CHAPTER FIVE

FORMATION AND DEVELOPMENT OF THEORY IN NON-CLASSICAL SCIENCE

MATHEMATICAL HYPOTHESIS AND ITS EMPIRICAL JUSTIFICATION

The strategies of theoretical investigation do not remain forever given and invariable; they are changing along with the evolution of the science.

Since Bacon and Descartes, philosophy and nature study used to believe that it is possible to find the only true strict way of cognition which could guarantee formation of true theories in any situations and concerning any objects. Foundations of the classical science included this ideal. Changeability and variety of concrete methods were not denied, but the aim of investigator was considered to be a united strategy of theory yielding. It was supposed that first the investigator was to find evident and obvious principles formulated as generalization of experience, and then, on their base, to seek for concrete theoretical laws.

This strategy was believed to be the only true way, the only method which leads to the true theory. As to investigations in physics, they required creation of an integral image of the reality studied, as a preliminary condition for the following employing of mathematical means to describe it.

The development of science in the 20th century has made people to reconsider these methodological attitudes. Even in the late 19th century, when historical changeability of the fundamental principles of science, relativity of their empirical justification accepted by the scientific community (empiriocriticism, conventionalism etc.) was discovered, the first critical observations towards the classical strategy of investigation were made. Certain doubts in the classical methodology as an absolute, reflected in philosophy of that historical period, may be regarded as the preliminary step in the formation of a new paradigm of theoretical cognition. But this paradigm itself was firmly established in science in a great part due to the development of modern quantum-relativisticic physics, the first of sciences that demonstrated non-classical strategies of yielding a theory.

A prominent Soviet physicist, L.I. Mandelstam characterized them in the following way: "Classical physics mostly acted so that determination of links between mathematical magnitudes and real objects preceded equations, i.e., establishing laws. Moreover, the derivation of equations was the main goal because the contents of the magnitudes in advance seemed clear, and scientists sought equations for them... Modern theoretical physics, though not deliberately, but historically it is true, has chosen a different way. It happened by itself. Now first of all we try to guess the mathematical apparatus operating magnitudes meaning of which (at least part) is entirely unclear".[1]

This mode of investigation, which has become domineering in 20th century physics, was connected with a broad application of a special method, which was called mathematical hypothesis or mathematical extrapolation.

General characteristics of this method are as follows. In order to find laws of a new area of phenomena, we take mathematical expressions for laws of a neighboring sphere, which are then transformed and generalized so that we could obtain new correlations between physical magnitudes. The obtained correlations are regarded as hypothetical equations describing new physical processes. After corresponding experimental verification, these equations either get status of theoretical laws, or are rejected as non-fitting to experience.[2]

The characteristic given reflects the most important feature of development of modern physical theories: in contrast to classical patterns, they start as if from top storeys – search for mathematical apparatus, and only when equations of the theory are found, scientists begin to interpret them and look for empirical justification. Though, we probably cannot extract more out of the characteristic of a mathematical hypothesis. Further specification of this characteristic requires that we determine how a mathematical hypothesis is formed in science and what the procedure of its justification is.

Here only first steps have been made yet. First, I should mention S.I. Vavilov's interesting observations about existence of regulative principles (correspondence, simplicity etc.), which give aim and direction to the search of adequate mathematical means.[3] S.I. Vavilov, who introduced the term "mathematical extrapolation", formulated a special group of problems connected with the discussion of the nature of corpuscle-wave dualism. It was said that specificity of mathematical hypothesis as method of today's physical investigation is not the fact that while creating a theory we transfer mathematical means from one field to another (this method has always been used in physics), but mostly in peculiarities of such a transfer itself – in today's mode.

S.I. Vavilov emphasized that mathematical extrapolation (in its modern variation) has appeared due to the fact that visual images, which used to be the basis for creation of mathematical formalism in classical physics, now, in quantum-relativisticic physics have lost their integrity and visuality. The picture of the world taken by modern physics reflects specific features of micro-objects by means of two complimentary representations – corpuscular and wave. Therefore, it looks impossible to work out a unified visual physical model of reality as a preliminary basis for a theory. We have to elaborate a theory concentrating on purely mathematical work connected with reconstruction of equations "dictated" by various analogue images. This is where we can see the unconventionality of mathematical extrapolation of nowadays. "Experience leads to our consciousness reflection on the spheres of the world, which are unfamiliar and alien to a common person. We lack familiar images for visual and model interpretation, but logic... in its mathematical form, still works and introduces order and links in a new, unwonted world".[4]

If we understand mathematical hypothesis this way, we have to ask a question: how does it regard the picture of the world which takes into account the specificity of new objects. It is evident that – in a hidden form – we are dealing with the problem of heuristic picture of the world as a preliminary base for search for adequate mathematical means employing in formulating laws of physics. All these problems need special discussion.

Peculiarities of modern forms of physical picture of the world and their role in putting forward mathematical hypotheses

The specificity of modern pictures of the world may give the impression that they emerge only after a theory has been formed, and so theoretical search nowadays is not directed by their influence.

Though, we may come to conclusions of such kind only after quite prompt consideration of modern investigational situations. More profound analysis discovers that in modern investigation the process of putting forward mathematical hypotheses may also be ruled by ontological principles of the picture of the world.

An example is the establishment of quantum electrodynamics (we'll speak about it in more details in further chapters).

Thus, it is important to emphasize that new strategies of cognition do not cancel the preceding classical models. The latter, though modified, may be reproduced in modern theoretical search as well. Non-classical strategies of investigation may co-exist along with the classical ones, interact with them and appear in a spectrum of variations – from evidently alternative (to the classical models) to hybrid ones, which combine various features of classical and non-classical investigation.

In obviously non-classical situations theories really are created before the new picture of the world appears. And still, the conclusion about disappearance of directional functions of the picture of the world seems hasty. We are to bear in mind two important circumstances.

The first one concerns the process of raising problems, the process which starts construction of fundamental theories. Special relativity theory and quantum mechanics were initiated by discovery of paradoxes in the system of physical knowledge which emerged when scientists tried to correlate new facts and new theoretical conclusions generated under direct influence of a previously formed picture of the world with this image itself. These paradoxes arose in terminological interpretation of corollaries of Lorentz's transformations and corollaries of Planck's law of radiation of absolutely black body. These paradoxes transformed into problems that encouraged theoretical research and led to construction of special relativity theory and quantum mechanics.

Though the new physical picture of the world appeared at the late stage of construction of these theories, its earlier version participated in raising problems. So we may say that certain aspects of directing role of the picture of the world remain also in modern research.

The other circumstance connected with the role of the picture of the world in construction of modern theories may be defined as reinforcement of significance of its operational aspects. We believe that this is the main feature of non-classical strategies of the construction of a new theory. Under modern circumstances, pictures of physical reality are created and reconstructed differently from the way which worked in the era of classical development of physics. They used to be created as visual patterns of structure and interaction of natural objects, i.e. types of measuring procedures, which gave an opportunity to reveal the corresponding objects, were presented in a veiled form. Nowadays the investigation uses a method which can be called – in certain aspects – contrary. The future picture of physical reality is fixed first as the most general pattern of measuring, and objects of a certain type should be inspected within its frame. The new picture of the world is given in its incipiency at this stage, while the structure of the physical reality studied is defined by means of the pattern of measuring: "nature has objective characteristics, recognized within the frame of such and such type of measurements".

By the way, these characteristics first are given as a quite approximate image of structure of the interactions studied, by means of fragmentary ontological ideas which are united in a system due to explication of an operational scheme. Only later does a relatively clear and "quasi-visual" idea appear, the idea of structural features of the physical reality, which is revealed in the type of measurements given and represented by the picture of the world. We can find examples of such way of investigation in the history of modern physics. Let us regard, for instance, Einstein's works of the period when he was working out the main ideas of the special relativity theory. It is well known that formation of this theory started from generalization of the relativity principle and creation of the scheme of spatial and temporal measurements which would consider finite signal propagation velocity necessary for synchronization of watches in inertial frames of reference. First Einstein explicated the scheme of experimental and measuring procedures which was the basis of Newtonian ideas of absolute space and absolute time. He demonstrated that those ideas had been introduced due to a recent postulate: watches, which are in different frames of reference, are correlated by means of instantaneous signal transmission.[5] Since no instantaneous transmission of signals exists, and any interaction propagates at a finite speed, Einstein offered another scheme of measuring space and temporal coordinates in inertial frames of reference, that have watches and rulers. Synchronization of the watches by means of light signals, spreading at a constant speed irrespective of the movement of the light source, was the central point of that scheme. Objective qualities of the nature, which could be also revealed through this type of experimental and measuring actions, were reflected in the ideas of space-temporal continuum, where space and temporal intervals, taken separately, are relative. But these ideas – in their "ontologized" form – were reflected in the physical picture of the world later, only after the special relativity theory had been created. At the early stage of yielding the new picture of the world the features of the physical reality mentioned were presented in direct connection with the operational scheme of investigation.[6]

The same specificity, in certain sense, can be traced in the process of the development of the quantum picture of the world. What is more, here the history of science lets us trace clearly, how the development of atomic physics led us to changes in the classical mode of construction of the picture of the world.

In the history of quantum mechanics we can single out two stages: the first one, which based on the classical methods of investigation, and the second, modern one, which has changed the very strategy of theoretical research.

However unusual the notions of the quanta of electromagnetic energy introduced by M. Planck were, they still did not break the very method of theoretical research. After all, Faraday's ideas of force fields were not less revolutionary than the idea of discreteness of electromagnetic radiation. So, when Planck's works introduced the idea of discreteness of radiation into the electrodynamics picture of the world, it was a revolutionary step, because the old picture of the world was blown up from the inside. But Planck's ideas did not exercise direct influence on the classical methods of yielding the picture of the world, which was created as a visual image of natural interactions. Further development of physics was related to efforts to create a quantum picture of reality in accordance with the ideals of the classical approach. Here de Broglie's investigations are characteristic. De Broglie offered a new picture of the physical reality, which included a statement about specificity of atomic processes, and introduced "visual" image of atomic particles as inseparably connected with the "waves of matter'. According to de Broglie, movement of the atomic particles is tied with some wave spreading in the three-dimensional space (the idea of a pilot-wave). Those ideas played a great role at the initial stages of quantum mechanics development. They gave basis to the natural analogy between the description of photons and electrons and provided transmission of quantum characteristics introduced for photons, to electrons and other elementary particles (de Broglie's picture of the physical reality provided us with the choice of analog models and working out certain theoretical schemes, which were to explain wave qualities of electrons).

Though, de Broglie's picture of the world was "the last of the Mohicans" of visual application of quasi-classical notions to the image of the physical reality. Schrödinger tried to develop this picture, introducing an idea of particles as wave packages in the real three-dimensional space, but failed, because his efforts provoked paradoxes in theoretical explanation of the facts (the problem of stability and reduction of the wave package). After M. Born had found the statistical interpretation of the wave function, it became clear that waves, a "packet" of which should have formed a particle, are "probability waves". Since that time physicists have more and more often regarded the efforts to introduce a visual picture of the world by means of classical models as an anachronism. It is becoming evident that the ideas of a corpuscle and a wave complement each other but are not compatible with each other within the same visual image.

The development of science showed that the object of the new type, studied by quantum physics, is extremely unlike the objects known, and, according to S. I. Vavilov, "we lack familiar images for a visual and model interpretation of its image". But a general image of the reality studied was still necessary, as it defined the strategy of theoretical search, directing the choice of analog models and mathematical means to put forward productive hypotheses.

Under these circumstances a turn to the new method of construction of the picture of the world was happened. Here a great part belongs to N. Bohr. The image of the physical reality was now built as an "operational scheme" of objects studied, and we may say that their characteristic is what is revealed within the scheme. Bohr's approach can be characterized not by introduction of hypothetical ideas of the structure of nature as foundation for new concrete theoretical hypotheses, which are to be verified experimentally, but by analysis of the scheme of measuring, which can help reveal the corresponding structure of the nature.

Niels Bohr was one of the first scientists who clearly formulated the principle of quantum-mechanical measuring, different from the classical pattern. The latter was based on extraction of a self-identical object of the material world. It was believed that the strict demarcation line separating the object from device would be drawn since in measuring it is always possible to take into account all details of influence of the device over the object. But the objects in the quantum sphere are quite specific, and detailing of influence of the device over the object can be accomplished only with precision determined by the existence of action quantum. Therefore the description of quantum phenomena includes description of essential interactions between atomic objects and devices.[7]

General features of a micro-object are defined by means of clear description of characteristics of two complementary types of devices (one is used, for instance, to measure coordinates, the other – to measure impulse). Complementary description is a method to reveal basic and profound features of a quantum object.

All these principles introduced "the operational scheme" which lay in the foundation of the new picture of the world created by quantum physics. Through such a scheme scientists could fix (as activity) essential features of a quantum object. This object, according to the new view, was presented as having a special "two-level" nature: a micro-object in its existence is stipulated by macro conditions, and they are inseparable. D. Bohm wrote that quantum mechanics makes us reject the assumption which lies in the foundation of many common statements and images: that we are able to analyze separate parts of the Universe, and each of them exists independently.[8] But this image of a quantum object has not been differentiated yet and not presented as a system-structural description of interactions in nature. So we can predict further development of quantum-relativistic picture of the world. Probably, it will lead us to notions of the structure of natural objects which include quantum characteristics as natural ones. The decisive part in such development will belong not only to new achievements of quantum physics, but also to philosophical analysis necessary to prepare usage of new system notions for description of the physical reality.

The approach to quantum objects as complicated self-organizing systems seems very fruitful. This problem has already been widely discussed in literature, including Russian literature. As early as in the 1970s authors tried to interpret the specificity of quantum mechanics description in terms of complicated systems. Yu.V. Sachkov, for instance, mentioned the two-level structure of quantum mechanics concepts: there are concepts which, on the one hand, describe the unity of the system, while on the other hand, represent typically random characteristics of the object.[9] The idea of such dismemberment of the theoretical description correlates with the idea of complicated systems, which are characterized by subsystems with stochastic interaction of the elements and, on the other hand, some "controlling" level securing integrity of the system.

The idea that quantum mechanics notions may be correlated with description of the reality in terms of complicated, self-regulating systems has also been postulated by G.N. Povarov[10] and V.I. Arshinov.[11] My works of the 1970s also promoted this idea.[12]

The foreign literature of that time present more or less detailed concepts alike in the works of such physicists as G. Chew, H. Stapp, D. Bohm, B. Hiley, philosopher F. Kapra and others.

In the conception of "bootstrap", which appeared on base of the S-matrix approach, G. Chew offered a picture of the physical reality in which all elementary particles obtain system integrity. They are as though laced together by generating reactions, but no one of them should be regarded as fundamental for others.[13] The American physicist-theorist H. Stapp worked with notions of the physical reality in the same direction. He paid special attention to ideas of non-locality, impossibility to combine requirements of causation and localization of micro-objects in a quantum mechanics description. Such incompatibility is expressed in the complementarity principle (complementarity of causal and spatial description). Correspondingly to these ideas Stapp outlines a new ontology, which states: the physical world is a system unity, irreducible to dynamical connections between its elements. According to Stapp, besides causal connections, the decisive part belongs to non-forced interactions which unite different elements and subsystems into a whole. As a result, we have an image of a weblike global structure of the world, all elements of which are interconsistence. Any localization, any individualization of elements in this global structure is relative, stipulated by general mutual dependence of the elements.[14] Stapp interprets the fundamental probability character of the results of measuring in quantum mechanics from the point of view of correlation of the local and the global.

G. Chew and H. Stapp emphasized the idea of system integrity of the world, but the problem of the level hierarchy of the elements – a very important characteristic of complicated self-regulating systems remained in the shadow. The idea of a web like network, where all elements and substructures are mutually correlated, did not generate enough stimuli for working out notions of their relative fundamentality and complexity of the elements and their connections which are found at different levels of the hierarchy.

Probably, these features of the "bootstrap" conception caused the decay of interest to it among physicists while the quark model of elementary particles has been being elaborated.

But the very idea of relativity of localization and individualization of physical objects and events, their stipulation by the qualities of a whole system became a necessary and important aspect taken into account in most modern efforts to build an integral physical picture of the world which would include quantum and relativistic notions.

This approach has been well presented in the works of D. Bohm, who tried to solve the problem of the quantum mechanics ontology. Bohm stressed the need of the system of notions of the physical world to overcome the classical approach that postulated existence of local elements and events, which are interconnected and may be isolated. The new image of the physical reality, according to Bohm, should be based on the idea of relative locality depending on the integrity of the Universe, on non-dynamic relations that (along with the dynamic ones) define the structure of the nature. Bohm compares the picture of the reality with correlated substructures and elements with a carpet, where parts of the decoration do not form a whole because they interact dynamically.[15] They are individualized through inclusion into the whole and their relation to other parts of the whole. Here Bohm's images of the reality correspond to those offered by Stapp. But Bohm has made a new step. He suggested to consider the world as some kind of order, a hierarchy of different levels. Every level, according to Bohm, is characterized by its own non-locality and non-force interactions. Bohm emphasizes that non-locality and non-force correlations can be revealed not only in the microworld, but also at the macrolevel. In the work written together with B. Hiley, D. Bohm gives an example of the experimental facts of correlation of distant atoms in super-fluid helium. These correlations disappear at high temperature, when the effect of viscid friction arises because of casual collisions of atoms, but they restore when the temperature is lower a certain threshold level.[16]

As to the conception of non-locality in the microworld, it is the most brightly expressed by the reduction of the wave function – which is the corner stone of quantum physics. Even in the 1930, at the time of Bohr's and Einstein's discussions, scientists formulated a so–called paradox of Einstein-Podolsky-Rosen (the EPR-paradox). The point of it is that a wave function is assigned to two interacting particles, and then the distance between particles becomes so considerable that their dynamical interaction can be ignored. But if we measure the magnitudes characterizing the state of one particle (for instance, its impulse or coordinate), we will see reduction of the wave function, and thereby the state of the other particle will automatically change. Einstein regarded this mental experiment as a paradox which proves that quantum mechanics is incomplete. But further discussions of the EPR-paradox, for instance, in the 1970s, showed that it leads to a contradiction if we latently accept the principle of locality, which assumes the possibility to separate system and to measure its spatially separated, distant parts independently.[17]

But if we reject locality as an absolute principle and think that it can be applied only relatively and limitedly, we will come to the probability of non-local interaction. The EPR-paradox may be interpreted as a display of non-locality.

Bohm's picture of the world postulates existence of some hidden order which organized all other types of orders in the Universe; this order is inherent in the net of space interactions. Bohm explains the idea of this hidden order by means of another visual analogy (like the example of a carpet ornament). He uses a metaphor of a hologram in which, if we throw light to any local part, we will be able to see the entire picture, though less detailed than in case of lighting of the whole hologram. Bohm tries to correlate the idea of the hidden order and hierarchy of orders with the notions of the structure of the space. Basing on the general relativity theory and interrelation with gravitating masses and curvature, he believes it possible that these ideas may be widened and generalized within the hypothesis of topological properties of the space correlated with the types of order in the Universe. Hiley and other Bohm's investigation program supporters have also developed these ideas.[18]

This program, as well as G. Chew's and H. Stapp's investigations, can be considered as variations of some general approach to the construction of a physical picture of the world, which would use the ideas of non-locality, non-forced interactions and notions of a complicated self-regulating system, where the features of parts and elements are stipulated by the features of the whole, and the probability causality is a basic characteristic.

The philosophical and methodological basis of such approach is the rejection of methodology of "elementarism", which was domineering in physics for a long time and assumed that the features of physical systems are completely described by characteristics of their elements.

The holistic approach, opposite to elementarism, is based on the idea that the features of the whole cannot be reduced to the features of the elements and their interactions.[19]

This approach was developed mainly in investigations of biological and social objects. Then it was transferred to the system of non-organic nature due to cybernetics, theory of information and the general theory of systems.

The way of investigation chosen (in various forms) by the conceptions of G. Chew, H. Stapp and D. Bohm is based on employment of the "organismical" methodology in the construction of the physical picture of the world. F. Capra says that Bohm's and Chew's conceptions are the two most philosophically sophisticated approaches to describe the physical reality.[20] He denotes their rapprochement – further versions of the "bootstrap" concept tried to consider elements of the S-matrix as types of orders and to link them with the space-time geometry. In Capra's opinion both of these conceptions understand the world as a dynamic network of relations and put the concept of order in the centre; they both use matrices as means of description, and topology – as means to determine categories of order more exactly.[21]

Then Capra emphasizes that Chew's, Stapp's and Bohm's picture of the world present elementary particles not as immutable bricks of the Universe, but as dynamic structures, "energy beams" forming objects which belong to higher levels of organization. According to Capra, for modern physicists matter is not passive and inert, but is always dancing and vibrating, and the rhythmic patterns of the dance and vibrations are determined by molecular, atomic and nuclear structures. The nature is in balance, but dynamic, not static.[22]

Here it would be right to stress that this image of the Universe as dynamics of the physical processes, their mutual correlation and hierarchy of orders, is more likely an image of a self-regulating system, where mass, stochastic interactions are controlled by the whole and reproduce the whole. The classical picture of the world as a simple device, which dominated in classical physics, is now replaced by the image of the Universe as a self-organizing machine.

Though, in this respect we are also to mention narrowness of such approaches to construct a modern physical picture of the world, which are adjoint to the images of a complicated self-organizing system reproducing the basic characteristics of the whole as a hierarchy of orders in dynamics.

Self-organization cannot be brought only to the processes of reproduction of dynamic order and level organization of the system, though this aspect is obligatory. The other aspect is irreversible change and development connected with appearance of new organization levels and transition from one type of self-regulation to another. If we take these aspects into consideration, we are to employ more complicated images of system organization, that is images of complicated, historically developing systems. The notions of such systems include the idea of dynamic balance, but only as one of the states of non-equilibrium processes characterized by changes of the types of dynamic balance, transitions from one type to another.

In the modern science the program, most adequate to such view, is the one connected with working out dynamics of non-equilibrium processes (I. Prigogine) and synergetics (H. Hacken, M. Eigen, G. Nicolis, E. Laszlo, S. Kurdyumov, G. Malinetsky, Yu. Klimantovich etc.). Differently from classical physics – in principle – the synergetic paradigm sees the place of non-equilibrium and irreversible processes and their correlation with equilibrium and reversible processes. While classical physics presented non-equilibrium processes as sort of declination from the standard situation, the new paradigm puts them into the focus of interest, considering them as a way to give birth to stable structures.

Stabilities appear not despite, but due to non-equilibrium states. In these states even small fluctuations, random influences cause attractors leading to new organization; at all levels, either level of macroscopic physics, or level of fluctuations, or microscopic level, the source of order is non-equilibrium. Non-equilibrium is what gives rise to "order from chaos".[23]

When we describe the behaviour of quantum objects in terms of self-organizing systems, we obtain new opportunities to build quantum mechanics ontology.

I. Prigogine emphasizes that we can explain features of quantum mechanics measuring connected with the reduction of the wave function as consequences of instability, immanent to the movement of micro-objects, and measuring – as an irreversible process of causing stabilities in dynamic chaos.

From the point of view of "order from chaos", the basically static character of predictions in quantum mechanics seems not to be the result of activity of the one who is doing the measuring, but to represent the essential characteristics of the nature itself.

Non-localities presented in the behaviour of micro-objects, according to I. Prigogine and C. George, are related to the growth of coherence of quantum ensembles in comparison with classical dynamics.[24] Coherence, in its turn, expresses a special quality of self-organizing systems, related to their non-linearity and ability to cause cooperative effects based on non-force interactions.

I. Prigogine and I. Stengers say: "In our approach, the world follows the same laws, with measuring or without measuring...";[25] "introduction of probabilities, in our approach, is compatible with physical realism, we do not need to identify it with incompleteness of our knowledge. The observer now does not play an active part in the evolution of the nature or, at least, his part is not more active than in classical physics. In both cases, we can put into action the information got from the outer world".[26]

S.P. Kurdyumov has found quite interesting solutions of problems connected with mathematical description of peaking regimes in a nonlinear medium. These regimes are an essential characteristic of behaviour of synergetic systems, and their mathematical description bases on nonlinear links of space and temporal coordinates. The apparatus developed in application to such situations is effective when applied to quantum mechanics problems. It allows to obtain Schrödinger's equation and to explain quantization as expression of the features of a nonlinear medium.[27]

Probably, with the development of all these approaches the quantum picture of the world will one day appear in objectivized form presenting the structure of the nature "by itself".

But in order to consider modern features of the theoretical research it is important that at initial stages of developing pictures of the world in modern physics the "operational aspect" of the vision of reality is accentuated. It is the operational side that mainly determines the search for mathematical hypotheses.

It is quite indicative that the modern theoretical-group approach directly connects the principles of symmetry based on various groups of transformations with characteristics of the measuring devices.[28] An attempt to use a certain mathematical structure in physics in this sense is determined by the choice of a measuring scheme as the "operational aspect" of the corresponding picture of the physical reality.

So far as the starting point of investigation – choice of the picture of the world as operational scheme – often presupposes quite radical changes in the strategy of theoretical research, it requires philosophical regulation. But, unlike classical situations, when introduction of the picture of the world was mainly directed by "philosophical ontology", in modern physical investigations epistemological problems are in the focus of attention. It is significant that in regulation principles, which facilitate the search for mathematical hypotheses, theoretical and cognitive statements (the correspondence principle, simplicity etc.) are evidently represented (in concretizing with reference to a physical research form).

It seems that only by analyzing these problems (while regarding the chain of relations: philosophy – the picture of the world – analog physical model – mathematics – mathematical apparatus of a physical theory) we can reveal at greatest length the mechanisms of developing a mathematical hypothesis.

From this point of view, the discussion of the method of mathematical hypothesis in philosophical and methodological literature has been valuable, not only due to verification that the fact really existed, but – to a greater extent – to the fact that the problems described above were formulated and first attempts to solve them were made.

