except that $\tilde{x}_{t+1} > x_{t+1}$. The
utility path $_1\mathbf{v} = (v_1, v_2, ...) = (u(\tilde{c}_1, \tilde{x}_1), u(\tilde{c}_2, \tilde{x}_2), ...)$ satisfies

$$v_s = \begin{cases} u(c_s, x_s) = u_s & \text{for } s = 1, ..., t-1, t+2, ... \\ u(c_{s+1}, x_s) > u(c_{s+1}, x_{s+1}) = u_{s+1} & \text{for } s = t \text{ since } x_t > x_{t+1}, \\ u(c_{s-1}, \tilde{x}_s) \geq u(c_{s-1}, x_{s-1}) = u_{s-1} & \text{for } s = t+1, \end{cases}$$

where $\tilde{x}_{t+1} \geq x_t$ follows from $\tilde{c}_t = c_{t+1} \leq g(x_t)$. Hence, there exists a utility path
$_1\mathbf{v} = (v_1, v_2, ...)$ that Pareto-dominates and thus, by Strong Pareto, is preferred to

$$(u_1, ..., u_{t-1}, u_{t+1}, u_t, u_{t+2}, ...),$$

which, by Weak Anonymity, is equally good as

$$(u_1, ..., u_{t-1}, u_t, u_{t+1}, u_{t+2}, ...).$$

By transitivity, the latter utility path is not R^S-maximal given x_1.

CASE 2: There does not exist $t \geq 1$ so that $g'(x_t + g(x_t)) \geq 0$, $c_t > c_{t+1}$, and
$g(x_t) \geq c_{t+1}$. This case clearly rules out $g(x_t) \geq c_1 > c_2$; hence, $c_1 > g(x_1)$. Suppose
there exists $t \geq 1$ such that $c_{t+1} \leq g(x_{t+1})$, and let without loss of generality t be the
first time at which $c_{t+1} \leq g(x_{t+1})$, so that $x_{t+1} < x_t \leq x_1$. Then, $g'(x_t + g(x_t)) \geq 0$ and
$c_t > g(x_t) > g(x_{t+1}) \geq c_{t+1}$, by the assumption that $g'(x_1 + g(x_1)) \geq 0$ and the
concavity of g (g is linear under Assumption 4 and strictly concave under
Assumption 5). This is also ruled out.

Hence, in this case there does not exist $t \geq 1$ such that $c_{t+1} \leq g(x_{t+1})$. Then,
since the resource stock is strictly decreasing, but bounded by a non-negativity
constraint, there is some $x^* \geq 0$ such that $\lim_{t \to \infty} x_t = x^*$ and $\lim_{t \to \infty} c_t = g(x^*)$.
Hence, since utility is increasing in c and x, and $g' \geq 0$ if x does not exceed
$x_1 + g(x_1)$, there is some $t > 1$ such that $u(c_s, x_s) < u(g(x_1), x_1)$ for all $s \geq t$.

Consider the alternative feasible consumption path $_1\tilde{\mathbf{c}} = (\tilde{c}_1, \tilde{c}_2, ...)$ satisfying

$$\tilde{c}_s = \begin{cases} g(x_1) & \text{for } s = 1, ..., t-1 \\ c_{s-t+1} & \text{for } s = t, ..., 2t-2 \\ x_{s-t+1} + g(x_{s-t+1}) - x_{s+1} & \text{for } s = 2t-1 \\ c_s & \text{for } s = 2t, 2t+1..., \end{cases}$$

with the accompanying path of resource stocks $_1\tilde{\mathbf{x}} = (\tilde{x}_1, \tilde{x}_2, ...)$ satisfying

$$\tilde{x}_s = \begin{cases} x_1 & \text{for} \quad s = 1,...,t-1 \\ x_{s-t+1} & \text{for} \quad s = t,...,2t-1 \\ x_s & \text{for} \quad s = 2t, 2t+1 \end{cases}$$

To confirm the feasibility of this path, we only have to consider time $2t-1$, where $\tilde{c}_{2t-1} = x_t + g(x_t) - x_{2t} > x_{2t-1} + g(x_{2t-1}) - x_{2t} = c_{2t-1} > 0$ since $x_t > x_{2t-1}$ and $g(x_t) \geq g(x_{2t-1})$. The utility path $_1\mathbf{v} = (v_1, v_2, ...) = (u(\tilde{c}_1, \tilde{x}_1), u(\tilde{c}_2, \tilde{x}_2), ...)$ satisfies

$$v_s \begin{cases} > u_{s+t-1} & \text{for} \quad s = 1,...,t-1 \\ = u_{s-t+1} & \text{for} \quad s = t,...,2t-2 \\ > u_s & \text{for} \quad s = 2t-1 \\ = u_s & \text{for} \quad s = 2t, 2t+1, \end{cases}$$

Hence, there exists a utility path $_1\mathbf{v} = (v_1, v_2, ...)$ that Pareto-dominates and thus, by Strong Pareto, is preferred to

$$(u_t, ..., u_{2t-2}, u_1, ..., u_{t-1}, u_{2t-1}, u_{2t}, ...),$$

which, by Weak Anonymity, is equally good as

$$(u_1, ..., u_{t-1}, u_t, ..., u_{2t-2}, u_{2t-1}, u_{2t}, ...).$$

By transitivity, the latter utility path is not R^S-maximal given x_1. □

Proof of Proposition 5. For given initial stock x_1, define x^* as follows:

$$x^* := \arg \max_{x \in [0, x_1]} u(g(x), x).$$

It follows by the properties of u and g under the assumptions of Models 2–4 that x^* is unique. Since, by definition of x^*, $x_1 \geq x^*$, we have two cases to consider: $x_1 = x^*$ and $x_1 > x^*$.