Still, though we do justice to actuality of the problems raised, when we accentuate heuristic value of the mathematical methods, we should not loose sight of another, not less important aspect of theoretical research: the process of constructing a theoretical scheme which allows us to interpret the mathematical formalism introduced. Inaccurate analysis of this aspect of investigation leads to hidden introduction of a series of simplifying notions, true only in their general formulating. If they are employed without enough specification, it may lead to incorrect ideas. Such notions include:

1. Assumption that experimental verification of a mathematical hypothesis and its transformation into a physical theory is a rather obvious procedure, which is just brought to mere comparison of all corollaries of the hypothesis with experimental data (the hypothesis is accepted if its corollaries correspond to the experiment, and rejected in case of contradicting); 2. Assumption that a mathematical apparatus of a developed theory can be created as a result of advancement in purely mathematical means, by mathematical extrapolation, without constructing any intermediate interpretational models.

We are going to try to demonstrate that such notions of forming of modern theory are not correct enough.

To begin with, we will analyze the situation of construction of local theoretical schemes, and then we will turn to the process of creating a developed theory. As the former we will consider the theoretical scheme which is the foundation of Dirac's relativistic electron theory, the latter – quantum electrodynamics (the theory of interaction of quantized electromagnetic and quantized electron-positron fields).

First we have to denote that the interpretation of Dirac's theory as knowledge corresponding to the level of local theoretical schemes can be employed only in case we take into consideration the fact that it has been assimilated by a developed theory – quantum electrodynamics – and has become a part of it as a fragment which describes one of the aspects of electrodynamics' interactions in the quantum area. In generality the theory of relativistic electron surpasses such classical local theoretical schemes and laws as, say, the system of theoretical knowledge about oscillation of the pendulum (Huygens's model) or Faraday's observations of electromagnetic induction.

But one of the features of the method of mathematical hypothesis is that it raises local theoretical schemes and laws to a new stage of generalization; it lets us start constructing a developed theory from synthesis of theoretical knowledge of a higher degree of generality – compared to classical examples.

The problem of empirical verification of a mathematical hypothesis

In classical physics the pattern of investigation was the following: a theoretical model was created (it was introduced as a hypothetical construction, then scientists proved that it included essential features of the generalized experimental situations) and only after that were mathematical expressions for the laws of the theory derived. The latter appeared as the result of revealing connections of abstract objects of the theoretical model and expressing them in the language of mathematics. So, introduced equations immediately got an adequate interpretation and connection with experience.

In such structure scientists had no difficulties in empirical justification of the equations. But in modern physics the situation is different. Using the method of mathematical hypothesis, physics began to create equations to construct rules of correspondence, which link magnitudes of the equations with the object of experience, and then emerge certain difficulties connected with the search for interpretation of the equations.[29]

We would like to emphasize that these difficulties essentially consist not in the fact that first a mathematical hypothesis is introduced without any interpretation at all. In that case the hypothetical equations could not be regarded as expressions for the laws of physics and would be only formulae of pure mathematics. Since certain symbols in the equations are considered as physical magnitudes, interpretation of the equations is indirectly assumed. But the problem is that, at the first stage, we, as a rule, inadequately interpret the hypothetical equations. The reason is the following. Formulating a mathematical hypothesis, we reconstruct the equations which used to express physical laws of some area. Such expressions were tied with a corresponding theoretical model, or scheme, which provides their interpretation. The magnitudes, tied in them, fixed attributes of abstract objects of the model given. But when the initial equation was reconstructed, it gave birth to new connections of the physical magnitudes and, consequently, new definitions within the new equations. Nevertheless, in a physicist's mind these magnitudes are still combined with the ideas of abstract objects of the old theoretical model. So, having carried out mathematical extrapolation along with physical magnitudes whose links are postulated in the equation, he borrows such objects and tries to use them to interpret the equations obtained. In correspondence with the new links of physical quantities in equations he sets down new attributes to the abstract objects and determines their correlations. This is the way to get a hypothetical model which then is showed as a representation of essential features of the new sphere of interactions. But in this quality it is not proved. We have not checked if it is possible to derive the element objects (with their new attributes) by means of idealization and passage to the limit from real object relations of the new area. Therefore it is very likely that the hypothetical interpretation of the new equations will be wrong. In this case, if we check the equations at once, comparing them with the experimental data, the results of the checking may lead to mismatch between the equations and experiment, even if the equations are productive.

To consider this aspect of the question at length, let us take an example which has already become classical: the justification of Dirac's relativistic equation. We know that Dirac had constructed – in complete correspondence with the canons of the method of mathematical hypothesis – a system of four linear differential equations of the first order for four independent wave functions and obtained, as one of the basic mathematical results, solutions which corresponded to negative value of rest-mass (complete energy) for a free particle.

It is usually believed that these results, when compared to the experience, led at once to prediction of positron. But reality was far more complicated. The initial comparison of Dirac's equations with the experience caused such predictions, after which it seemed impossible to save the equation.

The most extravagant and obviously contradicting to the experience were conclusions about possibility of spontaneous collapse of electrons and, as a consequence, about instability of hydrogen atoms.

It is easy to see that such conclusions contradicted to all experimental treasures of atomic physics so brusquely, that they were enough to reject Dirac's equation as an unsuccessful mathematical extrapolation. But the point is that the results mentioned were pressed not by the qualities of Dirac's equation, but by its initial interpretation. Since the equation was obtained out of classical correlation between mass and energy for one particle, and contained an ordinary expression for quantum mechanical operator of the impulse of that particle, then it seemed very natural that Dirac's equation described the behaviour of a separate quantum mechanical particle under condition that non-relativisticic restrictions are removed.[30] In other words, the transformation of traditional quantum mechanical equations into a relativistic equation for electron took place along with the introduction of a new system of abstract objects, which were taken from theoretical models of non-relativistic quantum mechanics and provided with new qualities. Solution of Dirac's equations indicated existence of areas with positive and negative energy, separated by the energy barrier $2mc^2$. Thus, the equations introduced the following system of abstract, theoretical objects: "particle" (in the sense of quantum mechanics, unable to move at relativistic speeds), "area of positive energies" and "area of negative energies". In accordance with the general principles of quantum mechanics, a particle with charge e and mass m got ability to move through the barriers between the areas mentioned under indefinitely small electromagnetic influence, and get into the area of negative energies. As Dirac's equation did not contain any "lowest" limitation of the possible quantity of negative energy ($-\infty < E \leq mc^2$), it followed that any particle, once in the area of negative energy and tending to the state with the lowest energy (the system stability principle), has to fall down into a bottomless energetic hole with a zero probability to return to the area of positive energies. It is easy to understand, that the above indicated paradoxical conclusions from Dirac's equation were somehow or other connected with the effect of particles' (electrons) disappearance "without trace" from the observed area when they got into the negative energy zone.

O. Klein was the first to find out these paradoxical corollaries soon after the theory of relativisticic electron had been published. They made many prominent physicists of the time be skeptical about Dirac's theory. For instance, W. Pauli stated that Klein's paradox (according to which, electrons are capable of overcoming barriers mc^2-order high and get from the area of positive energies into the area of negative energies) is a corner stone difficulty of Dirac's theory.[31]

Pauli wrote that states with negative energy have no physical sense. Nevertheless, unlike classical relativisticic mechanics, in Dirac's theory it is impossible to eliminate in general case states of negative energy for free electrons.[32]

The example of Dirac's quantum-relativisticic equation is quite instructive methodologically. It shows that the initial theoretical model introduced together with mathematical extrapolation may result false and dangerous even for productive equations. Hence we may draw an important feature of justification of a mathematical hypothesis. At the first stage verification of the equations by the experimental data does not let us determine whether the equations are fit for description of a new sphere of phenomena. Even if the conclusions from the equations do not agree with the experiment, it does not necessarily mean that they should be rejected as a fruitless hypothesis. Mismatch with the experiment is just a sign that in the integrity "equations plus interpretation" some part is inadequate to the new sphere of phenomena. The investigator does not know in advance, which part (we may speak about productivity of equations only in retrospective, when we already know their role in the history of physics, as, for instance in the case of Dirac's equation).

Nevertheless, as the initial interpretation of the equations is hypothetical, it is quite probable that it bears responsibility for contradictions between corollaries of the equations and experimental data. So, if we discover mismatch of the equations and experiment, it is the start of the second stage of empirical justification of a mathematical hypothesis. Here the initial interpretation is being changed; the initial hypothetical model, which used to serve the equations, is transformed into a new model. To illustrate characteristic features of this process, let us return to the example of Dirac's equation.

After mismatch of the equation and experiment had been discovered, Dirac reconstructed its initial interpretation. He refused to treat the equation as description of one particle's behavior. The theoretical model, due to which Dirac's mathematical formalism turned into an effective apparatus, was connected with the idea of many-particle systems. In this model the area of negative energies was forbidden for free particles, though presence of two signs for energies was a direct mathematical corollary of the strict solution of the equation. Such exclusion was obtained thanks to Pauli's principle formulated, as we know, for a system of electrons. Within the new interpretation, all negative energy states were considered as totally filled by electrons. Such "quasi-continuum" of electrons, according to Pauli's principle, could never manifest itself externally, because the electrons transfer (moving) inside the continuum, as an indispensable condition of its experimental discovery, stipulates change of the electrons energy, which is impossible because all energetic levels are already full.[33] The only possibility to find out at least one particle of the continuum was to transfer the particle to the positive energy zone, where there were free levels. It was possible to reach under energetic effect not weaker than $2mc^2$ (volume of the energy barrier). But when an electron is extracted in such a way from the continuum, there appears a "free place" (a hole) which behaves as a state with positive charge and positive energy (since to eliminate this state we have to, by definition, place an electron, negatively charged, there). This "non-filled state" can already be experimentally revealed. The "hole" in the electron continuum may be filled by an electron from the neighboring cell of the continuum where an electron from another cell can "jump" etc. Efficiently this process should be appeared as basically observable motion of a positive charge with positive energy. So, the very qualities of the new model naturally caused prediction of the positron.

Though, interpretation of the "hole" also required some creative efforts. At the early stage Dirac associated the "hole" with proton. But soon R. Oppenheimer proved that if the "hole" was interpreted as proton, this would preserve the conclusion from Klein's paradox of instability of hydrogen atoms (according to which the lifetime of a hydrogen atom was to make about 10^{-10} sec). To find a solution of the contradiction, Oppenheimer suggested that we should consider the "holes" as positive electrons, different from protons. It was Oppenheimer who introduced the term "positron".[34] H. Weil proved that the mass of the holes has to coincide with the mass of electron. About three years after Dirac's new interpretation of the quantum mechanics equation for electron, in 1932 C. Anderson discovered positron experimentally.

According to the new interpretation of Dirac's equation, any "hole" (positron) which appeared in the continuum, may be destroyed when an electron from the zone of positive energies enters it. Such transition of electron must cause discharge of quanta of energy (no less than $2mc^2$), in the same way as energy is discharged when an atom, which has lost an electron from one of the internal shells, captures a free electron. It is easy to notice that the properties of the new theoretical model directly led to the idea of annihilation.

Dirac's reinterpretation of his equation removed mismatch of the latter with the experiment. The equation was not only put in concord with experiments, but also enables scientists to predict most unexpected phenomena: positrons and annihilation and pair creation effect.

The new theoretical scheme providing an adequate link of quantum relativistic equation for elecron and experiment in correlation with physical picture of the world introduced basically new ideas of electromagnetic interactions. In the physical picture of the world new notions of electron-positron vacuum as a specific state of the physical world were appeared, actively reflected in interactions of electrons, positrons and photons.

The new interpretation of Dirac's equation, after all details of its physical sense had been clarified, was recognized by the scientific world quite soon. The physicists who had been skeptical to Dirac's theory first, reconsidered their positions. A characteristic example here is W. Pauli. He paid attention to Dirac's grace in his new interpretative scheme of prohibition principle and recognized perspectives opened by the notions of physical vacuum as potential generator of particles.

In his Nobel Prize lecture delivered December 13, 1946, Pauli, considering Dirac's discovery from historical distance, said: "P. Dirac's response led to what could really happen if we employed the prohibition principle". In his Stockholm lecture, Dirac himself spoke of his proposal of new interpretation of his theory, according to which in the true vacuum all negative energy states are to be filled, and we can consider as observable only deviations from this minimal energy state, i.e., holes in the sea of the filled states. The prohibition principle is what guarantees stability of the vacuum in which all negative energy states are filled. What is more, the holes possess all the qualities of particles with positive energy and positive charge, since they can be born and destroyed in pairs in external electromagnetic fields. Indeed, thus predicted positrons, exact mirror images of electrons, were found experimentally.

It is evident that in principle the new interpretation does reject the point of view proper for one-particle problem, and from the very beginning it considers the problem of many particles".[35]

This example, to our mind, allows us to distinguish a few peculiarities of experimental justification of a mathematical hypothesis connected with construction of new interpretation of equations. In general sense, it is well known that when an experiment does not confirm a mathematical hypothesis, the investigator starts searching a new interpretation. But we would like to draw our reader's attention to the following mechanisms of the search.

The first important thing is that the initial material for new interpretation consists of abstract objects of the model initially introduced. Constructing the new model, Dirac used abstract objects "particle", "area of positive energies" and "area of negative energies" which already existed, and only the last object was changed (the feature "to have free energy levels" was eliminated).

The investigator does not yield his new interpretation "out of nothing", but uses abstract objects introduced before, while constructing the mathematical hypothesis, as his building material.

The second important factor directing construction of the new interpretation is the following requirement: the theoretical model should be justified as an idealized scheme of interactions which are observed in real experimental situations. That is what makes the investigator reconstruct abstract objects, finding correlatives of their features in real interactions observed in experiments. As early as in primary experimental verification of mathematical hypotheses it becomes clear which of the abstract objects do not meet this requirement. This is how non-constructive elements in the primary interpretation are discovered, and the ways of its changing are indicated. So, when the primary model in which Dirac's equation was held, was mapped on experimental situations in the atomic area, such mapping showed that its contradicting to the experiment was caused by the notions of the negative energy zone.

But, just as the equations required that such abstract object should be introduced, so there remained only one way: to provide "the area with negative energies" with features which would prohibit electrons to enter this area. This is probably the source of the right conjecture on electron continuum, which allowed to shape a productive interpretation of the equations.

It is characteristic that, introducing a new system of abstract objects (continuum of electrons filling all states with negative energy and free electrons in the positive energy zone) instead of the previous model, Dirac justified this system as an idealized scheme of experimental measuring situations of the atomic area. He found reason for features of the abstract objects in experimentally observable situations. Such abstract objects as "electron" and "area of positive energy" were justified easily enough (in principle all preceding development of atomic physics proved lawfulness of their introduction). The task was harder in case of "electron continuum". Nevertheless, this abstract object also got a correlative in real interactions fixed by experiments in the atomic area. The idea of continuum was a result of analysis of all theoretical and experimental material of physics connected with studies of electron shells of atoms. Dirac introduced continuum of electrons as an analogy to filled shells of an atom which also could lose electrons at external shells. Having imagined such shells in extremely idealized form, Dirac interpreted them as a sort of system of fermi-particles in general. After that the electron continuum turned justified by all experimental measuring situations in which investigations of many-electron systems were held. Then such justification allowed to use effectively Pauli's exclusion principle in constructing a new theoretical model.

So the process of empirical justification of a mathematical hypothesis includes a number of procedures, complicated enough. We may point out the following: 1) explication of a hypothetical model introduced initially along with new equations; 2) mapping of this model on experimentally observable interactions of natural objects; 3) comparison of the "equation plus model" system with the experimental data; 4) reconstruction of the primary model in case of mismatch with experiment; 5) constructive justification of the new model; 6) new experimental verification of the system "equations plus their new interpretation".

Only when all these operations are completed, one may decide whether the equations (introduced by method of mathematical hypothesis) are fit for description of the sphere of interactions. As to the statement that the judgment about the hypothetically introduced equations is passed by means of their comparison with the experiment, it is true only in case we take into consideration all peculiarities of the empirical justification of the equations. But if we simplify it – "equations are rejected if they are not confirmed by the experiment, and are accepted if they coincide with the experimental data" – it may turn out false: mismatch with experiment at the first stage of empirical justification of a mathematical hypothesis is not a sufficient reason to reject the equations.

From all said above we may conclude that the main difficulties in creation of a non-contradictory system of theoretical knowledge are not over when equations are found. What is more, here a theorist faces the hardest and most important stage of his work.

P. Dirac wrote: "It is easier to discover mathematical form necessary for some fundamental physical theory than find its interpretation. It is true because the number of objects we deal with while discovering formalism is strictly limited, but, dealing with physical interpretation, we may find strikingly unexpected things".[36] We do not think it would be an exaggeration if we postulate: at the current stage of development of theoretical knowledge, when the investigator's first steps are connected with mathematical hypothesis, construction of a theoretical scheme which provides interpretation of the equation and their comparison with experiment still remains the key stage of the investigation.

HOW A DEVELOPED THEORY IS FORMED IN MODERN SCIENCE

Considering genesis of a theory in modern physics, it is important that one should not forget about differences in levels of theoretical organization of knowledge. Plain extrapolation of construction methods of a local theoretical scheme to all cases of theoretical research may lead to erroneous notions of ways of today's theoretical research. Such extrapolation makes one think that mathematical apparatus of a developed fundamental theory can be obtained thanks to continuous series of mathematical hypotheses, like the way Dirac, for instance, got his equation for relativistic electron.

Even if we agree with the statement of universality, assume that means of construction of mathematical apparatus for all primary theoretical schemes of modern physics (such as

Dirac's scheme) are the same, we still cannot conclude that mathematical apparatus of a developed theory should be obtained in the same manner.

This apparatus is a more complicated system of mathematical means, first of all because it allows to get – due to certain methods – the regularities characterizing local theoretical laws from the basic correlations. There are no reasons to believe that such apparatus can be worked out merely by means of continuous series of mathematical hypotheses. The contrary is more likely. If every stage of creation of apparatus of a developed theory ends at putting forward a hypothetical equation, consequently, the investigator has to justify the legitimacy of this equation before taking it for initial base for putting forward the next mathematical hypothesis. Philosophical literature has always taken somehow or so this circumstance into consideration discussing the problem of mathematical extrapolation. It is evident enough that only a hypothesis which has been verified empirically has got "the right to live due to dictate of experiment" and gets the role of "starting point for a new hypothesis which will inevitably replace it".[37] Though we have seen that the procedure of comparing mathematical hypothesis with experiment turns a complicated system of operations aimed at constructing a theoretical scheme, which provides interpretation of the equations.

If we take this circumstance into account, we will come to a non-trivial conclusion: forming of mathematical apparatus of a developed theory should be interrupted by intermediate interpretations, which would direct every new series of mathematical hypotheses. Naturally, this conclusion is to be checked. But if we accept it as a preliminary assumption, we will see a parallel between process of theoretical synthesis in classical physics (which has already been discussed) and situations of construction of a developed theory in modern physics. We should not be surprised by such analogy, because the process of evolution provides succession between higher and lower levels of development.

The very idea of evolution in scientific thinking claims for seeking not only specific, but also repeating, invariant contents in historically changing methods of construction of theory. However greatly the past is transformed in the present, their genetic link always lead to reproduction in compact of the main features and specificities of their historical development. That is why history of scientific cognition should be analyzed in two aspects: revealing of specific features of the investigation characterizing the modern stage of evolution of physics, and search for invariant contents inherent in both classical and modern forms.

Now let us consider modern situation in construction of a developed theory from this point of view. To reach this goal, we reconstruct logically the process of settling of quantum electrodynamics. Even cursory comparison of classical and modern situations of theoretical search show up several characteristic features of theoretical activity nowadays.[38]

One of these features is the fact that developed theories of high community degree now are elaborated by research groups, and the duties are distributed among them clearly enough. For instance, we of course can regard the creators of quantum electrodynamics W. Heisenberg, W. Pauli, P. Dirac, P. Jordan, N. Bohr, L. Rosenfeld, L. Landau, R. Peierles, V. Fok, S. Tomanaga, J. Schwinger, R. Feynman, F. Dyson and others as a "collective creative subject" who executed all logically necessary operations which led to construction of a new theory. Just for comparison, we would like to remind the reader that for classical theory of electromagnetic field all operations of the kind were carried out by one investigator – J. C. Maxwell. For classical physics it was more a rule than exception; of its three most important theories – mechanics, electrodynamics, thermodynamics – only the latter can be looked at as production of a "collective creative subject".[39]

In quantum relativistic physics, after creation of general relativity theory, we cannot find a situation when a developed theory was constructed by creative efforts of one investigator. The objects studied became far more complicated; construction of a theory now requires far greater quantities of information, so each of the investigators carries out only some of the logically necessary procedures which provide construction of a new theoretical system.

In this respect the following example is characteristic. N. Bohr, who, together with L. Rosenfeld, did the main work on interpretation of the mathematical apparatus of quantum electrodynamics, joined the creative group working on the new theory, when its mathematical formalism had already been basically built. According to Rosenfeld, Bohr not only had taken no part in creating this formalism, but even did not know its basic principles at the early stage. Rosenfeld recollected: "Bohr's state of mind when he attacked the problem reminded me of an anecdote about Pasteur. When the latter set about investigating the silkworm sickness, he went to Avignon to consult Fabre. "I should like to see cocoons," he said, "I have never seen any, I know them only by name." Fabre gave him a handful: he took one, turned it between his fingers, examined it curiously as we would some singular object brought from the other end of the world. He shook it near his ear. "It rattles," he said, much surprised, "there is something inside".[40]

L. Rosenfeld continued: "My first task was to lecture Bohr on the fundamentals of field quantization; the mathematical structure of the communication relations and the underlying physical assumptions of the theory were subjected to unrelenting scrutiny. After a very short time, needless to say, the roles were inverted and he was pointing out to me essential features to which nobody had as yet paid sufficient attention".[41]

Another important specificity of modern theoretical-cognitive situation is that fundamental theories more and more often are created without a well-developed layer of primary theoretical schemes and laws, which could characterize certain aspects of the new area. In this respect it is significant, for instance, that quantum electrodynamics, as preliminary knowledge of microstructure of electromagnetic interactions, had only fragmentary theoretical laws and models which characterized quantum properties of radiation and absorption of light by the matter. The other intermediatory links, necessary for construction of the theory, were created in the course of theoretical synthesis.

Last but not least, the third specificity of construction of modern physical theories is application of the method of mathematical hypothesis considered above. This method allows to pass in compact the stage of forming primary theoretical schemes and laws, finding at once equations of some vast object domain and then getting on their base the corollaries – theoretical laws which characterize particular aspects of this area.

In order to imagine visually the peculiarities of this way of theoretical investigation, let us consider the following hypothetical situation. Suppose Maxwell, while working on the electromagnetic field theory, did not have laws of electromagnetic and electrostatic induction or Coulomb's interaction of charges. Imagine then, Maxwell's theory was being created through introduction – by method of mathematical extrapolation – of generalizing equations for blocks of electromagnetic induction, electrostatics and others, which were derived out of Coulomb's, Faraday's and other laws, i.e. laws experimentally verified. In this case synthesis, leading to Maxwell's equations of electromagnetic field, would have been carried out on base of the mentioned generalizing laws.

Something of this kind is happening in construction of modern physical theories, and quantum electrodynamics is a typical example. It was formed in complete accordance with the requirements of mathematical hypothesis, and "intermediate" theoretical knowledge necessary to construct the new theory was created in the course of theoretical synthesis, which led to the system of its fundamental equations.

The main stages of development of the mathematical apparatus of quantum electrodynamics

The process of creation of the mathematical apparatus of modern quantum electrodynamics can be conventionally divided into four stages.

The first stage: apparatus of quantized electromagnetic field of radiation (field not interacting with the sources). The second stage: mathematical theory of quantized electron-positron field (quantization of sources of the field). The third stage: description of the interactions of the said fields within framework of the disturbance theory in first approximation. The fourth stage: apparatus characterizing interaction of quantized electromagnetic and electron-positron fields and taking into account the second and further approximations of the disturbance theory (development of renormalization method which allowed to describe the interacting fields in highest orders of the disturbance theory).

Each of these stages also consisted of several logically necessary steps which led to the corresponding equations of quantum electrodynamics. From this point of view, for instance, the first stage – construction of the apparatus of free quantized electromagnetic field – could be executed only due to preliminary investigation of quantum properties of radiation.[42] On this base scientists formed the notion of electromagnetic field of radiation as a specific quantum system which, on the one hand, has continual characteristics (frequency, wave vector), and, on the other hand, can be presented as set of photons in different quantum states. In the aspect of wave properties the field traditionally has been described by Maxwell's equations. Thus, there emerged the problem to transform the equations so that to take into account corpuscular properties of free electromagnetic field as well.

In order to do this, the magnitudes bound in Maxwell's equations, by analogy with now customary quantum mechanical approach, were regarded as operators subordinated to transposition correlations. So Maxwell's equations were transformed into equations of quantized electromagnetic field. Taken together with the commutation rules for operators (transposition correlations), they formed mathematical apparatus describing this field.[43]

The next step in investigation of the microstructure of electromagnetic processes stipulated an account of interaction of the radiation field with quantized sources (densities of charge-current). It required development of mathematical formalism describing quantum qualities of electron system in relativistic area. The solution of such a problem led to notions of electron-positron field. Finally the initial problem of quantization of sources of electromagnetic field was reformulated as problem of mathematical description of quantum properties of electron-positron field. Its solution marked the second stage of working out the apparatus of quantum electrodynamics.

From the point of view of logic of cognitive motion, the initial point of this stage is Dirac's relativisticic quantum mechanics of electron. We would like to emphasize again the fact that Dirac's theory, which opened for physics the area of electron-positron interactions, served as a kind of intermediate "pack" of knowledge for construction of modern quantum electrodynamics. It was a typical example – how, in the course of theoretical synthesis, investigators introduced missing links (local theoretical schemes and laws), which provide successful progress toward future fundamental equations of the theory.

Generalization of Dirac's equations was connected with quantization of the electron-positron field. This object, introduced within the scope of electron relativistic mechanics, was considered in the same way as previously had been considered the electromagnetic field of radiation subject to quantization. It was presented as some integral dynamic system having both wave and corpuscular qualities. Quantum nature of this system was described by introducing operators that influenced the wave function (state vector) of the system, which had been defined as a function in the space of filling numbers (particles corresponding to numbers – electrons and positrons, which were in certain quantum states and formed electron-positron fields). Wave functions $\psi(x)$ and $\overline{\psi}(x)$, which characterized states of electrons and positrons in Dirac's equations, were considered as the main operators of the field. Influence of these operators upon the field state vector changed the filling numbers; that corresponded to description of the field in terms of creation and annihilation of electrons and positrons in certain quantum states.[44]

Thus scientists created the mathematical theory of free quantized electron-positron field. The notion of such field made them reformulate the problem of theoretical description of quantized electromagnetic field interacting with the sources. Now it emerged as the problem of interaction of corresponding quantized fields.

Foundations of the mathematical apparatus describing this interaction were found at the third stage of forming quantum electrodynamics. The said apparatus consisted in a system of equations which united equations for quantized electromagnetic and electron-positron fields (correspondingly Maxwell's and Dirac's equations for operators of the fields). Besides, it included methods of their approximate solution by means of the perturbation theory which had been developed within non-relativisticic quantum mechanics and then transposed to the sphere of interaction of quantized fields. In quantum electrodynamics such interaction is presented as scattering of corresponding particles (electrons, positrons and photons) connected with their mutual transformations.[45] First the processes of dispersion were described only in first approximation of the perturbation theory. This became foundation for the theory of interaction of quantized electromagnetic field with charges. The theory allowed to describe and explain two types of processes: 1) transition of electron (or positron) from one state into another with emitting a photon and 2) formation or absorption of electron – positron pairs accompanied by absorption or emitting of photons.

Attempts to explore interaction of quantized electromagnetic and electron-positron fields in other approximations of the perturbation theory not only failed to make the results more precise, but even led to mathematically meaningless expressions. Observable magnitudes for characteristics of electrons and positrons, i.e. charge, mass and other connected magnitudes got infinite expressions in the form of divergent integrals.

The problem of construction of mathematical apparatus, which would take into account higher approximations of the perturbation theory, was solved only at the fourth, final stage of evolution of quantum dynamics. S. Tomanaga, J. Schwinger, R. Feynman, F. Dyson in their works developed the perturbation theory in relativisticic invariant form and suggested the renormalization method, which eliminated deviations by replacing formally computed infinite values of physical magnitudes by finite values known from experiments.

In the issue the sphere of processes, described and explained by quantum electrodynamics, considerably widened. It became possible to solve problems of scattering electron by electron, photon by electron, predict interaction of electron and vacuum, scattering of photon on photons etc.

This is the history of quantum electrodynamics taken in the aspect of forming its mathematical apparatus. It is easy to trace clearly expressed internal logic of its construction: first formalism, describing free quantized fields, was created, then on base of it the apparatus characterizing interaction of fields was constructed.

Outwardly the whole process (in its main part, at least) looks like a series of mathematical extrapolations leading to a system of equations for interacting quantized fields and methods of solving such equations. Wonderful achievements of quantum electrodynamics can be interpreted as one more evidence of efficiency of the modern method of constructing a theory. It is enough to say that the equations preceded such unexpected predictions as the one of electromagnetic vacuum (the state of electromagnetic field with the lowest energy which, despite absence of photons, influences upon charges behaviour, for instance, electron in atom). Predicted effects of vacuum polarization (effects connected with formation – due to an electromagnetic field – of virtual pairs, which cause certain distribution of charges in space, like polarization of dielectric, and have opposite action upon the external field, screening the primary charge creating this field) were quite unusual.

Nevertheless, speaking about heuristic functions of the method of mathematical hypothesis, we cannot stop at a trivial statement that in modern physics construction of a theory starts with attempts to “guess” its future mathematical apparatus.

Reflection of creators of new theories evokes a lot of judgments of this kind.[46] But this is only the first step toward understanding genesis of the theory. The main goal is to see logically necessary operations, leading to construction of new systems of theoretical knowledge, behind external features of modern investigation. In this respect we would like to pay attention to two important factors which refer to the process of becoming of quantum electrodynamics: 1) stipulation of putting forward mathematical hypotheses by the picture of physical reality preliminarily accepted by the investigators and 2) correlation between construction of the apparatus of the new theory and creation of a theoretical scheme which provides interpretation of this apparatus.

Quantum mechanical picture of the world and its role in forming the mathematical apparatus of quantum electrodynamics

Tracing the shifts of mathematical extrapolations in the history of quantum electrodynamics, we inevitably face the problem of initial ideas, bases for this or that extrapolation. Here it becomes clear that the putting of theoretical problems and indication of the ways of their solving were generated (at starting point, at least) by physical picture of the world grown out of the development of quantum mechanics. In that image the physical reality was depicted as two linked layers: macro and microlevels, and microlevel physical systems were considered as objects included in certain macroconditions and expressing their wave-corpuscular nature. In “operational” aspect the idea of wave-corpuscular features of microobjects was revealed by means of the complementarity principle. An object was regarded as a physical system which essential aspects, expressing in macrocircumstances strictly fixed by certain devices, could turn out mutually eliminating.

But that they were regarded as some kind of projections of an integral whole, united within one and the same method of description as complementary characteristics, discovered the specificity of the microobject.

The investigator who accepted this picture of physical reality had to take into account two possible aspects of considering physical systems: from the directions of their macro and microstructure. Correspondingly, he should apply a certain method of description of the system (classical or quantum mechanical). The connection between macro and microlevels of physical reality stipulated the connection between mentioned description methods within the correspondence principle[47].

We may find the decisive role of such picture of the world in putting initial problems of quantum electrodynamics, if we take into consideration the following. The program of quantizing fields was based on extrapolation of methods of quantum mechanics of points to a new sphere – fields and their interactions. But, in order to realize such extrapolation, scientists first had to see resemblance of fields with already studied quantum mechanical systems. Such view of fields was not at all evident because known and familiar quantum systems, physics had dealt with before quantum electrodynamics was constructed, in classical limit could be regarded as systems of a finite number of particles (systems with a finite number of degrees of freedom). Here, in a quantizing field, a classical analog was a continuum medium which could be compared with a dynamic system with an infinite number of degrees of freedom. That is why extrapolation of quantum mechanical description to the new area required certain justification. It could be provided by the quantum mechanical picture of the world which fixed the most general features of discernment of quantum objects. Previously collected empirical and theoretical knowledge of microstructure of electromagnetic interactions revealed such features of electromagnetic field (dualism of wave-corpuscular qualities). On this basing electromagnetic field was considered as an integral system which had quantum nature. Then this type of consideration was extended to electron-positron field. But such transfer was as well connected with functioning of quantum mechanical picture of physical reality, as consideration of an electron system in the image of electromagnetic field stipulated non-standard vision of it. The electron system now acts not as a mere multitude of quantum mechanical particles, but as an integral object – field whose separate quanta are particles belonging to the system.

Such vision was unusual since there was no classical analog for such an object (unlike quantized electromagnetic field which has a classical analog, the idea of electron field is meaningless in classical physics: in classical language electrons are particle with a finite – in principle – number of degrees of freedom).

We may follow T. Kuhn and characterize such approach to new consideration of electron system as a sort of gestalt-switching caused by change of model of vision in investigational situations. It is important that the latter was prepared and happened due to an already formed picture of the physical reality.[48]

Just as the picture of the world identified field and set of quantum mechanical particles as objects of the same nature, having the same combination of qualities (wave-corpuscular dualism), so it was possible to choose any of these objects as a model for considering the other (possibility to consider field as a system of particles, or to define a system of quantum particles as field).

Thus, the picture of the world in physics contributed to the idea of fields as special quantum objects which are to be theoretically described. This was the foundation for formulating initial investigational problem, which led to creation of quantum electrodynamics. The picture of the world served as stimulus to put forward such a problem, and it also pointed out the ways to solve it. These ways were founded in transfer of mathematical structure of quantum mechanics of points to the new area (fields and their interactions). Field was to be quantized in the same way as non-relativistic quantum mechanics did with systems of particles. On this base the method of secondary quantizing was developed. It provided transition from equations describing classical electromagnetic fields, and the ones describing quantum mechanical particles, to equations of quantized fields. Taking into consideration what was said about the role of physical picture of the world in constructing mathematical apparatus of quantum electrodynamics, it would be interesting to compare the modern way of investigation and models of theoretical investigation in classical physics, for instance, method of constructing a theory used by Maxwell (described above). The comparison shows that, at least in initial points, there is no sharp rupture between traditional and modern ways constructing a theory, despite the fact that in 20th century physics theories are constructed by the method of mathematical extrapolation. In both cases the investigator first "guesses" new equations due to directing influence of the picture of the world, which defines the putting of theoretical problems and points at the sphere of mathematical means, which would provide construction of a theory. The new element in modern investigation, along with explication of operational aspects of the picture of the world, is more active reverse influence of even early studies of mathematical synthesis upon the picture of the world. In the history of quantum electrodynamics we can see examples when the mathematical apparatus being created made scientists correct the quantum mechanical picture of the world from the point of view of relativistic ideas. The need in such correcting was caused by the requirement of Lorentz-invariance of the equation created (Lorentz-invariance of classical electrodynamics equations, when synthesized with the formalism of quantum mechanics, should be transferred to the equations of quantized field). But after the general relativity theory had emerged, to require Lorentz-invariance meant to accept relativistic notions of space-time. Consequently, such notions were to enter the quantum picture of physical reality in hidden form. Though the program of joining of quantum and relativistic notions within the framework of an integral physical picture of the world was accepted by all investigators after quantum mechanics had been completed, the first real steps toward its realization were made only in the process of constructing relativistic quantum mechanics and the quantized fields theory. In any case, it was stipulated by the very character of the mathematical formalism of the new theory, and that is why creation of the latter may be regarded as a considerable contribution to construction of the quantum-relativistic picture of physical reality.[49]

Paradoxes of the theory created and the problem of interpretation

The second important aspect of modern investigation is connection between mathematical hypotheses and procedure of construction of theoretical schemes.

In analysis of modern theoretical activities this side is usually lost sight of, because search for mathematical structures, especially at the early stages of formation of a theory, becomes the cognitive task number one. The problem of interpretation emerges only when the mathematical apparatus is already quite developed.

So we come to an impression that mathematical formalism of a developed theory is created independently from its interpretation, by a series of mathematical hypotheses realized in succession. Apparently the history of quantum electrodynamics proves it is right. But deeper analysis reveals that if we agree with it, we will have to make a very strained statement.

As we have emphasized above, equations of physics cannot exist outside of connection with theoretical schemes. Otherwise they would be purely mathematical statements but not expressions for laws of physics.

Since the process of reconstruction of equations taken from already formed spheres of theoretical knowledge into a new sphere always stipulates translation and redefinition of the corresponding abstract objects, then any mathematical hypothesis inevitably introduces a model which is supposed to be the theoretical scheme of the new sphere of physical processes. This model is reflected in the picture of the world and obtains ontological sense. It determines the initial semantic interpretation of the created formalism of the theory. At this stage, usually there is no empirical justification, so empirical sense of many magnitudes linked in the equations may be unclear. But their semantic interpretation doubtlessly should exist. Until some moment this interpretation encourages development of the mathematical formalism of the theory. The process of working out the mathematical apparatus of quantum electrodynamics is a good illustration. Let us take, say, the first stage of development of the apparatus. In the course of quantizing of electromagnetic field, the quantities of Maxwell's equations were tied in a new network of relations, in accordance with the principles of quantum mechanical description. Correspondingly, abstract objects transferred from classical electrodynamics and quantum mechanics to the new area of theoretical knowledge, also get new features. This was how, along with mathematical formalism, a preliminary theoretical scheme characterizing microstructure of electromagnetic field was created. Its authors introduced fundamental theoretical constructs: states of electromagnetic field and classical observables, whose probabilities of numeric values are correlated with the state of field. It was supposed that field described by the wave function (state vector) ψ_{nk} can be defined through superposition of some elementary states k', k'' etc., and to each of them these correspond photons (quanta of field) which are in the given state ($n'_{k'}$ photons in state k', $n''_{k''}$ photons in state k'' etc.). The field state vector allows to fix probability of emergence of photons in every "elementary" state.

In ontological aspect, which corresponds to reflection of this scheme in the picture of the world, it corresponded to the idea of electromagnetic field as a system with a varying number of photons, which appear in certain state with certain probability.

At the same time the theoretical scheme expected that the field state vector should be connected with some probability of observation of classical field components in a point. It followed from the basic principles of quantum mechanical description, in accordance with which the apparatus of quantized electromagnetic field was composed. According to these principles, operators of the field should be juxtaposed with physical quantities whose numeric values can be determined exactly at macroscopic registration level by a device set for measuring the corresponding value. The probability of these quantities is determined by the field state vector (or, squared modulus of the wave function). For example, field could be characterized by field strength operators $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$, so the experiment should give values $\mathbf{E}$ and $\mathbf{H}$, corresponding to mathematical expectations of these operators.

The considered theoretical scheme at the early stages was accepted without the procedure of its empirical justification. For instance, there was no special verification of the question: how legitimate is to transfer such idealizations (abstract objects) as field in point, taken from classical electrodynamics, to a new area of interactions. The ideas of classical electrodynamics that the states of field can be characterized by strengths **E** and **H** in a space-time point were conserved within the framework of quantum mechanical description of electromagnetic field. Such description introduced only one evident change to the classical notions: it claimed to use classical observables in order to characterize the field state taking into account basically the statistic character of expectation of the concrete values, but put no limitations for possibility to determine exactly each of the values for each separately taken value measured. Therefore at the early stages the preliminary theoretical model of quantized field of radiation, as its determined semantic interpretation of corresponding equations, was accepted as legitimate from the point of view of empirical sense as well. In any case, empirical interpretation of the magnitudes linked in the equations seemed then obvious and easy to realize according to patterns of standard quantum mechanical description.

The conviction that the introduced theoretical models are quite reliable for some time encouraged development of mathematical formalism of quantum electrodynamics. It is enough to say that immediately after quantizing electromagnetic field attempts were made to build a similar apparatus for description of electron field.

But successful progress toward generalizing equations of quantum electrodynamics was stopped, when in the very foundation of the theory paradoxes were found. It became clear that classical field strengths in a point cannot have exact value. If the field consists of separate quanta, appearing and disappearing with certain probability in different quantum states, chaotic fluctuations of every component of the field in a point are always possible.

Thus, two equally fundamental principles of the quantum field radiation theory, which seemed completed – 1) state of the field can be characterized by classical values of components of the field in a point and 2) the field is a system with a varying number of photons filling certain “elementary” states whose superposition characterizes the field – resulted contradictory. Emergence of such contradictions destroyed the primary theoretical scheme and made the corresponding mathematical apparatus physically meaningless.

For methodological analysis this circumstance is of paramount importance. It leads us to the conclusion that at a certain stage of constructing apparatus of modern theory it is mandatory that mathematical hypotheses should be supported by analysis of theoretical schemes and their constructive justification. In other words, progress in the plane of mathematical formalism can be relatively free only until some stage, and then it can continue only in case it is correlated with movement in the plain of physical contents.

Paradoxes discovered in the primary variant of the theory of quantized electromagnetic field were one of very characteristic moments of modern theoretical investigation. Mathematical hypothesis, altering connections between theoretical constructs of preceding equations, provide such construct with new features, and one feature may rule out another. Exactly that happened when constructing the apparatus of quantized radiation field, when scientists tried to synthesize equations of Einstein-Lorentz electrodynamics with a quantum mechanical method of description.

The paradoxes of quantized radiation field were a signal of emergence of constructs with mutually eliminating features in the theory.

This situation was similar to already discussed paradoxes of Rutherford’s model of atom and Dirac’s relativisticic electron theory. As to history of classical electrodynamics, we analyzed a similar situation speaking about the period in Maxwell’s work when he tried to introduce an equation for electromagnetic induction basing on the model of stationary force lines.

Naturally the first efforts to eliminate paradoxes had to be aimed at finding non-constructive elements inside of the theoretical scheme, introduced together with the apparatus of quantized radiation field at the stage of mathematical hypothesis. It was necessary to do some selecting among theoretical objects, to discover the ones "responsible" for paradoxes and replace them by new abstract objects which would be fit to the procedure of empirical justification.

The first part of this problem was partly solved in the work by V. Fok and P. Jordan,[50] and more completely by L. Landau and P. Peierles.[51]

Strictly speaking, the paradoxes could be caused either by definition of the field state vector as (in contrast to familiar quantum mechanical approach) superposition of states with a varying number of particles (photons), or by a hidden assumption that observable magnitudes are strengths in a point.

Since the idea of field as a system with a varying number of photons allowed to explain well known dependencies of absorption and emission of light quanta by atoms, then the corresponding characteristics of vector-state were based empirically and got constructive meaning. Then it was to be checked whether classical observable fields possess such meaning in a point. Certain intellectual experiments we held to understand if we could, introducing the said observables into the new area, conserve their main quality – fundamental measurability (i.e., possibility to get exact values of every observable magnitude using a classical device). Intellectual experiments made by Fok-Jordan and Landau-Peierles revealed that if both quantum and relativistic effects are taken into consideration, measuring strengths of quantized field in a point is impossible.

They came to this conclusion by the following reasoning. According to the approach typical for classical theory, intensities of **E** and **H** are determined through influence of the field upon a charged test body. In case of component **E** that influence is measured through impulse passed to the experimental charge, in case of component of **H** – through moment of magnet or some distribution of charge-current. Just as we are to measure the field in a point, so the experimental body should also be a point. Suppose, the task is to determine component E_x. For this we need a point charge. Thought experiments by Fok-Jordan took an electron accelerated by the field, while experiments by Landau-Peierles admitted a point particle of any nature (it could have, a particle with, for instance, greater mass than electron).

Measuring the field component means that the impulse by the experimental particle from the field should be registered by a classical device. In this case the value of this impulse will let us determine the value of the corresponding field component exactly.

Thus, the procedure of thought measuring the field components in a point in moment t stipulated two conditions: 1) localization of the experimental particle in the given point of the field at moment *t*, where the particle gets an impulse, 2) exact registering this particle by a classical device.

Since the experimental particle submitted quantum laws, both conditions were basically impracticable. The first was impossible because of indeterminacy relationship: localization of the particle in a point led to its fundamental indeterminacy Δp in the value of its impulse. Consequently, the value of the field strength can be accurate up to no more than Δp.

The second condition is impracticable because of two reasons. First, it was impossible to register exactly the impulse of the point experimental particle because of quantum laws of energy-impulse exchange of the particle with the device. Since there is indeterminacy relationship $\Delta\varepsilon\Delta t \sim \hbar$ (ε – energy, t – time), so collision of the particle with the device when during time Δt the former gives its energy to the latter causes indeterminacy $\Delta\varepsilon$ in the value of this energy. The connection between energy and impulse causes corresponding relation between time Δt and impulse measured P_x. This relation is expressed by formula $|v''_x - v'_x|\Delta P_x \Delta t \sim \hbar$ (1),[52] where v'_x and v''_x – velocities of the particle before and after measuring, Δt – time of measuring, ΔP_x – indeterminacy in the value of the impulse of the particle.

Accounting of relativistic effects stipulate that $|v''_x - v'_x|$ should not exceed light speed c. Consequently, on the base of (1) there emerges relation $\Delta P_x \Delta t \geq \frac{\hbar}{c}$, according to which, the less time measuring the impulse of the particle takes, the greater indeterminacy in the value of the impulse is measured.

In measuring component E_x in a space-time point it is stipulated that the impulse of the experimental particle should be registered practically immediately. We are to reduce the period of measuring $\Delta t \to 0$ infinitely to avoid side effect upon the impulse of test particle. But in this case ΔP_x will increase infinitely. So observance of one necessary condition, which would provide exact measuring field strengths in a point (practically immediate registering the impulse of the experimental particle) leads to fundamental impracticability of the other condition, as much again necessary condition (exact measuring of that impulse by a classical device).

Secondly, exact registering the impulse of the experimental particle is impossible because the particle is radiating at the moment of collision with the device and starts interacting with its own radiation. The influence of the particle's own radiation can be taken into account only with a basically irremovable error.[53]

Thus, measuring the field component by a point experimental particle we face three irremovable types of indeterminacy: because of its localization in a point of the field; because of its interaction with the device during time Δt; because of its interaction with its own radiation.

In its turn, indeterminacy of impulse of an experimental particle means fundamental impossibility to measure every component of quantized radiation field strengths in a space-time point. Consequently, the theoretical constructs (of field in a point) are meaningless when extended to the area of quantum processes. From the point of view of methodology, it is important to pay attention to the structure of the intellectual experiments which led to this conclusion. It is significant that they took into account not only quantum, but also relativistic effects which were expressed when the field components changed, and because of this they expressed – in idealized form – characteristic features of possible experiments and measurements in the new area. Analysis of measurability of field in a point shows whether we can introduce the mentioned abstract objects as idealizations basing on real specificities of experimental-measuring activity in quantum field studies. Here we can easily see characteristic features of constructive introduction of abstract objects.

The negative result meant that the objects mentioned are non-constructive elements in the preliminary theoretical scheme. Discovery of such elements was the first necessary step toward rebuilding the theoretical scheme on a constructive base. Further task was to change it so, on the one hand, to conserve the constructed apparatus of the theory, at least in its basic characteristics, on the other hand, to justify the theoretical scheme introduced by idealization of experiments and measurements related to the new area of interactions. In the history of quantum electrodynamics this problem was solved due to cognitive activity known as Bohr-Rosenfeld measuring procedures.

Idealized procedures of field measuring and interpretation of the apparatus of quantum electrodynamics (the initial idea of Bohr-Rosenfeld procedures)

Bohr-Rosenfeld measuring procedures occupy a special place in settling quantum electrodynamics, because it was thanks to them that a non-contradictory interpretation of its mathematical apparatus was developed. At first Bohr and Rosenfeld interpreted the apparatus of quantized radiation field, and then revealed the physical meaning of the formalism which described interaction of the field with quantized sources. We will try to show that Bohr-Rosenfeld procedures are a typical example of stage-by-stage shaping of a constructively justified theoretical scheme in the modern epoch of theoretical investigation.

First we would like to describe the historical situation in which the cognitive activity took place. After Landau and Peierles had proved that it was meaningless to apply the idea of field in a point for description of quantum processes, quantum electrodynamics entered a period of crisis of its foundations.

First, it was entirely unclear, how to change the theory in order to get non-contradictory interpretation of the mathematical apparatus introduced. What is more, nobody knew if it was possible in principle. Only retrospectively (we retold Landau's and Peierles's work mainly from the point of view of its logically necessary contribution to construction of the new theory) can we see that the only right position in those circumstances was the desire to reconstruct the initial theoretical scheme so that it could allow only to reject use of field quantities in a point but conserve the idea of classical observables (field strengths).

But this step was not at all easy. In any case, the investigators who had discovered paradoxes of impossibility to measure the field components failed to do the necessary work themselves.

At that stage of development of electrodynamics Landau and Peierles regarded their results not as a proof of limitedness of the initial interpretation of the mathematical apparatus of the theory, but as evidence that this apparatus was worthless and basically could not bear any physical meaning. It seemed their point of view had solid ground. The state of electromagnetic field in classical theory was characterized by strengths **E** and **H**. As to quantum mechanical description, it contained a well known principle: quantizing of a system limits simultaneous measurability of complementary (in Bohr's sense) pairs of quantities, but puts no limitations to measurability of a separate magnitude (classical observable). So, Landau and Peierles believed it was impossible to get the exact value of strengths **E** and **H** taken separately, it meant that there are no ways to apply quantizing methods to such an object as radiation electromagnetic field.

Later Landau and Peierles extended this conclusion to quantizing field sources. They showed that determination of state of electrons, provided that they are measured by means of a point experimental particle during a very short period of time, led to irremovable indeterminacies of each of the separate quantities characterizing the state of electron.[54] It could be automatically concluded that it was impossible to create a quantum mechanical description of the field sources, or, what is equivalent, to construct a quantized electron field theory.[55]

Last, Landau and Peierles appealed to numerous difficulties which had emerged in quantum electrodynamics with efforts to find the physical meaning of its apparatus, extended through a series of mathematical extrapolations. They meant difficulties with interpretation of Dirac's equations (they included solutions with negative energy values) and difficulties in search for sense of so called zero fluctuations of electromagnetic field. The former have already been discussed. We are only to remind the reader that though Dirac had already proposed an interpretation of his equations, a lot of investigators who worked on the quantum theory of field first took his model of "holes" as quite artificial[56] (especially since at the early stages there existed a tendency to connect the "holes" with presence of proton, which led to contradictory conclusions in calculations of mass-energy of particles; only later there appeared the hypothesis of positron, empirically proved only in 1932). Under those circumstances Landau's and Peierles's thesis that quantum mechanical methods cannot be applied in the relativistic area did not at all seem unconvincing nor illogical.