CASE 1: $x_1 = x^*$. Consider the path

$$_1\mathbf{u} = (u_1, u_2, ...) = (u(g(x_1), x_1), u(g(x_1), x_1)...).$$

Since $_1\mathbf{u}$ has constant utility, it is Suppes-Sen maximal if and only if it is efficient. Therefore, suppose there exists a path $_1\mathbf{v} = (v_1, v_2, ...)$ (with $_1\tilde{\mathbf{x}}$ and $_1\tilde{\mathbf{c}}$ as accompanying paths of consumption and resource stocks) that Pareto-dominates $_1\mathbf{u}$. Without loss of generality we can assume that $v_1 > u_1$. This entails $\tilde{c}_1 > g(x_1)$ so that

$\tilde{x}_2 = x_1 + g(x_1) - \tilde{c}_1 < x_1 = x^*$. By the definition of x^* and the properties of u and g under the assumptions of Models 2–4, it now follows that if $v_t \geq u_t$ for $t = 2, 3, \ldots$, then the per time period depletion of the resource is positive and bounded away from zero (with $\tilde{c}_1 - g(x_1) > 0$ as a lower bound). Thus, since the resource stock must be non-negative, such a path is infeasible. Hence, $_1\mathbf{u}$ is R^S-maximal given x_1.

CASE 2: $x_1 > x^*$. This case can only occur in Models 2 and 4, for which g satisfies Assumption 5. Consider the path $_1\mathbf{u} = (u_1, u_2, \ldots)$, where

$$u_t = \begin{cases} u(x_1 + g(x_1) - x^*, x_1) & \text{for } t = 1 \\ u(g(x^*), x^*) & \text{for } t > 1 \end{cases}$$

By definition, $_1\mathbf{u}$ is Suppes-Sen maximal if and only if it is efficient and there does not exist an alternative path Pareto-dominating a finite permutation of $_1\mathbf{u}$.

To show that $_1\mathbf{u}$ is efficient, suppose there exists a path $_1\mathbf{v} = (v_1, v_2, \ldots)$ (with $_1\tilde{\mathbf{x}}$ and $_1\tilde{\mathbf{c}}$ as accompanying paths of consumption and resource stocks) that Pareto-dominates $_1\mathbf{u}$. However, if $v_s = u_s$ for $s = 1, \ldots, t-1$ and $v_t > u_t$, then $\tilde{x}_{t+1} < x^*$. In line with the proof of Case 1, it now follows that it is infeasible to keep $v_s \geq u_s$ for all $s > t$.

To show that there does not exist an alternative path Pareto-dominating a finite permutation of $_1\mathbf{u}$, it is sufficient to show that there exists no finite permutation of $_1\mathbf{u}$ (since then no path Pareto-dominating such a permutation is feasible either). By Assumption 5, g is strictly concave and satisfies $\forall x \in (0, \overline{x}), 0 < g(x) \leq \overline{x} - x$, and it follows that $x + g(x)$ is a strictly increasing function of x. Since $x_1 > x^*$, we therefore have that $u_1 = u(x_1 + g(x_1) - x^*, x_1) > u(g(x^*), x^*) = u_2 = u_3 = \cdots$. Consequently, any finite permutation amounts to a path $_1\mathbf{v} = (v_1, v_2, \ldots)$ (with $_1\tilde{\mathbf{x}}$ and $_1\tilde{\mathbf{c}}$ as accompanying paths of consumption and resource stocks) where

$$v_s = \begin{cases} u_1 = u(x_1 + g(x_1) - x^*, x_1) & \text{for } s = t \\ u_t = u(g(x^*), x^*) & \text{for } s \neq t \end{cases}$$

and t is some period after period 1. Since x^* is the unique maximizer of $u(g(x), x)$, so that in particular, $v_1 = u(g(x^*), x^*)$ only if $\tilde{c}_1 > g(x_1)$, it follows that $\tilde{x}_t < x_1$. By the properties of u under the assumptions of Models 2 and 4, this implies that $\tilde{c}_t \geq c_1$. Since $x + g(x)$ is a strictly increasing function of x, we have that $\tilde{x}_t + g(\tilde{x}_t) < x_1 + g(x_1)$. Hence, $\tilde{x}_{t+1} = \tilde{x}_t + g(\tilde{x}_t) - \tilde{c}_t < x_1 + g(x_1) - c_1 = x^*$. As we have argued above, it now follows that it is infeasible to keep $v_s = u_s$ for all $s > t$. We have thus shown that there exists no finite permutation of $_1\mathbf{u}$, and consequently, no alternative path Pareto-dominating a finite permutation of $_1\mathbf{u}$.

Hence, $_1\mathbf{u}$ is R^S-maximal given x_1. □

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