Besides, there were more difficulties connected with paradoxical corollaries of the mathematical apparatus describing quantized radiation field. According to them, the energy of zero energy level of the field was infinite.[57]

Landau and Peierles linked those corollaries with the idea of fundamental incommensurability of the field components in a space-time point. They indicated that it follows from the expression for indeterminacy of each of the components **E** and **H** $\Delta E \geq \frac{\sqrt{\hbar c}}{(c\Delta t)^2}$ and $\Delta H \geq \frac{\sqrt{\hbar c}}{(c\Delta t)^2}$ (where ΔE – indeterminacy in the value of electrical intensity, ΔH – indeterminacy in the value of magnetic intensity, Δt – indeterminacy in the time of measuring, c – light speed, $\hbar$ -Planck's constant), that if we decrease the time of measuring Δt to zero (to realize measurement of the field in time point t_1) correspondingly ΔE and ΔH will tend to infinity. From this position the conclusion of infinite values zero energy level of the quantized field was presented as a special type of incommensurability paradoxes.[58]

Taking all this into consideration, we may understand why there appeared a tendency to preserve quantum mechanics methods only within the sphere of non-relativistic processes.[59]

The crisis of the early 1930s in quantum electrodynamics provides one more proof that fundamental theories of higher degree of generality are constructed differently from the way it seems when we use a simplified approach to mathematical extrapolation. Usually for such theories it is impossible to build mathematical apparatus at once by means of a continuous series of mathematical hypotheses and then find interpretation of the ready formalism. Quite long progress in mathematical means enlarges the danger of hidden introduction and accumulation of non-constructive objects in the theory. So it is urgent that we should use special analysis of physical sense of already constructed links of the mathematical apparatus and their interpretation as early as at intermediate stages of forming of the theory's fundamental laws.

In such periods the central point of the research passes to the area of search of theoretical models which could provide interpretation of the equations introduced.

Let us consider the logic of this search at the period of struggle against crisis in quantum electrodynamics.

First of all, to provide progress in the development of the theory, it was necessary to formulate the theorem correctly. To do this, the investigators had to see in the incommensurability paradoxes only limitations for classical idealizations of the field strengths, but not prohibition to use quantum mechanics methods for description of relativistic processes.

Correspondingly, the investigation task was to be formulated as search of classical observable, which would be fit for characterizing wave properties of quantized electromagnetic field (without using field strengths in a point). But after Landau's and Peierles's work many investigators would have regarded such formula as inherently contradictory.

Here we have come to a very important aspect in evaluation of the crisis caused by incommensurability paradoxes. The fact is that Landau and Peierles, speculating on unsuitability of quantum mechanical description in relativistic area, unwittingly accepted one ill-founded assumption which caused their too categorical conclusion. We mean the supposition that the test particle, used for measuring field quantities, is always a point particle and is of quantum mechanical nature. Such idealization of the test particle was legitimate when the problem dealt with measuring instantaneous value of **E** and **H** because of the problem itself. Indeed, if we measure the force which is to influence upon the test particle in a point of the field, it means that the particle should be located in that very point at the moment given. But for this the particle itself should be regarded as point. Naturally, in measurements in very small areas only microparticles which submitted quantum mechanics laws, could satisfy these requirements.

But then the idea of quantum mechanical test particle was automatically transferred to any situation of idealized measuring field magnitude in quantum area. Landau and Peierles concentrated on its interaction with the device and discovered that here that increasing indeterminacy of the impulse of quantum test particle inevitably appears, if measurements take short periods of time.

For determination of the magnitudes characterizing state of quantum system in relativistic area only short periods are necessary, because here the state of the system can change rapidly enough during the time of measuring. So, it would be easy to conclude: it is impossible to register the corresponding parameters of the test particle exactly, and, consequently, to determine classical observables characterizing quantum system in relativistic area.

This conclusion would be logically immaculate only in one case: if we assume that the means of measurement is a point quantum test particle.

It just never occurred to the majority of scientists to throw doubt on that assumption. But its critical analysis led to decisive clearing on the situation. It was N. Bohr who carried out this analysis. Bohr put forward an idea which provided overcoming the crisis: he proposed to use in the intellectual experiments testing measurability of field quantities a classical experimental body instead of point quantum mechanical particle. Historians of quantum electrodynamics, including Bohr's co-author L. Rosenfeld who brilliantly depicted that "heroic" (Rosenfeld's word) period of the development of quantum physics, usually emphasize great productivity of Bohr's idea, but they rarely reflect the logic of its emergence. Though, from a methodological point of view, understanding of this logic is of extreme importance, because here Bohr's idea is presented not only as a product of highly gifted intuition and a "spontaneous guess" but also as a logically necessary step of theoretical investigation. Probably, the main condition for this step was analysis of the notion of experimental body in the aspect of specificities of quantum mechanical measuring. Let us examine this point in more detailed way.

It is well known that the most part of investigations connected with experiment stipulate use of a special physical agent – a means to transfer the information about the state of the object measured to the observer. The role of such agent may be played by, for instance, a well charged body in experimental measuring of electrical field strengths, some volume of liquid in experimental measuring of temperature, a polarized beam in experiments with crystals etc. All agents of this kind are concrete variations of experimental bodies.

The construction of correspondence rules (operational definitions) is based on thought experiments which are just idealizations of real experimental-measuring activity. In this connection theoretical discourses of physics start using a special idealized object – experimental body. Its general features are derived from analysis of functions of concrete variations of experimental bodies in experiment. Such analysis lets us distinguish three basic and necessary features of experimental body: 1) it should interact with the physical system studied, changing its state in correlation with the state of this system: 2) it should translate the accepted state until interaction with the register device;[60] 3) its interaction with the register device should give the observer enough information about the state of the experimental body, that he could judge the state of the physical system studied (in this case the observer comes to conclusions about values of physical quantities characterizing the state of the system measured, basing on the data from the device).

The features mentioned of experimental bodies can be easily illustrated by simple examples. Suppose, we are measuring temperature with a mercury thermometer. The role of experimental body belongs to a volume of mercury in a glass vessel. The possibility to use it as an experimental body is conditioned by the following: 1) change of the volume of mercury (state of the experimental body) is correlated with the change of temperature of the bodies observed: 2) within certain limits we always can fulfill the requirement than, until observation of the scale (register device) which fixes the height of the mercury column, either the height (volume of mercury) will not change at all under external influences, or, if such change still takes place, it can be taken into account using corresponding equations (for instance, the heat balance equation); 3) when the height of the mercury column is registered by the observer, this act by itself does not change the state of the experimental body so, that it could prevent the body from transferring information about the temperature measured (this condition is practicable because we can, for instance, ignore the influence of light upon the mercury column, take into consideration in the very construction of the thermometer in graduation the change of the volume of mercury caused by its heat exchange with the scale etc.). In other words, really we can use a container with mercury as a means of temperature measuring, because the criteria of correlation, translation and possibility to register the state of this experimental body as the result of interaction with the object measured are observed. It is easy to see that requirements of this kind are observed in any experiment concerning any experimental bodies. They are common and significant features of the whole class of experimental bodies, which is why they form the sense of the corresponding idea.

In experimental-measuring situations of classical, quantum and quantum relativistic physics the indicated features are specified in several special assumptions.

For example, classical physics assumed that, first, the experimental body does not influence upon the state of the object studied during their interaction; second, that perturbing influences upon the experimental body from the register device at the moment of registering can be ignored. Of course, both assumptions are idealizations, but they take into account circumstances of real experiments and measuring in classical area. No doubt, perturbations caused by the experimental body always exist, and the experimental body itself also is object of influences from the register device during the period of time which is needed for measuring (it starts with interaction of the experimental body with the device and ends with finish of the device's indication). But in such experimental-measuring situations, where elements of the system – experimental body and register device – are classical objects, it is always possible either to provide such conditions of experiment that these perturbations would be negligible, or to take these disturbances into account by means of calculations and corrections.

In the measurements of quantum objects all these assumptions lose their legitimacy. In such measurements the physical system whose state is measured is always a microsystem, while the device registering quantities, which characterize the state of that system always belongs to macrolevel. The experimental body, as mediator between the microsystem measured and the experimental body, should interact with the former as a microsystem. Existence of quantum of action prevents us from ignoring the reverse influence of the experimental body upon the object measured, so in quantum area we should avoid an idealized image of a register device, which does not influence upon the object of measuring. This rejection means that in quantum mechanical measurements, unlike classical situations, we cannot identify the state of the system before and after measuring. Reproducing the same conditions and repeating the same measuring of the "prepared" state of the system, we will get different results every time. But each of them can be expected with a certain probability, if we characterize the state of the system before measuring by some wave function. Such connection between mathematical expectation of the results of measuring and characteristics of the state of the system measured allows us to predict (as we know the wave function) the results of measuring (measurements of quantum systems are not repeatable but predictable).[61]

Thus, quantum mechanical character of interaction of the experimental body with the object measured does not prevent the observer from receiving information about the state of the object. The experimental body takes part in quantum interactions and changes its state in correlation with the state of the system studied (though the characteristics of the state are different from those in classical physics). In this sense the first feature characterizing experimental bodies is still valid, when their interactions with the object measured submit quantum laws.

But there exists one more interaction: the experimental body transfers information about the object to the register device. If the experimental body interacts with the device also in accordance with quantum laws, how can it influence the functions of the experimental body? Can it, being a quantum particle, first, translate its state in interaction with the system measured until interaction with the registering device, and, second, transfer without errors the information about the system measured to the device?

In non-relativistic area, when the state of the quantum system is constant during a period of time comparable with the period of measuring, it is possible to fit both conditions.[62] But in relativistic area the situation is entirely different, as Landau – Peierles investigation proved. Here the function of experimental bodies belongs to quantum particles, and observation of one condition automatically excludes the other. The test particle enters interactions in which the state of systems changes during period of time comparable with the period of measuring. After interaction with the system measured the test particle – before it transfers information to the register device – may undergo a new type of influence from the system, since interaction in relativistic area is related to creation of new particles, generated by both the system measured and the experimental point particle itself. The longer is the time of measuring, the harder is the influence of the particles mentioned upon the experimental particle whose state is being transformed. Hence, it is necessary to register the state of the experimental particle as soon as possible after its interaction with the system measured. But, as we have already mentioned, observance of this condition leads to irremovable increasing errors in determining magnitudes characterizing the state of the test particle. Thus, requirements of translation of the state transferring information about the system measures, and requirement of registration of this information without errors are mutually eliminating for a point quantum mechanical particle used as experimental body in measurements in relativistic area. Measurements made through such particles resulted unpredictable.

Investigators saw that a point particle, when used in relativistic area as experimental body, loses its features which could make it belong to the class of experimental bodies. This was the key moment in transition from Landau-Peierles analysis to Bohr-Rosenfeld procedures. From Landau-Peierles intellectual experiments the only conclusion could be drawn: a quantum mechanical particle cannot be experimental body in measuring quantized field, but from this it did not follow that methods of quantum mechanics are inapplicable in relativistic area. Such conclusion considerably changed the situation. Now the task was put into practice idealized procedures of measuring in quantum relativistic area without quantum mechanical experimental bodies.

There was only one way to reach this goal: to return to classical experimental bodies. This approach automatically eliminated all problems connected with translation of state of the experimental particle and its interaction with classical device. If the experimental body is a classical object, in description of its interaction with the register device, it is absolutely correct to apply classical idealizations, which allow either to ignore the perturbing influence of the device or to take it into account by means of corresponding corrections. The only question to solve was that of interaction of the experimental body with the quantum object.

Evidently, such interaction should proceed in accordance with quantum laws. How can it be, when the experimental body is not a microparticle, but a classical object? The answer was simple: quantum systems always include description in terms of macroscopic parameters, and quantum interactions by definition should have in their last stage interaction with a classical device. The latter can be accomplished as early as at the first step (Mandelshtam’s words), when we deal with direct measurements, and through a series of further links, where the measurements are indirect.

Application of classical experimental bodies as means of obtaining information on quantum systems in relativistic area may be carried out in two variations: 1) investigator abstracts himself from detailed examination and calculation of atomic structure of experimental bodies, considering the latter as a special part of a classical meter unit adjusted to measuring corresponding field quantities and 2) the said structure is taken into account, i.e., the experimental body is considered as a kind of aggregate of microparticles (for instance, distributions of electrons in certain volume forming experimental charge), which is set for interaction with the object and then interacts with the device, presenting itself as a classical object.

In the first case the measurements are direct, but, unlike direct measurements in non-relativistic area, here we should bear in mind the measured quantum objects’ ability to change their state during period of time comparable with the period of measuring. Because of this there are restrictions first marked by Landau and Peierles (but these restrictions now concern not experimental bodies, but the objects measured and are their immanent characteristic). These restrictions consist in the following: to measure a separate classical quantity determining the state of the system, we need time, not longer than the period during which the state described by the quantity measured can be disturbed. If this is beyond our possibilities, measuring not pairs but a separate quantity will give a certain indeterminacy (for instance, for coordinate q and impulse p of a point particle in relativistic area there emerge indeterminacies $|\Delta p \sim \frac{\hbar}{c\Delta t}|$ and $|\Delta q \sim \frac{\hbar}{mc}|$).

In the second case, when atomic structure of the experimental bodies is taken into account, measurements are more like indirect ones. Here we can trace quantum effects of interaction of the object measured and the experimental body, say, some distribution of charge accounting microstructure of this distribution. Such interaction in relativity area causes creation of new particles, and that makes certain contribution to macro–effects fixed by the register device.

So, a classical experimental body used in quantum measurement has dual nature: at microlevel it interacts with the object measured, at macrolevel with the register device. Thanks to this it transfers information about the object measured to the observer and works as means of measuring quantum systems.

The given analysis may be regarded as logical reconstruction of the cognitive activity which secured transition from Landau-Peierles conclusions to Bohr’s fundamental idea.

We would like to draw the reader’s attention to the fact that analysis of functions of experimental bodies in idealized measurements is a special investigation, which uses metatheoretical language as the language of quantum electrodynamics (or any other concrete physical discipline: classical mechanics, non-relativistic quantum mechanics etc.). This is the language of logical-methodological analysis, an instrument of analysis of common features of experimental bodies and understanding of the very idea of “experimental body”.

This peculiarity is important because it discovers exit (characteristic for investigation) to the area of methodological problems every time when science comes across seemingly unsolvable paradoxes. Solution of the paradoxes (or justification of impossibility to solve them with further reconstruction of previously suggested investigational program) is provided by metatheoretical investigations connected with analysis of the most general features of objects studied and comprehension of methods of theoretical cognition.

In this respect let us mark that analysis of function of experimental body was purposeful, on the one hand, by general methodological condition to link basic quantities of the equations with experiment by means of corresponding idealized measuring, on the other hand, by specificity of quantum mechanical objects, which require that for their description classical idealizations should be applied. The fact that it was Niels Bohr who succeeded in this analysis has profound foundation. We should take into account Bohr’s decisive part in revelation of conceptual foundations of quantum mechanics, his permanent attention to the key problems of quantum mechanics measurement theory, his methodological erudition which let him grasp the very core of such problems and find solutions. All this gave Bohr the opportunity to be the first who overcame the psychological obstacle which had appeared due to blind using a point quantum object as experimental particle[63]. But these factors refer more to psychology of scientific creative work. In respect of logic of investigation, it is important that there existed logically necessary transition from Landau-

Peierles thought experiments to the fundamental idea of Bohr-Rosenfeld procedures. From this point of view we may say that once the problem of quantizing of fields had been raised and difficulties in interpretation of the introduced equations were found, so if not Bohr, then somebody else had to make the described steps toward the program of idealized measurements by means of classical experimental bodies.[64]

Reconstruction of the theoretical model of quantized electromagnetic field and justification of its consistency

After N. Bohr's program had been put forward, scientists started its accomplishment. The work was done in several stages.

First of all it was necessary to interpret apparatus of quantized radiation field within the framework of idealized measurements with classical experimental bodies. In case of success of this part of the program, it should be extended to the area of quantizing of sources of the field and then – to the area of interaction of the quantized field with quantized sources.

Naturally, no one could guarantee that Bohr's program of interpretation of the quantum electrodynamics equations will successfully solve all problems of the new theory. Only concrete investigation could demonstrate it. But still there was progress, as it became clear how to overcome contradictions of the previous period of development of quantum electrodynamics.

The very formulation of Bohr's basic idea showed concrete ways to positive reconstruction of previously introduced theoretical scheme of quantized radiation field.

First, it became clear, what observables were to be introduced into the scheme instead of field strengths in a point. Measurements of field components should be performed by means of classical experimental body which always occupies certain volume V, while displacement of the experimental body measuring the field strength takes certain period of time τ. So the field strengths could be determined exactly in thought experiments with classical experimental bodies only in area $V\tau$, but not in a point. The conclusion suggests itself: these magnitudes should be observables characterizing the state of the quantum field.

Introduction of such observables meant decisive change of the previous scheme (there appeared a new abstract object, and correspondingly all connections among all other elements transformed). Naturally, the new scheme gave new semantic interpretations of the equations of the theory: it meant that only quantum field strengths averaged in some space-time area (not in a point!) should have physical sense.

Clearly, such interpretation still remained a hypothesis. It could turn out that it is inconsistent with the structure of already built formalism, or requires such corrections which contradict to general foundations of quantizing fields. Instead of the past paradoxes of the theoretical scheme there might emerge new ones, and interpretation might be logically impossible. The possibility of such paradoxes and mismatches at the stage of reconstruction of the initial theoretical scheme is easy to explain, if we take into consideration the basic specificities of structure and functioning of such schemes.

First, a new element introduced to the scheme always changes correlations among all other elements. Just as such correlations are described in the equations, so first of all one has to check whether the offered modernization of the theoretical scheme will be fit for already shaped mathematical formalism or the latter should be transformed.

Second, the change of correlations among abstract objects which form the theoretical scheme can hiddenly provide the objects with such new features, which would be incompatible with the previous ones, those undergone constructive justification. So it is necessary to clear up, whether the new object destroys that constructive and heuristic meaning which was loaded into the theoretical scheme by previous development of the theory.

Clear, successful execution of this operation does not guarantee correctness of the new (reconstructed) scheme.

Even if it is proved that it corresponds to the theory apparatus and is inherently consistent, still the scheme remains a hypothetical construction. It will leave this status behind only by procedures of constructive introducing abstract objects, when the scheme is validated as a generalized model of corresponding experiments and measurements.

In this sense the final semantic interpretation of the theory apparatus appears only when its empirical interpretation is built. Their separation and consideration out of mutual influence is possible only up to certain limits. But, just as the procedures of constructive justification require a lot of work, so, before starting them, it is necessary to make sure the way of their realization is expected to be fruitful. That is why we verify correspondence between the theoretical scheme and the theory apparatus, and verify its object's inherent consistency. We will call such verification "potential interpretation", since final ("actual") semantic interpretation is formed only due to finding empirical sense of the basic quantities linked in the theory equations.

Analysis of the history of quantum electrodynamics shows that the first steps toward realization of Bohr's program of idealized measurements really were connected with potential interpretation of quantized electromagnetic field equations. Having proposed to reconstruct the initially introduced theoretical scheme into new one, in which the place of the observable components of field in a point was occupied by other observables (field components averaged on finite space-time area), Bohr first of all checked how such scheme conforms with the mathematical formalism of the theory, and then, together with Rosenfeld, justified inherent consistence of the new scheme.

Verification of the first type showed that there is complete agreement between the main idea of the new interpretation and character of the mathematical apparatus of quantized electromagnetic field.

Analyzing this apparatus, Bohr proved that there idealizations of field in a point are used only as a formal auxiliary construct and does not have real physical meaning, while field components averaged on some finite space-time area do have such meaning. It followed from the very character of commutation relations for field operators $\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$. The fact is that the commutation relations were expressed through generalized functions of the kind of δ-function introduced by Dirac in construction of commutation relations in a continuous spectrum. The fundamental feature of this function is its ability to be reduced to zero in all points except one, where it equals to infinity. The field quantities in a point should have behaved correspondingly. But δ-function has one more remarkable property: being integrated over all values of the variables, it turns into one. In commutation relations the role of arguments of generalized functions expressed through derivatives of δ-function belonged to space and time coordinates. Hence, integration with respect to some part of space-time area gave finite values for the right side of commutators of field values and corresponding uncertainty relations for these values. In other words, integrals of field components taken over finite space-time area got unambiguous meaning.

As followed from the structure of the mathematical formalism of quantum electrodynamics, that physically meaningful statements are not those of fields in a point, but those of average values of field components taken over finite space-time areas. This was the first sign of fruitfulness of the reconstructed theoretical scheme and, correspondingly, of Bohr's program of idealized measurements of quantized field components by means of classical experimental bodies.

We have to mention that the described above period of Bohr's cognitive activity is usually related "topsy-turvy" in the history of physics. It is believed that first Bohr discovered that in the mathematical apparatus only averaged field strengths are meaningful, and only then, on base of these specificities of the apparatus of the theory, came to conclusion about application of classical experimental bodies. Such statements can be found, for example, in L. Rosenfeld memoirs of his work with N. Bohr. What is more, the original text written by Bohr and Rosenfeld, dedicated to analysis of measurability of electromagnetic field, offers us a similar version[65]. No surprise that authors of historical essays follow the same way; describing development of quantum electrodynamics, they usually retell recollections of the investigators who built interpretation of the equations of quantized electromagnetic field. Yet when we regard some reproduction of a theory made by its creators, we are to keep in mind that logic of rendering results of the investigation and logic of obtaining these results do not usually coincide. Deductive posing usually starts with statements which were final results in the investigation itself. Therefore real historical progress of thought leading to certain result rarely is rendered without swerves in a scientific text rendering the obtained result. As to retrospective historical analysis of a discovery made by its authors, we should never forget that numerous publications of the obtained results, which searched for the easiest and most compact logic of rendering, are capable of quite considerable deformations of notion of ways to the desired result. We have to be very careful with historical testimonies made by creators of a theory. In this respect A. Einstein said: "If you want to learn something about their methods from physicist-theorists, judge deeds not words". Of course, it does not mean that the creators' reflection cannot give us any more or less valuable historical evidences. We only mean that not every such evidence should be treated as an undisputable historical fact, moreover, retrospective analysis in memoirs normally reproduces only key moments of the creative work, but not the progress of thought which led to them. The latter remains "behind the scenes" of empirical history of science and needs special reconstruction. No doubts, the discovery that only field averages, and not fields in a point, have physical meaning in the structure of mathematical formalism of quantum electrodynamics, was one of the key moments in construction of adequate interpretation of this formalism. But to fix the said circumstance which, by the way, was missed by almost all investigators, it was necessary to approach analysis of the mathematical apparatus from very special positions. We may say N. Bohr possessed brilliant intuition, but this is not enough to explain why other investigators (including such theorist of highest rank as W. Pauli and W. Heisenberg) who paid close attention to the discussion dedicated to problems of measurability of field, did not notice this circumstance. The reason, probably, is that Bohr's intuition was attributed to a special point of view which allowed him to see what remained unseen by other physicists. Above we tried to show that that special point was formed by preliminary analysis of the idea of experimental body in the aspect of the corner stone of quantum mechanics description – the question of relation of quantum object and classical device. This analysis reached the top intensity probably in February 1931 in Copenhagen in discussions between Bohr, on the one hand, Landau and Peierles, on the other hand. In L. Rosenfeld's mentioned works dedicated to the history of quantum electrodynamics we can find a bright description of the emotional atmosphere of those discussions[66]. Rosenfeld's text clearly shows that discussions of the foundations of measuring procedures in quantum electrodynamics and talks on status of experimental bodies preceded Bohr's decisive statement that field components in space-time points are used in the formalism of the theory as an auxiliary idealization which has no direct physical meaning. Analysis of the idea of experimental body showed that quantum particle used in thought experiments on measurability of quantized fields are not fit for the basic definitions of experimental body. From this the hypothesis of classical experimental bodies followed. In its turn, it logically led to the hypothesis of averaged field components which were to replace field in a point. The latter was what stimulated the corresponding analysis of the mathematical formalism of the theory.

Bohr postulated consistency between mathematical apparatus and reconstructed theoretical scheme of quantum electrodynamics; that allowed transition to the second stage of verification of such scheme within framework of potential interpretation. This stage consisted in fixation of inherent mutual consistency of objects forming the theoretical scheme. In particular, it was necessary to find out whether the idea of field as a system with a variable number of particles does not contradict to the idea of field averages. Both characteristics were equally indispensable for description of quantum fields, because one of them appointed corpuscular qualities (field as a system of particles able to appear and disappear in corresponding quantum states with certain probability), and the other – wave qualities (field as an integral system, described by classical wave quantities, observable values of which form spectrum of values of corresponding field operator).

Preliminary analysis showed that field strengths averaged on area $V\tau$ must undergo fluctuations because of effects of creation and annihilation of photons in this area and, consequently, cannot have exact values. Landau and Peierles had also paid attention to this peculiarity, emphasizing that fundamental indeterminacy of field components in a point extends over the averages on some area field components. Landau and Peierles saw here confirmation of their thesis of fundamental inapplicability of the term "electromagnetic field" in quantum sphere.

It might seem that the new theoretical scheme reproduced paradoxes of the old one: the idea of field as a system with a variable number of particles and the idea of field as a system characterized by classical components of strengths averaged on some space-time area are incompatible.

Yet N. Bohr and L. Rosenfeld proved that the situation with field strengths in a point and the situation with averaged field strengths were radically different. As opposed to the first situation, the second one does not lead to logical contradictions, even if we accept the idea of fluctuations. After scrupulous analysis of the apparatus of the theory, Bohr and Rosenfeld showed that in measurements of averaged field components we are to distinguish two cases: 1) when time interval of averaging τ multiplied by speed of spread of electromagnetic wave c is large enough in comparison with linear sizes L of volume V, over which averaging is carried out (i.e., $L \leq c\tau$), and 2) the opposite case, when time τ multiplied by c is small in comparison with L (i.e., $L > c\tau$). In the first case we cannot ignore fluctuations while determining averaged over area $V\tau$ field strengths. It occurs through the fact that during the time of measuring photons, emerging due to radiation, can spread from other areas to the space area V, on which strengths are averaged. Abstraction from fluctuations is possible in this case only if we accept degeneracy of quantum electrodynamics into classical electromagnetism theory.[67]

Entirely different is the situation when field quantities are averaged over area where $L > c\tau$. In this case the averaging area is not connected with neighboring areas by light signals, that is why it contains only photons which have penetrated there before (the light wave covers distance smaller than L during the time of measuring). This allows us to ignore fluctuations while determining averaged field components without losing the field's quantum features. Values of such fluctuations every time will be included in values of strengths determined in area $V\tau$ and with $L > c\tau$ they can be minimized.

Availability of such version is the decisive circumstance which makes the old and new situations of measurability of field components entirely different. It is easy to see that at consideration of the field quantities in a point the described variant ($L > c\tau$) disappears by definition (since $L \to 0$). Therefore paradoxes of incommensurability here are fundamentally irremovable.

Inherent consistency of the objects of the reconstructed theoretical scheme was the second signal of efficacy of Bohr's program. Now, after verification the theoretical scheme of quantized radiation field from the point of view of its consistency and correspondence to the character of the mathematical formalism, there were all opportunities to start the decisive moment of interpretation: the procedure of constructive introduction of abstract objects forming the mentioned theoretical scheme.

The proof of measurability of quantized radiation fields

Let us consider more thoroughly the main features of the procedures of constructive justification of Bohr's scheme of quantized radiation field. Starting their work, N. Bohr and L. Rosenfeld stipulated those initial features of abstract objects which were introduced as their definitions within the framework of the theoretical scheme of quantized radiation field and which now were to be obtained as result of idealized measurements. Such features corresponded to the main correlations of abstract objects inside the theoretical scheme and could be settled through analysis of fundamental dependences of the mathematical apparatus.

After in the theoretical scheme field strengths had been replaced by strengths averaged on a space-time area, the basic mathematical dependences of the theory which had direct physical sense were commutation rules for operators $\overline{\hat{\mathbf{E}}}$ and $\overline{\hat{\mathbf{H}}}$ of averaged fields. They occupied the places of commutation relations for the field operators in a point and formally were easily derived from them by means of integration with respect to corresponding areas of space-time. Then, from the commutation rules for $\overline{\hat{\mathbf{E}}}$ and $\overline{\hat{\mathbf{H}}}$ it was easy to get correlations of indeterminacy for averaged field components. From here followed:

1) one can always determine exactly the value of separate components of the field strengths, averaged on some space-time area (it was supposed that in measuring one can always get exact value of each separate component $\overline{\hat{\mathbf{E}}}$ and $\overline{\hat{\mathbf{H}}}$, a value which belongs to the spectrum of its operator's eigenvalues and, consequently, should be expected in the experiment with certain probability);

2) field components of the same name, for example, $\overline{E'_x}$ and $\overline{E''_x}$, averaged on two different, not coinciding areas of space-time, can be determined together only up to $\hbar$;

3) two field components of different names $\overline{E'_x}$ and $\overline{E''_x}$, averaged over two different, not coinciding areas, can be determined within any accuracy.

Constructive justification of the theoretical scheme meant that observables, having the features listed, should be introduced as idealizations based on real specificities of the experiments and measurements in quantum relativistic area.

The first step on that way consisted in checking measurability of a separate averaged field component. This goal was reached by a theoretical experiment, where an experimental body, which volume V coincided with the boundaries of the averaging area of the field measured, was put into this area and, during the time equal to the averaging time, got an impulse from the field. The impulse should be registered by the device. According to conditions previously set for verification of the theory ($L > c\tau$), it was supposed that linear sizes of the experimental body are greater than time of measuring τ multiplied by light speed.

Bohr and Rosenfeld were to prove that all fundamental difficulties which had emerged in Landau-Peierles thought experiments with point experimental bodies are eliminated in the idealized measuring procedures of the new type.

If we study more or less thoroughly Bohr's and Rosenfeld's arguments, we will see that the proof was presented by means of scrupulous analysis of details of the intellectual experiment on base of continuous juxtaposing of theoretical corollaries and real possibilities of the experimentation. The arguments which helped solve this problem leave a deep impression (emphasized by many historians of science) of graciousness of investigators' thought, which found the exit from seemingly unsolvable paradoxes, but, what is more, can serve models which provide adequate interpretation of mathematical apparatus of modern theory.

In these judgments we can trace, how operational definitions, or (in Mandelshtam's terms) connection receipts of physical quantities of mathematical apparatus with experiment are created, and how in the process of creation of such receipts conceptual structure of a modern physical theory is formed.

The knots of the matter in the proof of fundamental measurability of a separate averaged field component were:

1) analysis of possibilities to locate the experimental body in area V during time of measuring τ;

2) analysis of the process of transmission of impulse from the experimental body to the register device;

3) exact account of fields radiated by the experimental body when measuring the field component.[68]

The typical method of Bohr's and Rosenfeld's judgment at this stage of analysis was the following: first they fixed difficulties and seemingly paradoxical corollaries revealed by theoretical analysis of field measurability, based on abstract notions of experimental bodies, then they showed how to overcome the difficulties if the notions of experimental bodies are adjusted and, correspondingly, conditions of idealized measurements are made specific, i.e. real specificities of physical experiments and measurements in quantum relativistic area are taken into consideration.

Moving this way – from general and abstract scheme of idealized measuring procedure to its detailed and concrete pattern, Bohr and Rosenfeld solved emerging questions of measurability of fields step by step.

It is indicative, for instance, the solution of the problem of localization of experimental body in space-time measuring area $V\tau$. In accordance with the main idea of the measuring procedure, the task was to determine exactly that particular impulse that the experimental body obtained in the area.

To do this, it was necessary that the experimental body should be isolated – as entirely as possible – from influences of neighboring areas during time and strictly fix the interval of measuring (otherwise the borders of the measured area would be blurred). To gain observance of this condition, the investigators had to determine impulse of the experimental body twice: one time just before its interaction with the field in area V, in the very beginning of period, the second time – at the end of that period, after the experimental body had interacted with the field in area V. Then the difference of impulse values p'_x and p''_x in the beginning and at the end of τ they could determine the value of the measured field strength. At the same time, to preserve the strictly determined period of time of averaging, the process of registration of the experimental body impulse p'_x and p''_x should take time Δt, which should be much smaller than general time of measuring τ.

Nevertheless, this refinement of the measuring procedure, though a necessary condition of localization of experimental body in space-time measuring area, by itself did not eliminate the main obstacles on this way.

For instance, there were difficulties connected with displacement of the experimental body in the course of measuring. The problem consisted in the following. Interacting with the field, and then with the register device, the experimental body every time should have received some recoil. Due to this, the body which initially occupied a certain spatial averaging area V, then it crosses the borders of this area and, if its displacement is considerable enough, it experiences perturbing influence from the field in neighboring areas.

In this case the difference p'_x and p''_x and impulses of the experimental body in the beginning and at the end of period τ one cannot determine the measured field strength in area $V\tau$. To avoid that, it was necessary to provide infinitesimal little displacements of the experimental body during the time of measuring. Bohr and Rosenfeld solved the problem by refinement of features of the experimental body and conditions of the measuring procedure. They supposed that the experimental body should have large mass which minimized its recoil[69]. For classical experimental bodies it is easy to gain (unlike point charges). We can easily see that this feature of experimental bodies is justified by idealization of real experiments in which the investigator can vary mass of the experimental body over a wide range.

In solving the problem of localization of experimental body in area there emerged more complicate difficulties, for instance, connected with correlation of indeterminacies between impulse and coordinate of the experimental body.

Just as the impulse of the experimental body should be measured exactly, there appears growing indeterminacy in its coordinate and, consequently, it becomes impossible to locate the experimental body exactly in a specified spatial field measuring area. Any exact measurement of impulses p'_x and p''_x of the experimental body in the beginning and at the end of the measuring period τ and, correspondingly, exact determination of impulse p'_x and p''_x means growth of indeterminacy Δx of the spatial measuring area. From here follows that a field component, say, $\overline{E}_x$, averaged on area $V\tau$ cannot be measured. Remembering that value of the component $\overline{E}_x$ is determined by the formula $\overline{E}_x = \frac{p_x^{''} - p_x^{'}}{\rho V \tau}$ (where ρ - density of charge of the experimental body),[70] and taking into consideration uncertainty relation, $\Delta x \Delta p_x \sim \hbar$ we can derive the expression for the indeterminacy to which the averaged component $\overline{E}_x$ will be measured every time, i.e. the expression $\Delta \overline{E}_x \sim \frac{\hbar}{\rho \Delta x V \tau}$.

At a glance it seems that we have returned to Landau's and Peierles's thesis of fundamental incommensurability of field. But Bohr and Rosenfeld demonstrated that classical nature of experimental bodies allows to overcome the paradox. Accepting indeterminacy Δx in position of the experimental body, one may make it much smaller than displacement of a heavy experimental body caused by change of its impulse. Then, at quite small Δx, one may increase density of the charge distributed over the volume of the experimental body. Then, as follows from the formula $\Delta \overline{E}x \sim \frac{\hbar}{\rho \Delta x V \tau}$, $\Delta \overline{E}_x$ will diminish. Basically, this way can always lead to change of a field component within the accuracy necessary for verification of the theory.[71] Thus, the problem is solved by introduction of a large charge experimental body compensating errors in field measurements caused by uncertainty relation. This feature of the experimental body, as a necessary condition for exact field measurements, like requirement of relatively large masses of the experimental bodies, was easily justified by real possibilities of physical experiment.

For classical experimental bodies varying of their charge basically can be realized (unlike point quantum particles used by Landau and Peierles in their thought experiments). But, admitting any densities of charge at their even distribution of charge over the volume of the experimental body, Bohr and Rosenfeld at once came across new difficulties. To assume even distribution of charge of any density over the volume of the experimental body is legitimate only in case when atomic structure of the experimental bodies is ignored. So a question arises: may one ignore this aspect of quantum area? Is it legitimate not to take into account quantum properties of the experimental charge in thought experiments on field measuring? This was a problem of principle, as it was evident that in interaction with field experimental bodies are added to the field sources, and their atomic structure should influence the quantum processes characterizing the field. Therefore there was a necessity to prove especially the legitimacy of abstraction from atomic structure of experimental bodies in measuring components of quantized field.

Bohr presented such proof on base of scrupulous analysis of specificities of the apparatus of quantum electrodynamics and reference to general principles of quantizing fields.

Mathematical apparatus of the theory of quantized radiation field did not introduce any universal scale of space-time sizes: formalism of the theory contained only two constants – $\hbar$ and c, "which could not form a characteristic length or interval".[72]

This exactly meant that in frameworks of quantized radiation field theory while describing field interactions with experimental bodies, the latter are to be considered just as classical charge distribution ignoring quantum specificities of such distribution.

Yet the reference to specificities of the theory was necessary but not sufficient for complete solution of the problem because, as Rosenfeld indicates in his review of quantum electrodynamics, "mathematical consistency of the formalism in particular was doubted at that time"[73]. Moreover, when Bohr-Rosenfeld procedures were performed, there already existed apparatus describing interactions of field with quantized sources. That is why it was unclear whether it was correct to execute field measuring procedures ignoring the more complicated process of quantum interactions in relativistic area.

Taking account of all this, Bohr supported the projected solution with methodological analysis of specificities and principles of quantizing electromagnetic field.

Rosenfeld emphasizes in his historical review that Bohr first paid attention to the basic side of the question – the fact that any characteristic of quantum system claimed on usage of classical idealization.[74]

Howsoever many intermediate links interaction of quantum systems included, the last link would require classical objects used as devices. Atomic structure of such objects is not taken into account by definition, they are described only by the language of classical physics. Hence there appeared fundamental possibility to abstract from atomic structure of experimental bodies, regarding them as a part of a classical aggregate.[75]

That such abstraction was necessary, followed from the very logic of the construction of quantum electrodynamics. Its mathematical apparatus was built so that quantizing of free fields preceded introduction of description of their interaction within the framework of the perturbation theory.

Construction of the interpretation should follow the same way. While equations for free quantized fields were not validated, it made no sense to interpret more complicated cases referring to interaction of such fields. But free fields require direct measurements (otherwise, if we introduce mediatory interactions between field measured and a classical device, the field is no longer free by definition). As we have already mentioned, observance of such conditions requires that we should consider the experimental body only as a fragment of a classical aggregate. Atomic structure of the experimental bodies should be taken into consideration only at the next stage of interpretation, in considering interacting quantized fields.

Thus, it was proved that at changes of free fields there is no need to include quantum qualities of the experimental charge. As a result, idealization of experimental charge of any, however high density was two-fold validated: from the point of view of real possibilities of the experiment and from the point of view of theoretical reasons of quantized field measurements.

Such two-fold justification of features of the experimental body is one of the important aspects of Bohr-Rosenfeld procedures.

Providing the experimental bodies with various ideal qualities, Bohr and Rosenfeld not only prove that such qualities can be obtained through idealization of real experiment, but also check whether the new idealization could destroy the fundamental conditions of measurement dictated by the principles of the theory.

For the most part, such verification did not require more or less complicated deductions and hence was not reproduced in the rendering of the results of the investigation. But cases with features of even distribution of charge of high density prove fundamental necessity and importance of such justification. The latter can guarantee correct synthesis of specificities of the theory verified with those of experimental practice.

When features of the experimental body had been specified, the thought experiment, which was to provide verification of measurability of the field, was seemingly developed enough to consider the problem of localization of the experimental body in measuring area as solved. Nevertheless, Bohr and Rosenfeld present one more proof: they demonstrate fundamental practicability of such experiment in real practice. In this respect it is very much characteristic that, having introduced such idealized assumptions as large mass and high density of the charge, Bohr and Rosenfeld at once work out a concrete scheme of an experiment including description of details of the device unit, which could provide localization of experimental body in measuring area $V\tau$.

It was supposed that the experimental body can be linked to a rigid carcass representing a spatial frame of reference. This allows to fix strictly its position before interaction with the field during time τ. In the beginning of this period tie with the framework is broken, and the experimental body experiences recoil under influence of the field. Then, at the end of the period τ, impulse is changed and tie with the framework is restored.[76]

Here we face with one more important peculiarity of idealized measuring procedures which throws light on their epistemological nature the method of their construction.

It is well known that concrete description of device units, which help study experimentally interactions in the nature, is usually inherent only to empirical schemes. In theoretical models it either is eliminated (as in quantum theory), or is replaced by abstract characteristic of the type of device, in correlation with which the vector of state of the quantum system is determined.

Introducing description of concrete details of structure and functioning of the measuring unit in discussion of the problem of localization of experimental body in measuring area, Bohr and Rosenfeld resort to notions which are characteristic for empirical schemes. But in return they get a guarantee that thought experiments with classical experimental bodies by their structure correspond to real specificities of physical experiments in the new interaction area. And this is that same condition which provides constructive justification of theoretical objects.

Thus, even the analysis of the first stage of proving measurability of a separate field component lets us conclude that in the process of idealized measurements Bohr and Rosenfeld perform many times repeated motion from the most general theoretical principles to concrete specificities of a physical experiment, and then, basing on these specificities, again turn to solving theoretical problems. In the course of this cognitive movement they reach such specification of the measuring procedure that it guarantees that specific features of quantum interactions in relativistic area are taken into account in the very process of idealized measuring.

The described process of analysis let Bohr and Rosenfeld successfully solve not only the problem of localization of the experimental body, but also two other key problems of field measurability: transfer of the impulse of the experimental body to the register device and account of disturbing influence of fields radiated by the experimental body.

In their analysis of the process of registering impulse of the experimental body, Bohr and Rosenfeld first of all demonstrate that using classical experimental bodies allows to avoid difficulties discovered by Landau and Peierles in their theoretical experiments with point particles. For classical bodies the impulse could be measured within such accuracy which would guarantee determination of the given averaged field component even with very little period of measuring Δt (much smaller than general time τ of measuring the field component).[77]

Though here emerge difficulties unknown in the theoretical experiments with point particles. For prolonged experimental bodies it becomes important that the speed of interaction is finite and it does not exceed the light speed. Because of this separate parts of the experimental body can transfer their impulse to the device not simultaneously but with certain retardation. If we accept an idealizing assumption that the experimental body is absolutely hard, the minimum time Δt, during which all its parts will pass their impulse (say, by means of collision with some membrane of the device), cannot be smaller than L/c (where L – linear size of the experimental body).

But, according to the conditions of measuring ($L > c\tau$), even general time of measuring, during which the experimental body interacts with the field and its impulse is measured, should be smaller than L/c, as to the period Δt of measuring impulse of the experimental body, it should be far smaller than τ. So, a new paradox emerges: measuring impulse of the experimental body requires time far exceeding permissible periods of measuring averaged field strengths.

Bohr and Rosenfeld found solution of this paradox using the method already tried. They reconsidered and refined characteristics of the experimental bodies and correspondingly specified the measuring procedure.

The first refinement which allowed to eliminate the contradiction consisted in rejection of idealization of absolutely hard experimental body. Instead Bohr and Rosenfeld introduced notion of experimental body as a system of small charged bodies which, interacting with the field and the register device, undergo approximately same recoil. To observe the last condition, it was supposed that the whole charge of the experimental body is evenly distributed among the elementary parts-components, and the density of the charge of each of them is also evenly distributed in its volume.[78]

Correspondence of such construction of experimental bodies to possibilities of the experimental practice was quite evident, if we take into account that systems of charges moving as a whole under influence of the field had been used in electrodynamics many times. As to idealization of even distribution of charge of high density in each of the elements of the experimental body, it could be easily justified by both empirical and theoretical reasons (for instance, by already proved possibility to ignore atomic structure of the experimental bodies). The described construction of the experimental bodies by itself allowed to transfer the whole impulse to the device unit during time Δt, given for registering impulse. To prove real practicability of this process, Bohr and Rosenfeld consider two possible ways to measure impulse: by collision of the experimental body with the diaphragm of the register device and on base of Doppler effect.[79] They introduce empirical schemes of possible experiments – corresponding to each of the ways, and then the idealized procedure of measuring the field component is justified as invariant contents of both types or real experiment and measuring. Having solved the problem of measuring impulse of the experimental body, Bohr and Rosenfeld proceed with the final stage of proving quantized electromagnetic field measurability – account of radiations generated by experimental bodies in the process of measuring.

At this stage the thought experiment, which provided measuring averaged field component, reaches highest possible completeness of development and justification – both from the point of view of theoretical reasons and from the point of view of real possibilities of physical experiments. To minimize radiations of the experimental body, which has reverse action upon its impulse, Bohr and Rosenfeld make new corrections in the measuring procedure. They propose to settle it so, that for the most part of the time τ of interacting with the field the experimental body remained stationary. To gain this, it would be enough to join the system of charges forming the experimental body with another set of charges, of the same distribution density but opposite in sign. Then, after a push from the field in the beginning of the period τ, during time Δt the experimental body would run some distance D_x (much smaller than linear sizes L of the field measuring area) and remain in this position during most part of the measuring time under influence of compensating charge, which could be fixed with hard framework of space-time frame of reference.[80] The displacement of the experimental body by itself would represent polarization of charges of a neutral (as a whole) distribution, combined from system of charges of the experimental body and compensating charges of the opposite sign. The value of such polarization would inform us of the field strength in the given averaging area. To measure this value, it would be enough either to determine value of displacement D_x, or register the impulse of the experimental body at the end of the time period τ, when polarization was removed, and the experimental body returned to its initial position under influence of neutralizing charge. The impulse could be measured during a very short time period Δt.[81]

The described scheme of experiment conserved all previously justified features of experimental bodies and conditions of measuring. In particular, it was easy to gain localization of the experimental body in the measuring area, for instance, necessary for this purpose joint and disjoint of the experimental body with rigid carcass of frame of reference were performed as if automatically – first under influence of the field displacing the experimental charge, then under influence of neutralizing charge which, being fixed with the framework, would attract the experimental body. It was easy enough to register its impulse during time Δt (the measuring could be carried out, for example, on base of the Doppler effect, lighting the experimental body with a beam at moments of its deviation and returning to the initial position).

At the same time the described thought experiment obtained minimal radiation of the experimental body in the measuring area. To determine the field component measured, the investigators only had to account this radiation and find a method of its compensating.

The main of the fields, radiated by the experimental body during time period τ and perturbing its impulse, referred to displacement, caused by interaction of the experimental body with the field measured. Bohr and Rosenfeld found a very simple way to compensate this radiation. For this they took a mechanical spring which elasticity corresponded in value to the force with which radiation caused by displacement D_x, will have reverse action upon the experimental body. Having fixed it at the experimental body and joined it with the rigid carcass, they could obtain such measuring unit that its construction had taken into account the perturbing influences of the radiation of the experimental body caused by its displacement under influence of the field measured.

It is clear that such compensation was possible only in respect to classical experimental bodies (it is impossible to fix mechanical springs to a point quantum particle).

By selection of compensatory springs they could also take account of fields appeared as a result of recoil of the experimental body during time Δt of its interaction with the device registering impulse. All these fields were computed by methods of classical mechanics, because the experimental body which radiated them was a classical charge distribution.

But there was one more radiation which was connected with account of quantum characteristics of measuring (relation of uncertainty between coordinate and impulse of the experimental body). This radiation was caused by displacement of the experimental body by distance Δx, which was to be accepted as minimal error in determination of position of the experimental body because of need to measure its impulse as exactly as possible.

It was obvious that this radiation could not be exactly taken into account in measurements because Δx is an indeterminate magnitude. But then there emerged an irremovable indeterminacy which cancelled all previously obtained proofs.

There appeared a situation which was characterized by Bohr and Rosenfeld as one of the most critical moments in physical justification of the apparatus of quantum electrodynamics[82]. It resulted that radiations of the experimental body can be compensated while they are considered within the framework of classical electrodynamics. But after transition to the area of quantum processes, where decisive role belongs to uncertainty relations, there appeared uncontrolled radiations of the experimental body which prevented investigators from exact determination of the field component,

If this problem had not been solved, Landau's and Peierles's conclusions about impracticability of quantum mechanical approaches to field description would have proved right.

But the solution, simple enough though unexpected, was found.

Bohr and Rosenfeld, estimating the highest possible value of disturbances, which radiation connected with Δx into the field measured, found out that in order of value the data of disturbance correspond to fluctuations of the field in the area of measuring emerging due to creation and annihilation of photons.

Processes of such creation and annihilation are the main and integral characteristic of quantized fields. One of remarkable features of these processes – the fact that they are statistically independent events (creation of photon in one of possible states, superposition of which form the radiation field, does not influence the probability of creation of other photons in other states and does not depend on the number of photons filling the said states). So in every possible state the number of photons, emitted by classical field sources and passed to measuring area $V\tau$, should oscillate about some average number in accordance with Poisson distribution. Such variations will cause small changes of energy in the measuring area, and, as energy is correlative to intensity, this will mean emergence of fluctuations in values of classical components of the field strength.

But if the condition of measuring $L > c\tau$ is observed, such fluctuations do not prevent exact measuring of quantized field component but, moreover, they are prerequisite for this measuring.

In this case the investigator will deal with statistical distribution of photons for various states characterizing the field only in area V. Fluctuations connected with statistically independent emission of photons by classical sources should, by definition, be included in the characteristic of the field. Their existence, on the whole, causes a statistical character of predictions of field quantities in quantum electrodynamics (unlike classical theory, what is predicted here is the probability of occurrence in the experiment of some value of the magnitude which belongs to the spectrum of the eigenvalues of the corresponding operator). This including of fluctuations into values of the field strengths measured in area $V\tau$ allows to obtain a certain set of values, and each of them can be expected in the experiment with a certain probability). But this would be that very set which coincides with the spectrum of the eigenvalues of the field strength operator.

In this respect fluctuations do not prevent exact determination of field magnitudes measured by a classical device. Quite another matter is that these magnitudes will somewhat differ from the values predicted by classical electrodynamics (field strength, predicted by the classical theory, is correlative to average energy of all photons which are in the measuring area, without account of fluctuations about average number of photons in each state; account of the photons leads to deviation from values predicted by classical electrodynamics). But this difference is that important property which allows to discover quantum characteristics in measurements of field magnitudes.

Thus, including fluctuations in area $V\tau$ (when $L > c\tau$) into observable field magnitudes not only conserves predictability of measurements, but also allows to discover those features of the field which are proper to it as a quantum system with a variable number of particles. From these positions it was easy to solve the problem of radiation connected with indeterminacy Δx in position of the experimental body.

Since disturbing influences of such radiation do not exceed fluctuations connected with the process of creation and annihilation of photons, then including of these disturbing influences into the measuring process makes us discover in the experiment those very values of the field component, whose mathematical expectation is predicted by the theory.

Perturbations caused by indeterminacy in position of the experimental body may lead only to difference between values calculated on base of the quantum field theory and those calculated by methods of classical electrodynamics. But this difference is the one which allows to predict measurable magnitudes with account of quantum specificities of the field – creation and annihilation of photons.

This way the last problem in the proof of measurability of a separate field component was solved. As a result, the main feature of Bohr’s theoretical model of quantized radiation got constructive meaning.

It is characteristic that here, at this final stage of the proof the idealized measuring procedure obtained such degree of specification that there could be no doubt, neither in its correspondence to the basic principles of the theory verified, nor in the fact that it accumulated essential features of practicable experiments and measurements in quantum relativistic area. But it was then that it was discovered that all essential features of the object studied (quantum radiation field) are automatically taken into account within the scope of the given procedure. All influences which prevented exact determination of averaged field strengths were eliminated or compensated in an experiment, planned in details, with classical experimental bodies. As to perturbations which could not be controlled or compensated, they were necessary conditions for determination of field quantities measured with account of quantum effects.

Emphasizing the importance of the last circumstance, L. Rosenfeld wrote that the impossibility to compensate or erase completely the perturbations introduced by the experimental body is not at all the result of imperfectness of an idealized measuring unit, used in the intellectual experiments with classical experimental bodies. On the contrary, “the impossibility of compensating or controlling them in any way, far from being an imperfection of the measuring device, is a property which it must necessarily possess to ensure that all the consequences of the theory are in principle verifiable by measurement”.[83] “The fact that the zero-field fluctuations are superposed on to the classical field distribution is indeed a well-defined theoretical prediction, and we see that we are able to suppress the perturbations arising from the manipulation of the test-bodies to an extent which just leaves scope for the test of this prediction”.[84]

The proof of fundamental measurability of field components, averaged on some space-time area, was a key moment in constructive justification of Bohr's theoretical scheme of quantized radiation field. The main argument of Landau and Peierles against employment of quantum methods in description of electromagnetic fields was eliminated.

But for complete justification of the quantum theory of electromagnetic field the investigators also had to verify the work of connection between the field strengths in two different space-time areas interacting by lights signals. This connection characterizes spread of the field in space in the course of time, so analysis of correlations between pairs of averaged field components, taken in different space-time areas, was necessary for investigation of dynamic characteristics of electromagnetic field in quantum area.

As we have emphasized, the theoretical scheme postulated impossibility to measure pairs of components of the same name together (for instance, $\overline{E'_x}$ and $\overline{E''_x}$ averaged over space-time areas $V_1\tau_1$ and $V_2\tau_2$ with accuracy exceeding Planck's constant by an order of value).

Bohr and Rosenfeld proved validity of this statement by their thought experiments with classical experimental bodies. Then they proved that features of joint measurability of two components of different types (for instance, $\overline{E''_x}$ and $\overline{H''_x}$ or $\overline{E'_x}$ and $\overline{H'_x}$ derived from thought experiments with classical experimental bodies, also coincide with the features postulated in the theoretical scheme of quantized radiation field).

Thus, the main abstract objects of this theoretical scheme (observable fields, averaged on finite space-time area) were introduced as idealizations based on real specificities of physical experiments. After all these procedures the theoretical scheme of quantized radiation field got its constructive meanings.

At the final stage of justification of Bohr's theoretical scheme the authors offered new development of details of thought experiment on measuring the field strengths. That was characteristic. It was required, in particular, due to the fact that at verification of measurability of pairs of field components of the same name, uncertainty relation for these components, derived from the measuring procedure, initially did not coincide with analogous relation given by mathematical formalism of the theory. But, remembering that between areas of field measuring there exists light signal exchange, basically it was possible to make corrections in the measuring procedure. These corrections should be connected with using such signals as "messages" automatically transferred by experimental bodies and carrying information of their mutual positions. Refinement of the thought experiment in this respect led to coincidence of predictions of the theoretical formalism verified and the results of idealized measurements.[85]

Later L. Rosenfeld described this stage of interpretation of the apparatus of quantum electrodynamics: "It is very striking indeed to see how the greatest accuracy compatible with the commutation law can only be achieved by exploiting to the utmost the possibilities, afforded by the physical situation of controlling the course of the measuring process".[86]

From these positions we can evaluate again heuristic function of the method used by Bohr and Rosenfeld in justification of their theoretical scheme of quantized electromagnetic field.

Consistently moving from the most general shape of the thought experiment, dictated by the mathematical apparatus and hypothetical model of its interpretation, to empirical schemes of a possible experiment, Bohr and Rosenfeld gained that idealized field measurements gradually accumulated essential features of real experimental measuring activity. In the framework of such measurements they traced the process of interaction of device units (including experimental bodies) with the field measured and discovered its characteristics. The latter were compared with the characteristics postulated by the previously accepted theoretical scheme. Coincidence of the field tokens obtained in two described ways proved that the given scheme was an adequate reflection of quantum specificities of electromagnetic radiation.

Thus they solved the main problem of theoretical search at the stage of interpretation of the theory's mathematical formalism: features of the abstract objects got their empirical justification.

We would like to pay attention to one important feature of the described method of investigation: its application no longer requires those real experiments, which provides verification of constructive meaning of the theoretical scheme, should be realized in practice. Enough if they are basically possible and practicable. The investigator can make sure that the latter is true when he develops analysis of measurability of theoretical quantities to concrete empirical schemes of real experiment, when possibility to realize one or another device unit and its interaction with the object measured becomes evident at least because similar device units and methods of their functioning are familiar by previous practice.

So, Bohr's and Rosenfeld's procedure of measuring a field component did not leave place to doubts in fundamental practicability of the corresponding experiment, because in previous physical experiments similar measuring devices and methods of measuring had been used many times. There was no sense to especially prove that the measuring unit might contain, besides experimental charge, a body carrying compensating charge; that the field would cause polarization of charges in a neutral (as a whole) charge distribution: that it was possible to settle rigid connection between the carcass of the frame of reference and compensating charge etc. – similar device units and methods of their functioning could easily be found in previous practice.

Taking into consideration the fact that in creation of a theory by method of mathematical hypothesis the layer of real experiments, where specificity of new interactions is seen, may be developed insufficiently (sometimes there can be no such experiments at all), we may say that the described way of investigation is probably the only possible way of justification of the theory at the modern stage of evolution of physics. Using it, the investigator as if shortens the way of development of the theory. He does not have to wait until a vast enough set of local theoretical schemes and laws justified by real experiments is created. He reproduces in thought empirical schemes of basically practicable intellectual experiments and develops analysis to the foundations where the possibility to realize experiment of the given type is quite evident. The latter only means that such and such type of device unit and the principle of its interaction with the object studied has already been realized in previous practice, so it would be redundant to repeat what has been done.

The necessity to develop and refine procedures of idealized measuring until they accumulate essential specificities of real experiments, which provide studies of corresponding object, Bohr often expresses as a requirement of fundamental controllability of interactions of object and device.

Rationally this requirement can be reduced to the following: any real measuring indeed stipulates a special set of conditions under which the investigator could eliminate (or take into account) perturbing external influences which distort real values of the magnitude measured. The possibility to eliminate such influences or to take them into account introducing corresponding corrections means that the investigator controls the condition of measuring.

Since thought experiments and measurements should be idealization of real experimental measuring activity, then the investigator also should completely discover in them the controllable conditions of measuring. From these positions he has to scrupulously check (basing on already known theoretical laws) consequences of every new detail in the mental scheme of the device unit and, at the same time, correlates the scheme with real possibilities of the experiment. Constructing idealized measuring procedures, the investigator step by step discovers those mentally fixed interactions of the object with the devices which could cause indeterminacies in values of magnitudes characterizing the object. Having revealed such interactions, he checks whether they refer to disturbing influences of the device unit which can be eliminated by its new refinement and application of compensatory devices.

Exhausting possibilities to control the conditions of measuring, the investigator makes sure that the idealized measuring corresponds as much as possible to the possibilities of real experimental measuring activity. If indeterminacies of magnitudes characterizing the object remain, it means that such indeterminacies should be considered as essential characteristics of the object itself.

In this respect everything what is fundamentally uncontrollable within the scope of idealized measuring, justified as scheme of a real experiment, should be included in the specificities of the object measured, since the measuring procedure itself is constructed in such a way that it reveals objective characteristics of the reality studied. Hence we cannot, of course, conclude that quantum characteristics appear due to uncontrolled interaction of the device and the microobject measured. The real structure of Bohr's cognitive activity and his method of construction of idealized measuring were not connected with the idea of uncontrollability in the sense above. They were based on an entirely opposite approach, according to which idealized measurements, structured in concordance with real specificities of quantum mechanical and quantum relativistic experiments, should reveal objective characteristics of the processes in atomic area.

Bohr's requirements of control over conditions of interaction of the object measured and the device were identical to requirements to construct idealized measuring drawing it as close as possible to real specificities of physical experiment. Then characteristics of a quantum object, which could be discovered within real experimental practice, undoubtedly should find expression in the results of idealized measurements.

Intermediate interpretations of apparatus of modern physical theory as a condition of its development

Constructive justification of the theoretical scheme of quantized radiation field automatically provided empirical interpretation of the formalism of the theory. Bohr-Rosenfeld procedures allowed to correlate field strengths from the equations of quantum electrodynamics with experiment indicating mechanism of such connection. This mechanism could be involved by means of description of Bohr-Rosenfeld procedures of thought experiments. The description itself formed a system of operational definitions for corresponding physical quantities.

In this respect the process of construction of idealized measurements in quantum electrodynamics can be taken as some model of activity which provides introduction of operational definitions at today's stage of development of physical theories. But Bohr-Rosenfeld procedures not only formed empirical interpretation of the equations of quantum electrodynamics. They discovered new aspects in characteristic of such field and urged to introduce corresponding corrections also in the semantic interpretation of the formalism of the theory.

The idea of field resulted to be applicable only to finite space-time areas and inapplicable to a point. Thus the idea of quantized field as transfer of electric and magnetic forces from point to point was destroyed. Such idea, acceptable within classical electrodynamics, was inapplicable in quantum area.

Then it became clear that, because of field fluctuations caused by creation and annihilation of photons, the connection between the field and its sources is more complicated than classical theory used to believe. The latter ties sources and fields in a strictly determinate way. At the same time in quantum theory Laplace's determinism of classical electrodynamics is replaced by a wider form of statistical causality. Fields are causally connected with sources only from the point of view of statistical predictability of field magnitudes measured in the experiment. Strictly determined connection, characteristic for classical physics, restores only when the field in the measuring area "consists" of a large number of photons, which, in accordance with Poisson distribution, oscillate about some average number in every of the possible states forming the field. As the average number of photons is large enough, we can ignore their fluctuations and turn to classical description of the field. All these field characteristics were revealed due to measurability procedures, because it was here where investigators determined the physical sense of influence of fluctuation field upon the magnitudes measured. The said fluctuations transformed traditional idea of radiation field determination by its sources.

Finally, in the process of idealized measurements the unbreakable link between radiation field and vacuum was justified. This is probably the most important consequence of Bohr-Rosenfeld procedures.

It may seem at the first glance that the idea of connection between quantized radiation field and vacuum was born due to mathematical apparatus of the theory and did not depend on the proof of the field measurability, as application of methods of quantizing to electromagnetic field automatically led to notions of infinite field energy in absence of photons.

But the matter of fact is that before justification of the field measurability it was entirely unclear whether it was possible to provide vacuum with real physical meaning or it should be accepted only as an auxiliary theoretical construct lacking such direct meaning.

Paradoxes with infinities push physicists to the latter conclusion. They supported opinion that for non-contradictory interpretation of quantum electrodynamics in general it was necessary to exclude somehow "zero field" from the "body" of the theory. We should remember, then, that Landau and Peierles linked the idea of vacuum with paradoxes of incommensurability, and in their analysis energy was presented as one of the evidences of fundamental inapplicability of quantum methods to description of electromagnetic field. Productively criticizing conclusions of Landau and Peierles, Bohr eliminated the last objection, but the question of physical sense of vacuum states still was not solved.

Only in the course of Bohr-Rosenfeld procedures was the problem clarified and connected with the discussion on the role of fluctuation of the field components in the measuring process. But there was one more aspect of the problem, which we have not yet touched for the sake of easiness of the account. Let us consider this aspect now.

Besides fluctuations connected with the presence of photons, there is one more variation of field fluctuations predicted by the apparatus of the theory. It is zero fluctuations which appear in absence of photons and connected with the zero energy level of the field. From the apparatus of the theory it followed that these fluctuations have finite positive value (nothing to do with the infinite energy of the field in zero state!).

As we have mentioned, Bohr and Rosenfeld proved that fluctuations connected with creation of photons should be included in values of the field components. They are discovered due to declinations of values of the field quantities predicted by the quantum theory, from the values calculated by methods of classical electrodynamics.

The empirical sense of fluctuations connected with creation of photons followed from the structure of idealized measuring of the field, since only taking them into account could the investigators determine exactly the averaged field component. But in that case fluctuations of the zero field also obtained empirical justification, as they were fundamentally inseparable from fluctuations connected with presence of photons.

As zero fluctuations were display of "zero field", the latter as well got real physical sense. It resulted that, if vacuum and zero fluctuations caused by it were removed, the very idea of quantized radiation field would become physically empty, because the averaged field component could not be measured exactly.[87]

As a result, Bohr-Rosenfeld idealized measuring procedures led to conclusion about real connection between the radiation field and vacuum and impossibility to obtain description of quantized radiation field without taking vacuum states into account.

In principle, the new vision of electromagnetic field caused by realization of the procedures of measurability is not something unusual or extraordinary in the development of theoretical knowledge. On the contrary, here we can see a certain pattern of epistemological nature; its manifestation we have already seen in the history of science (for instance, in analysis of the history of classical electrodynamics). The essence of it is the following: realizing constructive introduction of abstract objects of a previously accepted theoretical model, the investigator fills this model with new physical contents, because he organizes real experimental research activity, revealing characteristics of the reality studied.

The obtained content is objectified due to mapping of the theoretical model on the picture of the world, and the result is a new vision of the object under study, which fixes its essential properties and relations. The last procedure finishes construction of interpretation of the corresponding phenomena of the corresponding equations of the theory, which are presented now as description of new essential characteristics of the physical reality studied. At this stage the theory obtains new physical notions, and its conceptual apparatus gets further development. Due to this the preliminary accepted semantic interpretation is refined and developed. Thus, constructive justification of the theoretical scheme leads to decisive development of the contents of the scientific theory. This is an accomplishment of the process of formation of its conceptual structure, started at the stage of mathematical hypothesis. Bohr-Rosenfeld procedures can present us a characteristic example of the process developing at modern stage of evolution of theoretical knowledge. After measurability of quantized radiation field had been proved, fundamental possibility to apply quantum mechanical methods in description of relativistic processes provoked no further doubts (unlike initial conclusions made by Landau and Peierles).

The foundation of quantum electrodynamics – the theory of free quantized electromagnetic field – became now a non-contradictory and experimentally justified system of knowledge.

Now the researches only had to interpret the fragments of quantum electrodynamics which described interaction of quantized radiation field with quantized sources (measurability of electron-positron field).

Bohr and Rosenfeld solved this problem was by at the second stage of realization of their research program. It was connected with construction of idealized measurements for sources (distributions of charge-current) interacting with quantized radiation field.[88]

First, they proved measurability of classical sources interacting with quantized electromagnetic field, and then presented a proof of measurability of field sources with account of creation of electron-positron pairs. Thus they completed the interpretation of mathematical apparatus of quantum electrodynamics describing free quantized fields and their interactions in the first approximation of the perturbation theory.

At this stage they not only formulated the correspondence rules, which connected all physical magnitudes of the equations of quantum electrodynamics with experiment, but also discovered early unknown characteristics of quantized fields. In particular, the procedures of quantized measuring allowed to raise the question of space-time boundaries beyond which the field approach to description of quantum properties of charge-current loosing its force.

From the mathematical apparatus of quantum electrodynamics it followed that, unlike fluctuations of electromagnetic field, the fluctuations of charge and current within any strictly limited space-time area are to be infinite. But the analysis of the situation of idealized measuring revealed new field specificities. It was discovered that in areas related to shell of finite depth consisting of experimental bodies (which served to measure the field sources), averaged on the same areas fluctuations became finite. If we infinitely reduce the depth of the shell, the fluctuations infinitely grew tending to infinity. When they are equal to mathematical expectation of the field quantities predicted by the apparatus of the theory, it indicates the limits of applicability of quantum electrodynamics.[89]

Thus, constructive justification of the theoretical scheme of interaction of quantized radiation field with quantum sources, providing empirical interpretation of the formalism of quantum electrodynamics, introduced new aspects into its semantic interpretation as well.

To sum up, we can now once more evaluate the way made by Bohr and Rosenfeld in construction of this interpretation.

Gradually justifying features of free quantized electromagnetic field, then interactions of this field with classical sources, and, lastly, with quantum sources, by means of idealized measurements, Bohr and Rosenfeld were creating a richer and richer theoretical model, which took into account new aspects of electromagnetic interactions in atomic area. This way of construction of interpretation reproduced the basic steps of historical development of the mathematical apparatus of quantum electrodynamics at the level of conceptual analysis.

No essential stage of its development was missed – the logic of construction of the interpretation mainly coincided with the logic of historical development of the mathematical apparatus of the theory.

In this respect, it is interesting to compare interactions of the mathematical apparatus and theoretical models in modern and classical situations in yielding of a scientific theory.

As we have shown above, in construction of classical electrodynamics every step toward the generalizing field equations (Maxwell's equations) was supported by a corresponding theoretical model, which was constructively validated even at the intermediate stages of the theoretical synthesis.

While quantum electrodynamics was being formed, the situation changed. Here for a quite long time mathematical apparatus was built without constructive justification of the theoretical models; there were only hypothetical schemes which introduced preliminary semantic interpretation of the equations. As to procedures of their constructive justification, which provided empirical interpretation of the formalism created, and then its final semantic interpretation, they were carried out later and were separated in time from construction of the formalism as such. Nevertheless, in those procedures investigation repeated all the main stages of development of the apparatus of the theory in brief. Step by step does it reconstruct the developed hypothetical models and, through their constructive justification, introduces intermediate interpretation which correspond to the most important stages of development of the apparatus. The accomplishment of this way consisted in clearing of the physical meaning of the generalizing system of equations of quantum electrodynamics.

So, the method of mathematical hypothesis does not at all reject the necessity of content-physical analysis at intermediate stages of forming the mathematical apparatus of the theory. The specificity of modern investigations is not that intermediate interpretations become redundant, but that the activity aimed at their construction becomes a continuous transition from one intermediate interpretation to another in accordance with the logic of development of the apparatus, which reproduces the history of its development in brief. Classical theory was constructed according to scheme: equation$_1$ → intermediate interpretation$_1$, equation$_2$ → intermediate interpretation$_2$ ... , generalizing system of equations → generalizing interpretation; in modern physics theory is constructed in a different manner: first equation$_1$ → equation$_2$ → etc, then interpretation$_1$ → interpretation$_2$ → etc. (but not equation$_1$ → equation$_2$ → generalizing system of equation and immediately accomplishing interpretation!). Clear, the shift of interpretations in modern physics does not entirely reproduce analogue processes of the classical period. We should not believe that we have only discrete transition from one intermediate interpretation to another replaced by continuous transition, only the number of intermediate links is changed. In modern physics it is as if packed, and therefore the process of construction of interpretation and development of conceptual apparatus of the theory takes cumulative form. There are at least two reasons for that.

First, as we have already emphasized, the process of constructing theoretical models reproduces the history of development of mathematical formalism not entirely, but in brief. Search for adequate interpretation requires verification only of those links of its historical development, which were accomplished by creation of equations included in the theory (for example, Bohr and Rosenfeld in their procedures of measurability of quantized radiation field, investigated the mathematical formalism created by Heisenberg, Jordan and Pauli on base of the initial variant, suggested by Dirac; this variant as such was not considered because it had been put away from a further, more perfect mathematical apparatus).

Second, the mathematical hypothesis by itself reduces the number of intermediate links on the way to generalizing equations of the theory (since at once there are introduced equations of generalization of great enough level – as basic dependences subject to further synthesis and generalization). In its turn, it leads to reducing of the number of intermediate stages on the way to the final interpretation of the theory formalism.

All said lets us conclude that, in comparison with classical models, in modern theoretical investigation the procedures of constructive justification of theoretical models and construction of operational definitions, which connect the formalism of the theory with experiment, are somehow packed. So we may state that at the modern stage of evolution of physics some features of theoretical synthesis, distinctive only of the classical period, are reproduced, but in a packed and pressed form.

In principle, that should be this way – if we take into consideration dialectical way of development: in self-developing systems (and scientific cognition is one of them) higher stages of evolution always repeat in their functioning some features of historically preceding forms. It is important to remember that such features can be both transformed enough or reproduced comparatively purely. The latter variant allows finding new aspects of interaction of mathematical apparatus and interpretation in development of modern theory. As we understand, at some stages of this development it is possible to see sort of return to classical scheme of theoretical synthesis, according to which advance in mathematical formalism should not happen before its exhaustive interpretation is created.

But such return is not the same as absolute repetition of classical methods. It goes on new basis and requires usage of modern methods of theoretical search.

Breakthrough in mathematical extrapolations usually takes place, when they have already helped to build quite rich formalism able to be base of the future apparatus of the theory. But the theory itself is not accomplished yet. The necessity of its further development at this stage may be evident enough, at least because necessary problems are solved only partly (there are theories which should be solved, according to requirements of the theory, but which are unsolvable by means which exist).

But not at all always it is clear, how to find new mathematical means. Moreover, there are doubts if such search is possible on previous basis, as existence of unsolvable problems can be evidence of inner contradictions in the formalism already created. Then we need content analysis of the foundations of the theory, proofs of consistency of the created apparatus and construction of its interpretation.

Development of mathematical formalism is relatively independent from its interpretation (including empirical aspects) only to certain extent. In modern physics there always are periods when further perfection of mathematical apparatus of the fundamental theory created entirely depends on construction of its consistent interpretation, which gives a new impact for further mathematical synthesis and accomplishing of the theory.

In this respect the history of quantum electrodynamics can be a most eloquent example.

Between the third and the forth stages of forming of its apparatus there emerged crisis of its foundation, caused by discovering of incommensurability paradoxes. Further generalization and elaboration of the formalism of quantum electrodynamics would have been impossible as the very principles of quantizing fields were doubted, if that crisis had not been overcome.

Bohr and Rosenfeld laid the way out of the crisis when they constructed a consistent interpretation of the created apparatus, which described processes of interaction of quantized electromagnetic and electron-positron fields in the first approximation of the perturbation theory. Only after that did it become possible for quantum electrodynamics to recover in the 1950s. That recovery was connected with construction of renormalization theory. Firm belief in fundamental applicability of quantum electrodynamics methods of description in the relativity area (shaken because of the crisis and restored thanks to success in solving the problem of measurability of quantized fields) was a necessary condition for search for theory of interaction of quantized fields with account of higher orders of the perturbation theory. The very setting of the problem was correct due to Bohr-Rosenfeld procedures, which had previously proved that the description of interaction of quantized fields in the first approximation of the perturbation theory was consistent.

But Bohr-Rosenfeld procedures gave an impact to further development of quantum electrodynamics not in this generally theoretical aspect only. They exercised concrete influence upon further evolution of the theory, as they revealed such new characteristics of electromagnetic interactions, the information about which made it considerably easier to elaborate the basic physical idea of renormalization.

We usually pay little attention to this circumstance, but still it is extremely important for understanding patterns of evolution of theoretical knowledge.

The general idea of renormalization appeared, as it is well known, due to understanding the limited nature of idealization of a free particle in respect to quantum relativistic area. Any particle is not free, in a strict sense of the word, because it interacts with vacuum, which corresponds to the lowest energy state of quantized fields. The result of such interaction is change of charge and mass of the particle, and then charge and mass of the particle observable in experiment become a summary of this interaction. For instance, if there are mass m_0 and charge e_0 of an electron not interacting with vacuum, in the experiment we observe other mass and charge which are equal to $m = m_0 + \Delta m$ and $e = e_0 + \Delta e$. The magnitudes Δm and Δe express changes introduced in charge and mass of the electron by vacuum.

It seems possible to calculate charge and mass of the electron (observable in the experiment) by means of determining corrections Δm and Δe for interactions with vacuum. But such corrections turned out infinite expressions having the form of divergent integrals. All this caused enormous difficulties in description of interaction of particles (considered as quanta of the field) by methods of the perturbation theory.

Renormalizations, which allowed to eliminate these difficulties, were based on a quite simple physical idea. Magnitudes m_0 and e_0 representing mass and charge of non-interacting (or "bare" in modern physical terminology) electron, as well as corrections, were considered as auxiliary theoretical constructs which had no real physical meaning, because a real electron always is in interaction with vacuum and never exists beyond such interactions. Then mass and charge of a free electron was identified with expressions $m = m_0 + \Delta m$ and $e = e_0 + \Delta e$ which are really observed in experiment. But since these magnitudes have finite values, finite values m and e were to be acquired through special selection of divergent values for Δm and Δe. The method of such selection formed the essence of the renormalization method.

It means that the renormalization method was based on the idea of observable magnitudes characterizing particles, which are considered as quanta of some field, as display of total result of interaction of these particles with vacuum.

But this very idea firmly occupied its place in physics due to the procedures of idealized measuring.

Let us recall that Bohr and Rosenfeld justified measurability of quantized radiation field, and this fact lead to a conclusion: there is a contribution of vacuum in the field observable magnitudes characterizing the state with presence of particles (photons). Further analysis spread this conclusion also on magnitudes describing electron-positron fields (for instance, on such dynamic variables of the field as charge and mass).

Beyond the measurability procedures the initial idea of observables having a contribution of vacuum looked no more than a hypothesis. But idealized measurements got the status of a validated theoretical statement for that hypothesis.

Since works of Bohr and Rosenfeld containing the results mentioned above were well known among the physicists-theorists of the 1940s,[90] we may quite naturally conclude that they prepared the necessary base for development of the idea of renormalization. In any case, we are to remember that the approach to observables, which became a necessary condition for the idea of renormalization, was prepared by Bohr-Rosenfeld procedures.[91]

It is characteristic that this stage coincided with new development of mathematical apparatus of quantum electrodynamics. Here we can see the reverse influence of Bohr-Rosenfeld theoretical model upon the search for new mathematical structures characterizing quantized fields. By the way, such influence can be seen even at quite late stages of development of quantum relativistic ideas. So, we would like to draw the reader's attention to the following important circumstance.

In axiomatic quantum field theory the mathematical apparatus from the very beginning is constructed, meaning that physical sense can belong not to fields in a point, but to magnitudes of fields averaged on some finite space-time area. The modern theory characterizes field not by operator functions (as it was at the earliest stage of development of quantum electrodynamics), but operator functionals, whose description openly contains the operation of averaging on finite space-time area. Such apparatus allows describing easily and briefly quantum processes in relativistic area. For reaching this goal, it uses mathematical structures of higher "information capacity" than those which were in foundation of the mathematical formalism of quantum electrodynamics of the 1930s–1940s.

It is obvious that the physical foundation for the application of new mathematical means were the specificities of fields uncovered by Bohr-Rosenfeld procedures. It means that the interpretation procedures prepare new development of the theory apparatus, encouraging search for more perfect mathematical structures.

To summarize all said above, we may formulate the following epistemological and methodological conclusions.

1. In modern physics the process of construction of a theory is even more autonomous in relation to new experimental data, than in classical physics. Mathematical hypothesis lets us move toward fundamental equations of developed theory even if the local theoretical laws, which are to be synthesized and which are based on real experiments, are presented scarcely enough.

2. Still an important directing role in theoretical investigation belongs to the picture of physical reality. It provides base for choice of principles of mathematical description of new area of physical processes. But, unlike classical models, its operational structure is accentuated.

3. A mathematical hypothesis is able to provide working out a quite developed apparatus, but only to certain extent, because equations manipulation is linked with corresponding transformation of abstract objects of theoretical schemes. If a series of mathematical extrapolations is quite long, it can cause accumulation of non-constructive objects with mutually eliminating features. So, for the development of non-contradictory

theoretical system of knowledge interpretation of mathematical formalism at intermediate stages of construction of the theory is required.

Creation of a theory keeps going on as an alternate correlated movement in mathematical means and plain of physical contents. But, in comparison with classical models, the relatively independent "run" at each of this level grows, and the movement from equations to interpretation and vice versa goes on in larger steps.

4. Construction of intermediate interpretations in modern physics goes on as procedures of idealized measuring and often without preliminary real experiments. Nevertheless, due to consistent development of details of the thought experiment – up to reproduction of empirical schemes of possible future experiment – the very idealized measuring procedures can be justified as schematized and idealized real experimental-measuring activity in the field of interactions. That is why they are capable of bringing to light objective characteristics of such interactions.

5. The idealized measurements not only verify characteristics hypothetically introduced in base of the specificities of the theory apparatus, but also discover new, unknown features of the physical processes studied. Hence the mathematical apparatus obtains new physical meaning, and the notion structure of the physical theory is reconstructed and presented as a deeper and more adequate reflection of the object area investigated. In turn, it raises foundation for search for new, more perfect means of its mathematical description.

6. Stages of development of idealized measurements, which end at construction of an adequate scheme of new area of interactions, reproduce the main stages of construction of the mathematical apparatus, as if repeating its history, but in brief. At the same time, idealized measurements of modern physics shorten the way of constructing the theory as well because they do not require long forming of preliminary theoretical models and laws based on real experiment. In the very process of construction of idealized measurements the investigation briefly passes the stage of forming of such models.

Thus, the evolution of physics at modern stage conserves some basic operations of construction of the theory characteristic for its past forms (classical physics). But is develops the operations, partly modifying them, partly repeating – on a new base – some features of construction of mathematical apparatus and theoretical models, appropriate to the classical models.

In modern investigation the process of theoretical search characteristic for classical physics is reproduced in transformed and pressed form – as it should be at higher stages of the evolution in relation to the historically passed stages.

MUTUAL CONNECTION OF GENESIS AND FUNCTIONING OF A THEORY.

THE CONSTRUCTIBILITY PRINCIPLE

If we compare specificities of development of a theory in classical and non-classical science, some common laws of the process of their development can be revealed.

Analysis of content aspects of the structure and genesis of a scientific theory demonstrate that in formation of its conceptual apparatus the key role belongs to procedures of constructing a theoretical scheme. Such construction is done as interaction between foundations of the science, mathematical apparatus, empirical and theoretical material generalized in the theory. First it stipulates transition from foundations of the science to a hypothetical variant of the theoretical scheme, and then – to empirical material.

This is the first cycle of the process of constructing the theory, connected with the hypothesis put forward. But then we face reverse movement – from generalized empirical and theoretical material to theoretical scheme and again to the foundations of the science. This is the second stage connected with justification of the hypothesis. Here the initially introduced theoretical schemes are reconstructed, saturated with new contents and actively influence upon the foundations of the science, preparing new changes in them.

The hypothesis suggested marks only the most general framework of the conceptual structure of the theory, which is formed – in its main features – with justification of the hypothesis.

Methodological literature usually characterizes the very process of suggesting hypotheses in terms of "discovery context". It is urgent to emphasize that transition from the foundation of the science to analog model and then to a hypothetical scheme of the interaction area studied makes a certain rational outline of this process. It is often described in terms of the discovery psychology and creative intuition. But such description, if it is supposed to be constructive, should, for sure, be linked with clearing of the intuition "mechanisms". It is characteristic that here investigators at once came across the so-called mechanism of gestalt-switching which lies at the base of intellectual intuition.[92]

Detailed analysis of this process shows that the intellectual intuition is considerably characterized by usage of some model ideas through which we examine the new situations. The model ideas stipulate the image of the structure (gestalt) which is transferred to new object area and organizes, in a new way, the before collected elements of knowledge of that sphere (notions, idealizations etc.).[93]

The result of such work of creative imagination is a hypothesis which allows to solve the problem offered.

Further consideration of mechanisms of intellectual intuition has marked clearly enough that the new vision of reality, corresponding to gestalt-switching, is formed due to substituting new elements – ideal objects – into the initial model-idea (gestalt), and it allows to construct a new model shaping new vision of the processes studied.[94]

Here gestalt is a kind of "mold" according to which the "model is molded".[95]

Such description of the procedures of generation of hypothesis corresponds to investigations of the discovery psychology. But the process of putting forward scientific hypotheses can be also described in terms of logical-methodological analysis. In this case its new important aspects will be uncovered.

First, let us emphasize once more the fact that the search for hypothesis cannot be reduced only to the method of trials and mistakes. In forming a hypothesis, a considerable role belongs to the investigator's foundations (ideals of cognition and the picture of the world) which aim the creative search, generating investigation problems and indicating the field of the solution means.

Second, the operation of forming a hypothesis cannot be entirely transferred to the sphere of individual creative work of a scientist. They are obtained by an individual, just as his thinking and imagination are formed in the cultural context absorbing samples of scientific knowledge and samples of their production activity. The search for a hypothesis, including choice of analogies and substituting new abstract objects, determined not only by historically developed means of theoretical investigation, into the analog model. This choice is also determined by translation in the culture of certain samples of the investigation activity (operations, procedures) which provide solution of the new problems.

T. Kuhn is right when he mentions that such samples are included into scientific knowledge and mastered in the process of learning.

Translation of theoretical knowledge in the culture means also translation of samples of the problem solution activity. Such samples reflect procedures and operations of generating new hypotheses (foundations of the science – analog model – substitution of new abstract objects into the model). That is why in the process of adoption of already obtained knowledge (formation of a scientist as a specialist) also some quite general schemes of intellectual work, providing generation of new hypotheses, are mastered.

Translation of schemes of intellectual work in the culture, which provide solution of the problems, allows considering the procedures of such generation, abstracting from personal qualities and abilities of a concrete investigator. From this point of view we can talk about the logic of forming hypothetical models as a part of the logic of forming a scientific theory.

Finally, summarizing specificities of the process of forming hypothetical models of science, it is important to emphasize that the base of this process is the combination of abstract objects from one field of knowledge with the structure ("network of relations") taken from another field. In the new system of relations the abstract objects are provided with new features, which makes appear, in the hypothetical model, new contents, which can correspond to not yet studied connections and relations of the object area, for description and explanation of which the hypothesis put forward is dedicated.

This feature of hypothesis is universal. It can be marked at the stage of formation of local theoretical schemes, as well as in construction of a developed theory.

As to procedures of justification of the hypothesis, they also have a quite complicated structure and internal logic. As it follows from reconstructions of development of classical and quantum electrodynamics, traced above, empirical justification of a hypothesis is not reduced to comparison of its corollaries with the results of experiments and observations. It includes procedures of constructive justification which is a condition and a premise of comparison of hypothetical models with experimental facts. Only after these procedures, does the theory get receipts of connections of its fundamental magnitudes with experiment – operational definitions, which guarantee efficiency of empirical verification of the theory. Further justification of hypothetical models and turning them into a theoretical scheme is connected with procedures of their correlation with disciplinary ontology (scientific picture of the world) and philosophical foundations of the science. When these procedures are completed, the ontological status of theoretical schemes as the core of the new theory is justified.

The process of justification of the hypothesis contributes to the construction of conceptual apparatus of the theory not less than the process of generation of the hypothesis. In the course of justification the contents of the basic notions of the theory are being developed. In turn, it creates premises for future theoretical search, as every new hypothesis stipulates usage of already developed notions and models as material for its construction.

If we take into consideration this specificity of development of scientific knowledge, it will be clear how incorrect the positivists were who strictly separated "the discovery context" and "the theory verification context".[96] The logic of discovery and the logic of verification are two aspects of one and the same process of the theory becoming, and there exists close mutual connection between them.

Historical approach to the problem of structure and genesis of the theory requires that we take into consideration not only mutual connections between different aspects of the theory genesis, but also the connection between the process of becoming and peculiarities of functioning of the theory.

Anti-historicism of the positivist analysis of scientific knowledge consists, for instance, in the fact that theory was considered only as given knowledge, without the peculiarities of its genesis. The result of such type of analysis was a quite poor idea of the process of functioning of a formed theory. Positivism could mark only some formally logical aspects of deductive development of theory and the process of theoretical explanation and prediction of events. Informal aspects of theoretical investigation were lost by the positivist history of science.

The interest to these aspects of theory emerged in the Western philosophy of science in connection with formation of post-positivist branches, whose representatives referred to analysis of the history of science. Studying informal aspects of theoretical investigation, they came across the connection between functioning of theory and its genesis. Probably the most interesting results, revealing this connection, were contained in Kuhn's conception of "model" problem solutions. Kuhn noted that operating models in the process of theoretical description and explanation of concrete events is analogous to the way of forming of new knowledge in the history of science.[97] In his analysis, Kuhn closely approached the question of reproduction of the peculiarities of theory's genesis in its structure and functioning. Still, he failed to determine clearly this problem and logical-methodological approaches to its solution. He tried to answer, how the first model problem solutions are created in a theory, appealing to the psychology of perception of the investigator included into the scientific community. At the same time objective origins and premises of formation of the "models" remained outside Kuhn's analysis.

Just as the problem of the "models" can be formulated as the problem of way of reduction of a fundamental theoretical scheme to local ones and transition from basic equations of the theory to their corollaries, so its solution is of greatest importance for understanding the laws of functioning of a theory. The key to the solution of this problem is to be sought in the logic of historical development of scientific knowledge.

Interaction of the operations of putting forward a hypothesis and its constructive justification is that key moment which allows to get the answer, how paradigmatic models of problem solutions appear in the theory.

Having raised the problem of getting models, the Western philosophy of science failed to find corresponding means to solve it, because it did not reveal and analyze, even in the first approximation, the procedure of constructive justification of hypotheses.

Discussing the problem of models, T. Kuhn and his followers emphasize only one side of the question: the role of analogies as basis of problem solving. The operations of forming and justification of meanwhile appearing theoretical schemes remain outside their analysis.

It is quite indicative that within such approach there emerge fundamental difficulties in trials to elucidate, what is the role of the correspondence rules and their origin. For instance, Kuhn believes that in the activity of scientific community these rules do not play such an important role as methodologists usually attribute to them. He especially emphasizes that the most important thing in solving problems is search for analogies between various physical situations and application of already found formulae on this basis. As to the correspondence rules, they, according to Kuhn, are a result of further methodological retrospective, when methodologist tries to ascertain criteria used by the scientific community in application of different analogies.[98] Kuhn is consistent in his views, because the question of procedures of constructive justification of theoretical models is not brought up in his concept. To detect this procedure, we need a special approach to investigation of structure and dynamics of scientific knowledge. It is necessary that we should consider theoretical models included into the theory as reflection of object in the shape of activity. Referring to a concrete investigation of nature and genesis of theoretical models of physics, such approach orients us to a special vision of them: theoretical models are considered as ontological scheme, which reflects essential characteristics of the reality studied, and at the same time as some kind of "closure" of object-practical procedures, within which in principle we can disclose the characteristics. That vision allows to discover and describe operations of constructive justification of theoretical schemes.

With other theoretical-cognitive basis the mentioned operations remain outside methodologist's field of investigation.

But, as it is constructive justification that provides appearance of the correspondence rules in theory, defining their contents and meaning, it cannot surprise us that Kuhn came across difficulties in determining the ways of forming and functioning of these rules.

It is characteristic that in discussion of the problems of samples Kuhn refers to the history of Maxwell's electrodynamics. Analyzing it only in the plane of application of analog models, he believes that the main results of Maxwell's investigation were gained without any construction of correspondence rules.[99] But, as we have seen, this conclusion lies far from real facts of the history of science.

We think that the above described analysis of procedures of construction of a theory allows getting answer to the question: where do model situations appear from in theory. Such model situations (examples of solution of theoretical problems) demonstrate methods of construction of local theoretical schemes on base of a fundamental one, and ways of transition from basic laws of theory to local theoretical ones. Forming and including such model situations into the theory take place in the course of its becoming.

In construction of a developed theory its fundamental theoretical scheme is created by means of consequent generalization of those theoretical schemes which either preceded the theory, or were constructed in the course of theoretical synthesis. This generalization is carried out by means of creation of several intermediate models, and each of them is aimed at representation of new, not considered before, characteristics of interactions studied, in the theory.

First the investigator introduces each of such models as a hypothesis and then gives its constructive justification. In the course of constructive justification of the model he works out two main proofs.

The first one determines that the model is able to express essential characteristics of situations being generalized. Such characteristics previously could be represented in cognition by local theoretical schemes. Now, when constructive justification of the model is done, the content of the mentioned schemes is included in the generalizing model.

During the second proof the investigator makes sure that in course of new generalization of the model its previous constructive content is not destroyed. This content corresponded to the local theoretical schemes which were assimilated by the generalizing model at previous stages of theoretical synthesis. To make sure this content is preserved, the investigator explicates it. From the generalizing model he derives corresponding local theoretical schemes which, in their content, are equivalent to the theoretical schemes assimilated in the theory.

Thus, in the course of the process of construction of a theory the investigator reduces the fundamental theoretical scheme being created to local theoretical schemes. The methods of such reduction reproduce, in their main features, the methods used for including of essential characteristics of concrete physical situations reflected in the theory, into generalizing model. Such including was executed by means of intellectual experiments based on real possibilities and peculiarities of the experiment. In the course of such experiments the investigator's thought traveled from model to experiment and from experiment to model, studying all main intermediate links between model and experiment. The same thought experiments in their main features are repeated in explication of constructive contents included into the model, when the latter is reduced to some local theoretical scheme. As in the process of justification of the model by new experiment, the investigator first considers concrete specificities of physical situations, and then imposes restricting conditions on the model and constructs a local theoretical scheme.

It is characteristic that at the final stage of theoretical synthesis, when the main equations of the theory are introduced and constructive justification of the fundamental theoretical scheme is accomplished, the investigator executes the last proof of correctness of the equation introduced and their interpretation: from the main equations he gets, in a new form, all generalized local theoretical laws, and then, on base of the fundamental theoretical scheme, he constructs local theoretical schemes corresponding to the said laws. A typical example of such justification is the final stage of formation of Maxwell's theory of electromagnetic field, when it was proved that on base of the theoretical model of electromagnetic field it is possible to obtain, as particular cases, theoretical schemes of direct current electrostatics, electromagnetic induction etc. and from equations of electromagnetic field – to deduce Coulomb's, Ampere's, Biot-Savart's laws, laws of electrostatic and electromagnetic induction discovered by Faraday, etc.

Final justification of the main equations of the theory and the fundamental theoretical scheme at the same time present as account of the "ready" theory. The process of its becoming is reproduced now in reverse order, in shape of deductive development of the theory, deriving corresponding theoretical corollaries from the main equations. Each conclusion here can be considered as account of some method and result of solution of a theoretical task.

Thus, the very process of constructing a theory forms and includes model situations of solving theoretical tasks.

Further functioning of the theory and expansion of its application area creates new examples of solving problems. They are included into the theory, along with those introduced in the beginning of its formation. With development of scientific knowledge and changes of previous form of the theory, the initial models are also modified. But, in their modified shape, they are normally preserved in all further accounts of the theory. Even the latest formulations of classical electrodynamics demonstrate methods of application of Maxwell's equations to concrete physical situation; the example used is derivation Coulomb's, Ampere's, Biot-Savart's, Faraday's laws from these equations. The theory, we may say, preserves in itself traces of its past history, reproduces – as typical problems and ways of their solution – the main specificities of the process of its forming.

Genesis of the theory is imprinted in its organization and determines its further existence. If we define genesis of the theory as intensive way of knowledge development, and functioning of the theory – as extensive way of such development, we will see that both ways are closely linked. Reproduction in a logic of unfolding the theory formed of the main specificities of its becoming is one of the sides of such mutual connection. But there is another side: active influence of the process of functioning of the formed theory upon future shapes of intensive development of theoretical knowledge.

After the theory is constructed, it enters the stage of explanation and prediction of new phenomena. Here the empirical basis of the theory is extended, it being known that the new empirical material is not only mechanically absorbed by the theory, but has active reverse action. The theory is now changing in the course of application to new situations.

One of the main reasons of such changes is difficulties emerging with solving new problems by old methods. To work out methods which would provide solution of wide range of such problems, we have to change mathematical means and develop new theoretical models of the reality studied. As the result we have reformulating of the existent theory: new mathematical apparatus is created, and its conceptual structure is developed.

The history of science presents us a lot of evidences of such development of a theory already settled. For instance, Newton's mechanics first was reformulated, on base of application of analytical methods, by Euler, and then reconstructed into Lagrange mechanics and Hamilton-Jacobi mechanics. Any such reconstruction was connected with application of mechanics to new physical situations and desire to work out general methods of solving various problems. Euler developed the analytical apparatus of mechanics to obtain universal methods of determining states of a material point or a system of such points under action of forces. The new methods enabled him to work out an absolutely new part of mechanics: solid body dynamics. Lagrange's, and later Hamilton-Jacobi's reformulating of mechanics were – to a considerable degree – caused by needs in description and explanation of complicated mechanical systems. Analytical methods, based on the accelerating forces principle, could not be applied in the process of solving quite a number of problems, as the value of forces applied to each body, which was a part of a complicated system, is normally unknown in advance. Lagrange's mechanics, and then Hamilton-Jacobi mechanics solved such problems successfully. In this process of development of mechanics its new mathematical apparatus were formed, its new principles (for instance, the smallest effect principle) were introduced, its new fundamental notions (effect, energy etc.) were formulated.

This kind of specificities of development of already settled theory can be traced also in other historical examples. Thus, predictions of electromagnetic waves and further application of Maxwell's theory to explanation of optical phenomena led to development of conceptual apparatus of electrodynamics (there appeared ideas of electromagnetic wave, electromagnetic radiation etc.). At the same time, as the sphere of empirical application of Maxwell's equations expanded, so it required that the mathematical shape of the theory should be improved. In H. Hertz's and O. Heaviside's works Maxwell's equations were expressed in a form close to a modern one, and then electrodynamics was accounted with help of modern methods of vector analysis.

Finally, we can refer to one more example of reconstruction of a settled theory: historical development of quantum mechanics. After it had been created in its initial version (by W. Heisenberg, E. Schrödinger, N. Bohr and M. Born), its application for explanation and prediction of a wider and wider set of processes in atomic sphere was accompanied by development of the apparatus and the conceptual structure of the theory. The stages of such development are, for example, Dirac's strict operational formulation of the theory in terms of q-numbers, von Neumann's axiomatic model of the quantum theory, Feynman's formulation of quantum mechanics (path integrals).

Reconstruction of a theory in the process of its functioning not only generates new methods of solving problems but also creates means for building new fundamental theories. Mathematical apparatus and conceptual structures, which are developing in the process of application of the settled theory to new physical situation, might be precisely those means needed whose employment in a new area of theoretical search provide intensive development of scientific knowledge.

Electrodynamics could not have been worked out, if mechanics had not formed the mathematical apparatus which provided solution of hydrodynamic problems. The development of quantum physics was carried out, in a great part, due to mathematical structures and notions formed in Lagrange's and Hamilton-Jacobi mechanics. The number of such examples can be increased.

Thus, means for future theoretical search and construction of new theories are created not only at the stage of becoming of the theory, but also, at even a greater extent, at the stage of functioning of a developed theory. This side of mutual connection of genesis and functioning of a theory was missed in Kuhn's analysis. In his conception of development of science the stage of extensive increase of knowledge is sharply opposed to its intensive development. In the real history of science these two sides are closely connected: genesis of the theory determines its functioning, while functioning of developed theories prepares basis for new theoretical structures.

Forming the conceptual structure of a new theory is the result of interaction of mathematical apparatus, theoretical schemes and experiment. Dynamics of such interaction is mostly determined by procedures of constructive justification of theoretical scheme. These procedures have practically never been analyzed in methodological and philosophic literature.[100] Meantime, their disclosure opens new perspectives for getting concrete methodological conclusions and recommendations. First of all, we can present the idea of "constructability" as a methodological rule, which indicates ways of construction of adequate interpretation of mathematical apparatus of the theory. This rule can be formulated in the following way: after a hypothetical model of explanation of empirical facts is introduced, new, hypothetical features of the abstract objects of the model are to be introduced as idealization based on a new layer of experiments and measurements, the layer which was intended to be explained with help of the model. Moreover, we have to make sure that the new features do not contradict to the features of the abstract objects justified by previous experience.

This rule does not mean the same as the requirement to verify theoretical knowledge by experiment. According to analysis of the historical material, verification of this kind stipulates (especially at modern stage) complicated activity connected with construction of adequate interpretation of the equations introduced. The core of such interpretation is constructive introduction of abstract objects. That is why the rule of "constructability" not only says that empirical justification of a theory is necessary, but also indicates how, in what manner such justification is done.

From the requirement of constructive introduction of abstract objects follow quite nontrivial methodological conclusions. One of them has already been discussed. It refers to connection between existence of non-constructive objects in the "body of the theory" and paradoxes emerging there. Since the presence of non-constructive objects can lead to paradoxes in a theoretical system (though not necessarily), then application of the "constructability" rule allows uncovering contradictions inside knowledge before they are uncovered in the spontaneous course of the investigation. This, in turn, can be a means to reconstruct the theory effectively, and to form a conceptual structure which would adequately reflect the new object. To find such a criterion is especially important in respect to modern knowledge, which is quite complicated in its system organization and where it is not always easily to find inconsistency.

The model of such activity aimed at analysis of inconsistency of knowledge by means of constructive justification of theoretical schemes may be Bohr-Rosenfeld procedures in quantum electrodynamics.

When we find non-constructive elements in a theoretical model, we can see weak points of the theory, which are – sooner or later – excluded through replacement of corresponding elements of the theoretical model and its constructive reorganization. This problem should be analyzed especially, as the requirement of elimination of non-constructive objects is close to the requirement of the "observability" principle. Here we are to discuss the question of relationship of ideas of "constructability" and "observability".

As we know, the "observability" principle meant that in construction of a theory the investigation should apply only magnitudes that have operational meaning, while ideas which cannot be verified in experiment should be eliminated from the theory.

Vast philosophical and physical literature gives us quite exhaustive analysis of the ideas of fundamental "observability". It shows that the "observability" principle, applied along with other principles of physics, had quite an important heuristic role in its development, but its usage took place differently in different investigational situations. The strict requirement to eliminate non-observable quantities from the theory has never been applied in physics. This requirement, if understood literally, prohibits us at all to use non-observable magnitudes, while without them we fundamentally cannot construct any hypothesis, because at the stage of such construction the investigator uses mostly non-observable objects (when he supplies the objects of the model with hypothetical features, he, usually, does not know which of them would be justified by experiment, and which of them not). Besides, in a theory already developed there always can exist auxiliary constructs (like "bare electron" in quantum electrodynamics) which are important for the development of the theoretical contents but which are fundamentally non-observable.

At the same time, in some investigational situations the ideas of "observability" unexpectedly turned out quite heuristic. For instance, in the period of construction of quantum mechanics elimination of non-observable electron orbits was a powerful impulse to development of the theory. A situation like this can be found in the period of construction of the special relativity theory, when elimination of non-observable absolute space allowed to develop new images of space and time.

All this is an evidence of certain part of rationality in the ideas of "observability", but, at the same time, of inadequacy of the very formulation of the "observability" principle, which does not include concrete directions: where and when it can be applied in the investigation, how we can tell observable quantities from non-observable ones, and at what stage of construction of the theory we are to eliminate non-observable objects.

Consequently, the regulative role of the "observability" principle was reduced to a trivial claim: to construct the foundation of the theory on magnitudes, tested by experiment, and to base on the intuition of the investigator who should find out, which magnitudes are to be considered as observable, and which ones are to be rejected as fundamentally non-observable.

The inadequacy of the very formulation of the "observability" principle was, in a major part, connected with its genetic, theoretical-cognitive origins. One of the first formulations, given by E. Mach, proceeded from false statements of his philosophy, that theory does not reflect the objective world, but experience and is not more than a brief reproduction of the facts observed. Later logical positivism tried to revive that idea in the form of the method of logical analysis. Positivism required that theory should eliminate all metaphysical ideas which have not been verified (checked up on base of reduction of the concepts to the data of observation). But a theory cannot be reduced to a brief summary of observations, and its notions cannot be treated as just fixation of phenomena observable in the area described by the theory: the theory reflects not the events, but the essence of processes in the real world, while scientific concepts have meaning not only within a certain theory, but they accumulate all preceding history of cognition which uncovers – step-by-step – new characteristics of the objective world.

The positivist interpretation of theory and following "linear prescriptions" of elimination of all non-observable concepts from science led to conclusion that no scientific theory could survive if "purified" in accordance with prescriptions of methodology of logical analysis.

No surprise that inadequacy of such statements to real specificities of scientific cognition led to a deep crisis in positivist philosophy of science.

At the end, the very positivist interpretation of the "observability" principle was put away. But at the same time there emerged an urgent problem of correct understanding of methods of empirical verification of a theory and discovering rational part of the "observability" principle, falsely interpreted by positivism.

In the course of this process investigators started to gradually understand that the abnormal hardness of the observability principle followed from the fact that theory is presented as result of purely inductive generalization of the facts observed. Understanding real methods of construction of a theory caused efforts to make a less hard formulation of the observability principle. We were to indicate, at what particular stage of development of the theory it could play the role of a methodological regulator.

A great part in the right formulation of this goal belonged to methodological investigation of the problem of "observability" made by classics of modern natural science A. Einstein, M. Born et al. What is especially interesting is the analysis of A. Einstein's comments of 1926 concerning W. Heisenberg's understanding of the "observability" principle. Einstein indicated that the very idea of "observability" depends on the theory. Only the theory determines what is observable, and what is not.[101] Einstein's criticism exercised influence upon Heisenberg's works of the 1930s, where the latter postulated that a considerable number of new conceptions should be introduced into a theory, and only then the nature will decide, whether to revise them or not – in every point. In this respect M.E. Omelyanovsky told a truth saying that for concretization of the ideas of "observability" we are to add: introduction of new concepts into a theory should take place at the stage of creation of the theory, and their verification should be done basing on new experience.[102]

Further investigation of the "observability" principle required analysis of the structure of the theory, methods of organizations of concepts inside the theory, distinguishing main and auxiliary abstract objects. Such analysis leads to ideas of constructive justification of the abstract objects of the theory.

After all above we can formulate the difference between requirements of "constructability" and the "observability" principle.

1. “Observability” stipulated inductive construction of the theory, while the “constructability” ideas are based on the opposite vision of genesis of the theory (from the very beginning they take into account that theoretical models are introduced from above, in respect to experiment, as hypotheses and only then are justified constructively).

2. The “observability” principle, at the best, only marks that at the stage of putting hypotheses forward we can use various notions, and only at the stage of justification of the hypothesis verify their empirical sense. The requirement of “constructability” clearly differs these to stages from the very beginning, meaning that constructive introduction of abstract objects into “the body” of the theory starts only after introduction of the supposed hypothetical model.

3. In the “observability” principle there is no differentiation of ideal objects of the theory, so it is not clear which of them are to be considered as observable, and which are non-observable. Criteria of such differentiation are transferred to the sphere of the investigator’s intuition. In the requirement of “constructability” we have an effort to introduce such differentiation (at least, in the first approximation). It is supposed that what should be constructively justified (i.e., introduced as an idealizations based on new experience) is abstract objects of the theoretical model which lies in foundation of the theory. Such model is pretty clearly indicated in any theory (so we can agree with Einstein that concrete structure of a concrete theory indicates what there should be observable and non-observable). Then, taking into account that a concrete theoretical scheme (model) and picture of the world should be distinguished, we may divide the problem into two parts: constructive justification of the theoretical scheme and constructive justification of the picture of the world. The latter can as well include non-constructive elements (visual auxiliary images which let us inscribe the created scientific knowledge into the culture of a certain period). These elements are eliminated from the picture of the world only in the long course of historical development. At the best, they can be fixed as non-observable essences, but “criticism of the pictures of the world” takes place only on the eve of their breach. As to abstract objects of concrete theoretical schemes, they are mandatory to be introduced constructively.

4. The “observability” principle, in its strict formulation, required that non-observable objects should be eliminated from the theory immediately after they are discovered. According to the ideas of “constructability”, the process of replacement of such objects can be executed as a long search for a new constructive meaning of the theoretical model. But the very fact that a non-constructive object has been found allows us to develop a consistent investigation. In this case the process of construction of theoretical knowledge can be run not by means of immediate elimination of the non-constructive object from the theoretical scheme, but by its localization and use of the theoretical scheme in further cognitive movement so that it could “work” only with its constructive elements. A characteristic example of such investigation is the process of development of knowledge based on the atom model, offered by Bohr and developed by Sommerfeld. That model included electron orbit (a non-constructive element), but Bohr, knowing that it is a “non-observable” object, constructed the system of postulates describing basic relations among main elements of the model, so that they “localized” the main paradoxical corollaries of employing electron orbits (it was supposed that electron, in its stationary state, does not radiate).

Considering the chance of this way of development of knowledge, we may come to conclusion that the very fact of discovering non-constructive objects provides progress of the theory, even if they are eliminated much later than they are discovered.

Thus, the method of constructive justification of theoretical schemes, indicating a concrete procedure of discovering non-constructive objects in "the body" of the theory, can make it easier to solve many investigation problems.

NOTES: CHAPTER 5

---

[1] Mandelstam (1972, p.329).
[2] Vavilov (1956, pp.156-157, 282-285). Mandelstam. (1972, pp.326-329), Kuznetsov (1975, pp.140-155).
[3] Vavilov (1972, pp.79-80).
[4] Ibid, p.80.
[5] Einstein (1965-1967, vol.2, pp.23-25).
[6] The process of becoming of the relativity theory and forming relativistic ideas of space and time in the physical picture of the world will be analyzed in more detail in chapter 6 "Scientific Revolution".
[7] Bohr (1970-1971, vol.2, p.510).
[8] Bohm (1952),
[9] Sachkov (1974, pp.71-72).
[10] Povarov (1972).
[11] Arshinov (1974), (1973).
[12] Stepin (1976, pp.290-300), (1982, pp.169-172).
[13] Chew (1966), Chew, Gell-Mann and Rosenfeld (1965), Chew (1968).
[14] Stapp (1971).
[15] Bohm (1971, p.28).
[16] See Bohm and Hiley (1977, pp.207-209).
[17] Nordin (1979, p.72).
[18] See Bohm and Hiley (1977), Philippidis, Dewdney and Hiley (1979).
[19] Concerning differences between those two strategies see Blauberg, Sadovsky and Yudin (1969, p.49).
[20] Capra (1994, p.298).
[21] Ibid.
[22] Ibid, p.174.
[23] Prigogine and Stengers (1984).
[24] George and Prigogine (1979, p.380).
[25] Prigogine and Stengers (1994, p.214).
[26] Ibid, p.215.
[27] Kurdyumov (1982, pp.235-236).
[28] Konopleva and Sokolik (1972, p.119), Vizguin (1975, pp.95-96).
[29] Mandelstam (1972, pp.329-337).
[30] It means that a particle is able to move at any speed — from zero to light speed (or, what is the same, energy of its movement is not necessarily small in comparison with the energy at rest).
[31] See Pauli (1964).
[32] Ibid.

---

[33] We would like to remind that, according to Pauli's principle, each energy state can include not more than one electron. In accordance with the identity principle, exchange effects are non-observable in such system.

[34] See van der Warden (1962, p.282).

[35] Pauli (1956, p.373).

[36] Quoted from Vavilov (1956, p.80).

[37] Kuznetsov (1975, pp.153-154).

[38] The term "modern situation" is used here in broad sense, in application to physics of the 20th century where quantum and relativistic theories prevail. In a narrower sense and within more differentiated approach it is desired that we should distinguish non-classical physics of the first and the second halves of the 20th century. In relation to the latter, special relativity and general relativity are sort of "quantum-relativistic classics". The image of physics of the last third of the 20th century is determined by investigation programs represented by quantum chromodynamics, the Grand Unification theory, theories of supergravitation and superstrings, in which investigators make efforts to come to synthesis of quantum and relativistic ideas, and create a whole theoretical description of strong, electroweak and gravitational interactions.

[39] Here we bear in mind construction of a developed theory in its first version (for instance, Newton's mechanics, Maxwell's electrodynamics), but not previous knowledge of some aspects of the object domain studied in theory, nor development and perfection of foundations of the theory already created (such as reformulation of Newton's mechanics made by Lagrange and then by Hamilton).

[40] Pauli (ed.) (1955, p.71).

[41] Ibid.

[42] The idea of quantum properties of radiation was historically the first fact which was integrated into foundation for development of quantum mechanics. But quantum mechanics for electromagnetic radiation (the theory of free quantized electromagnetic field) was created later than quantum mechanics for atom and atomic particles (electrons, nuclei etc.). This is due to the fact that atomic particles have different from zero rest mass, so for them there is energy area where it is possible not to take into account the effects of the relativity theory. As to photon, its rest mass is zero, so it does not have non-relativistic area. That is why the idea of electromagnetic field as a system of photons could be theoretically expressed in corresponding apparatus only after the quantum theory for non-relativistic particles had been created.

[43] The logic of construction of mathematical apparatus of quantized radiation field can also be traced in its "historical realization". P. Dirac started building this apparatus, then P. Jordan, W. Pauli and W. Heisenberg kept on working at it. In 1926–1927 Dirac suggested the first version of the quantum theory of electromagnetic radiation, which already included the method of quantizing of free field typical for modern physics. The base for transition to the quantum theory was a special method of classical description of the field. The classical radiation field was considered as a set of plane transverse waves, encased in a large but finite space volume. Correspondingly, the classical field equations were expressed through Fourier's transformation and then written down in a shape analogue to canonical equations of mechanics (Hamilton's equations). The expression for energy (Hamilton's functions) of each of the waves whose superposition as the field of study coincided with Hamilton's function for oscillator, which allowed to correlate a set of waves and an analogue set of oscillators. This method of description of electromagnetic field was known even to classical physics. Using it and then applying the quantizing rule for oscillator, Dirac carried out quantizing of the radiation field; its Fourier-components, preliminarily presented as canonical variables (generalized coordinates and impulses) were considered as operators which obey commutation relations. The influence of these operators upon the wave function characterized processes of creation and annihilation of photons in various quantum states. Formally it was expressed in the following way. The wave function (state vector) was defined as a function in space of filling numbers, i. e. photon-particles in various quantum states. Acting on it, the operators, corresponding to the field's Fourier-

components, either increase, or reduce by one the filling numbers, which means either creation, or annihilation of a photon in the given quantum state (correspondingly, these operators are called creation and annihilation operators).

This apparatus, mainly worked out by Dirac, allows us to explain many facts of interaction of electromagnetic radiation and matter (in particular, investigators got corollaries from it: well known rules describing emitting and absorption of electrons).

Jordan, Pauli and Heisenberg perfectioned Dirac's theory of electromagnetic radiation. They constructed the apparatus of the theory in the form satisfying Lorentz's transformations. Here we are to say that Dirac, who developed a perspective method of field quantizing, still failed to create equations relativistically covariant. Jordan and Pauli were the first to overcome this obstacle when they found a Lorentz-invariant expression of commutation relationships for the field operators (see Jordan and Pauli (1928)). It became possible, within the new formalism, on base of initial creation and annihilation operators, to create other operators which would correspond to various field quantities, answering the requirements of the theory's relativistic invariance.

[44] In construction of the mathematical apparatus of quantized electron-positron field Dirac's equations played a role similar to the one of Maxwell's equations in construction of the apparatus of quantized electromagnetic radiation field. The wave functions for electron and positron, in Dirac's equations, were presented as magnitudes characterizing electron-positron field and then regarded as operators satisfying anticommuting transposition relationships (this method, based on presentation of wave functions as operators, was then called the secondary quantization method).

[45] To find probabilities of the quantum effects characterizing dispersion of particles, which form electromagnetic and electron-positron field, we build a so-called dispersion matrix, or *S*-matrix. Squares of modules of this matrix's elements characterize probabilities of transition of the system described from some initial state to some final state. To find the *S*-matrix, we solve a connected system of operator equations which describe interacting quantized fields. The exact solution of this system is unknown, but we may find an approximate solution by means of the perturbation theory. In the framework of this theory interaction is considered as perturbation of the state of one free field by another in some area of interaction. Such visualization corresponds to consideration of particles, which interact only in the process of collision, while before and after collision they move independently. The states of non-disturbed system (of non-interacting photons and electrons, in this case) represent some basic integrity of quantum states. Perturbation (interaction of fields) leads to quantum transitions between these states (to changes of the number of the particles, their energies, impulses etc.). In the perturbation theory the dispersion matrix is presented through operators of free quantized fields and is computated in expanded form under the interaction constant which, in case of electromagnetic interactions, has the form of a dimensionless quantity $\alpha = \frac{e^2}{\hbar c} = \frac{1}{137}$ where α – electromagnetic interaction constant (or thin structure constant), e – charge of electron, $\hbar$ – Planck's constant, c – light speed.

[46] Feynman (1968, p.180).

[47] The correspondence principle has two aspects. The first one can be defined as generally methodological. Here the correspondence principle plays a specific form of connection between old and new theories (see Kuznetsov (1948)). The other aspect of the correspondence principle marks peculiarities of quantum mechanical description: the quantum object theory cannot be constructed without the language of classical mechanics. This aspect, though tied with the first one, cannot be reduced to it. It expresses the special nature of quantum objects: their physical being, characterized by physical magnitudes, is determined by macroconditions, the way of interaction of a quantum object with a classical body (see Kuznetsov (ed.) (1967, p.105-109)).

[48] We would like to remind the reader that, according to T. Kuhn's views, the change of vision of investigation situations is always stipulated by changes of some models, as "patterns", which indicate how to consider the said situations. From this point of view, the transition from vision of the system

---

of electrons as of a set of particles with quantum nature; their vision as of a field could be explained by choice of a new "pattern". The latter is understood as quantized electromagnetic radiation field, through which the investigator sees also other objects, for instance, he evaluates the system of electrons as a set of quanta of some field. Still, this approach, correct to some extent, leaves some important sides of the investigation process in the dark. It does not take into consideration the above mentioned difficulty of transfer of ideas about the system of photons as a field to a system of electrons (presence of a classical pattern in the first case and its absence in the second one). To carry out such transfer, we, previously, are to refer them to some general class and only then consider one object in the image, after the likeness of another. In other words, to compare, we are to have a base for comparison; to assimilate one image to another, we need a scheme of image distinguish. In this case the role of such a scheme belonged to the picture of physical reality which introduced an extremely general notion of the nature of quantum objects. Correlation of electromagnetic field and system of electrons with it was a base for further representation of one of the objects as a model of the other.

[49] The modern stage of quantum relativistic picture of the world is connected with elaborating the program of Grand Unification which is aimed at synthesis of the four main types of interaction: strong, weak, electromagnetic and gravitational. A considerable success of this program was construction of the electroweak interactions theory.

[50] Fok and Jordan (1931, S.206).

[51] Landau and Peierles (1965).

[52] The given relationship was first obtained by N. Bohr in 1928. Landau and Peierles give a derivation of that relationship (Landau and Peierles (1965, pp.59-61)). Energy and impulse exchange between the particle and the device should follow the conservation laws of impulse and energy. The impulse conservation law provides the following dependence between change of the particle impulse P and the device impulse p before and after measuring: $p'' + P'' - p' - P' = 0$ (1) where p' and p'' – the state of the device before and after the exchange of impulse with the particle, P' and P'' – corresponding states of the particles. The energy conservation law requires the same dependence for energy exchange between the particle and the device during measuring time Δt. Considering the relationship $\Delta\varepsilon\Delta p \sim \hbar$, this dependence looks $\varepsilon'' + E'' - \varepsilon' \sim \hbar/\Delta t$ (2), where ε' and ε'' – energy of the device before and after measuring, E' and E'' – corresponding values for energy of the particle. Values of p' and p'' and ε' and ε'' as related to the device, are always known within any accuracy. So, the equations (1), (2) give relationships $\Delta P' = \Delta P''$ and $\Delta E'' - \Delta E' \sim \hbar/\Delta t$ (3) for impulse and energy of the test particle. In accordance with the correlation between energy and impulse, $\Delta E' = \frac{\partial E'}{\partial P'}\Delta P' = v'\Delta P'$ (v' – speed of the particle before collision), and $\Delta E'' = \frac{\partial E''}{\partial P''}\Delta P'' = v'\Delta P''$. Substituting these values to (3), we get $|\, v''-v' \,|\, \Delta P \sim \frac{\hbar}{\Delta t}$. Thus, this correlation appears because any measuring takes some period of time Δt, during which there appears indeterminacy in the exchange of energy-impulse between the measured quantum particle and the classical device.

[53] It is connected with the need to control the change of speeds of the particle at the moment of its collision with the device — to compute the disturbing influence of its own radiation field upon its impulse. But such control, in turn, stipulates new measuring (determining velocities v' and v'' before and after collision of the particle with the device), measuring during infinitesimal time period. The situation is repeated also due to $\Delta P\Delta t \sim \frac{\hbar}{\Delta t}$, if $\Delta t \to 0$, then $\Delta P \to \infty$, i.e., any control over the disturbing influence of the field radiated by the particle, upon its impulse, leads to increase, not decrease, of indeterminacy of this impulse.

---

[54] To avoid analysis of disturbing effect of charged experimental particles on the electron, Landau and Peierles, treating photons as such particles, constructed their thought experiments in accordance with the scheme of experiments based on Compton's effect. In that case it was important that the impulse of photon, colliding with electron and transferring information of its state to the device, can be measured during time period Δt only with indeterminacy ΔP which cannot be made smaller than $\frac{\hbar}{c\Delta t}$ (according to the relationship $\Delta P\Delta t \geq \frac{\hbar}{\Delta t}$). If we take this circumstance into account, it means that a classical device can fix the magnitude, characterizing the state of the electron, with the corresponding indeterminacy.

[55] The quantum mechanical description of densities of the charge-current stipulates their representation as a set of separate electrons. The latter can be interpreted as quanta of electron field. According to a postulate of quantum mechanical description, classical quantities characterizing the system should be used also as observables in description of its quantum properties. Sources of the field were characterized in classical electrodynamics by vector of density of charge-current in a space-time point. When we determine this magnitude in the process of measuring it is taken that the time period, required for measuring, should be infinitely small. But in this case, quantum effects taken into account, it is impossible to get the exact value of this fundamental quantity, which contradicts to the quantum mechanical description postulate, which sets no limitations to exact measuring of one observable.

[56] The evidence is W. Pauli's skepticism expressed in 1932 (in. Collected Scientific Papers by W.Pauli, in Two Volumes. Ed. by R. Kronig and V.Weisskopf. New York-Sydney, Interscience Publishing, 1964. (.P. 284 — 286)).

[57] We would like to remind to the reader that the initial model for quantizing the field was the idea of it as of an infinite set of oscillators, each of them is subject to quantizing. The field energy was written down as sum of expression for energy of each oscillator. These expressions meant that the energy values of zero oscillations of all field oscillators are different from zero. At the same time, the said expressions showed that the state studied cannot include photons, i.e., physically it should be pure vacuum. As the number of the field oscillators was infinite (according to the number of the degrees of freedom), we had that, without photons, instead of the expected zero energy there emerged infinite energy which should be attributed to vacuum. That conclusion was so unexpected that initially it could well be regarded as evidence of profound defects of the theory created.

[58] Landau and Peierles (1965, p.69).

[59] Ibid.

[60] Here the term "translation" means that the state of the experimental body during time $t_1 - t_2$ between interactions with the object measured, on the one hand, and the register device, on the other hand, either does not change, or changes in time in accordance with the known law, on base of which the observer can determine the initial state of the experimental body, which is an indicator of the studied state of the measured object.

[61] Landau and Peierles (1965, p.57).

[62] In this case we can always operate so that the experimental body, once having interacted with the measured quantum system, would move as a free particle, without any more influences (translation of its state would follow Schrödinger's equation, and at any moment we could receive information about this state on base of the said equation). As to perturbing influence of the register device upon the state of the experimental particle during time Δt (time of registering this state), we can minimize emerging indeterminacies by means of corresponding choice of Δt. If we bear in mind values of energy ε or impulse P of the experimental particle as characteristics of its state, indeterminacies $\Delta\varepsilon$ and ΔP (caused by quantum effects which emerge with transmission of energy-impulse of the experimental particle to the device) can be reduced by increase in measuring time Δt (in accordance with correlations $\Delta\varepsilon\Delta t \geq \hbar$ and $|v''_x - v'_x|\Delta P_x\Delta t \geq \hbar$). All this makes measurements in the area of non-relativistic quantum interactions quite predictable, even if the experimental particle interacts with the

---

register device as a quantum object. Analysis of such measurements, when we get information about state of quantum systems not through their immediate interaction with the device (direct measurements), but through a number of intermediate links – quantum mechanical particles (indirect measurements), and justification of fundamental possibility of such measurement in non-relativistic area can be found, for instance, in L. Mandelstam's lectures on quantum mechanics (Mandelstam (1972)).

[63] By the way, the discussions of incommensurability are very close in time to two Solvay congresses of 1927 and 1930, where the famous disputes on foundations of the quantum theory between Bohr and Einstein took place. The corner stone of these disputes was specificity of quantum mechanical measuring and clearing of special role of classical device in determination of states of the quantum system measured.

[64] Appearing psychological barrier and overcoming it is one of the characteristic features of the psychology of discovery in science. A detailed discussion of this aspect of scientific creative work can be found in B. M. Kedrov's writings.

[65] Bohr (1970-1971, vol.1, pp.125-131).

[66] Rosenfeld's memories of his work together with Bohr start from a later period (late February, 1931), when Bohr's discussions with Landau and Peierles were over. We believe, at that time Bohr had already come to the general idea that it is mandatory that classical experimental bodies should be employed in idealized measuring procedures. Describing the corresponding period of the history of electrodynamics, Rosenfeld intended to reproduce the main stages of the measuring procedures which led to justification of fundamental measurability of the components of quantized field. Naturally, he pays closer attention to the procedures, and not to the preliminary period. That period is mentioned by Rosenfeld without any specific details. No wonder that the logic of thoughts, which led Bohr to his remark about the field averages, remains at the background. Reconstruction of that speculation never was among the aims of Rosenfeld's essay.

[67] Bohr (1970-1971, vol.2, p.130).

[68] It is easy to understand that here we see that very set of questions that made Landau and Peierles to come to the conclusion of fundamental impossibility to measure field. Bohr and Rosenfeld return to discussing these questions, but on a fundamentally different base: analysis of the measurability problem within thought experiments with classical experimental bodies.

[69] Bohr (1970-1971, vol.2, p.132).

[70] This formula is easily deduced from Lorentz's equation $F_x = \frac{1}{\rho}\frac{dP_x}{dt}$ for the force of the field acting on the charge ρ at moment t in direction of x-axis. Turning to integral form of this expression for force component, affecting a charged body of volume V during time t averaged over area $V\tau$, and taking into account that the force of the field action upon a charged body, by definition, gives the value of the field strength, we get the formula $\bar{E}_x = \frac{P_x}{\rho V\tau}$, where $P_x = p''_x - p'_x$.

[71] Bohr (1970-1971, vol.2, pp.132-133).

[72] Bolzman (1929, p.121).

[73] Pauli (ed.) (1955).

[74] Ibid.

[75] The conclusion that it is possible to present the experimental body as a part of the device was, probably, prepared by analysis of the functions of experimental bodies. N. Bohr carried out this analysis while constructing his program of idealized measurements.

[76] Bohr (1970-1971, vol.2, pp.141-142).

[77] Certainly, here we will have some error in the impulse ΔP_x due to the relation $\Delta P_x \Delta t \sim \frac{\hbar}{c}$. But, with fixed Δt, such error has a certain order of magnitude. As Bohr and Rosenfeld have demonstrated, it exactly corresponds to the value ΔP_x which appears with fixed indeterminacy Δx in position of the experimental body along with its displacement caused by interaction with the register device. The presence of indeterminacy ΔP_x, when Δx is fixed, does not prevent us from exact measuring the averaged on $V\tau$ field component, since, as we have proved, that error can be compensated by increase of the density of the charge of the experimental body (for more details see Bohr (1970-1971, vol.2, pp.137-138)).

[78] Bohr (1970-1971, vol.2, p.137).

[79] Ibid, pp.139-140.

[80] Ibid, pp.142-143.

[81] Ibid, pp.142-143.

[82] Bohr (1970-1971, vol.2, p.149), Pauli (ed.) (1955).

[83] Pauli (ed.) (1955, p.78).

[84] Ibid.

[85] Bohr (1970-1971, vol.2, pp.153-158).

[86] Pauli (ed.) (1955, p.76).

[87] In this case we would have to consider the radiation, caused by displacement of the experimental body by Δx at measuring its impulse, and which cannot be compensate, as that perturbing influence, which basically prevents us from exact determination of the field component.

[88] Bohr (1970-1971, vol.2, pp.434-445).

[89] Pauli (ed.) (1955).

[90] The first Bohr's and Rosenfeld's publication dedicated to the problems of measurability of quantized electromagnetic field was made in 1934. The work referring to measurability of densities of current charge was published, in its final version, in 1952, but its first edition, as a review, was prepared in the mid 1930s and was quite well known for the majority of theorists who worked at the problem of field quantizing (see. L. Rosenfeld's memories in Kuznetsov (ed.) (1967, p.76)).

[91] In modern exposition, the need to consider the observables as summary of interaction of a bare charged particle with vacuum is often corroborated by references to vacuum polarization (interacting with vacuum, electron gets polarization “cover” made of virtual electrons and positrons, which an outside observer perceives as effective reduction of the electron charge). But we are to remember that the very discovery of vacuum polarization was a quite late achievement (compared to Bohr-Rosenfeld procedures) and, by itself, needed the preliminary idea of physical reality of vacuum and possibility to observe effects of its interaction with charged particles in an experiment. Such ideas were formed due to idealized measurements of quantized fields.

[92] In Kuhn's conception of paradigmatic models of solutions of problems, new non-standard solutions, leading to perspective hypotheses, are described in terms of gestalt-switching (see Kuhn (1962)).

[93] See Karmin and Khaikin (1971, pp.36-39).

[94] See Bransky (1978, pp.40-41, 36-39).

[95] Ibid, p.40.

[96] Reichenbach (1961, p.6-7).

[97] Kuhn (1962).

[98] See Kuhn (1974).

[99] Ibid.

[100] They were discovered and first described in Stepin and Tomilchik (1970), Stepin (1972, 1976).

[101] Heisenberg (1969, S.91-92).

[102] Omelyanovsky (1973, p.99).