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ANCIENT SANSKRIT MATHEMATICS: AN ORAL TRADITION AND A WRITTEN LITERATURE

ABSTRACT

The originality of India's mathematical texts is a consequence of the refined culture of the scholars who produced them. A few examples display clearly some salient features of the habits of exposition and the methods of thought of ancient and medieval Indian mathematicians. The attitude of the traditional learned man, called "pandit", is the same, whether he works on literary or technical matter. Propensity to orality, use of memory, brain work are his specific qualities. Composition in verse form, use of synonymous words, metaphorical expression, which are unexpected processes for the exposition of technical matter, have been the rule in all the vast Sanskrit mathematical literature. The present article analyses a technique of memorization of the text of the Vedas, the earliest exposition of geometry rules in the context of Vedic rites of building brick altars, the numeration system, the arithmetical and geometrical concept of square.

India's mathematical texts are highly original in more than one way: not only do they embody original research methods, they also emerge from a refined culture with which every scholar was deeply imbued. When one examines a Sanskrit scientific text, it is proper not to forget the Indian environment of the author, even if the contents of his work consist of positive knowledge existing also in other civilizations at the same time. The purpose of this article is to draw attention to the intellectual background of ancient and medieval mathematicians. A salient feature is that in ancient times they appear to have been working in an environment of pure orality with original intellectual activities, memorization etc. In the period that followed they worked in an environment where the habits of orality were maintained and the tool of writing was used. We will consider briefly these intellectual habits and give a few examples of mathematical expositions.

Indian mathematicians of ancient and early medieval periods whose works have come down to us can be defined as Sanskrit pandits (*paṇḍita* "learned man").[1] What this means is that they underwent an intensive training in handling Sanskrit language, literary composition and in practicing several intellectual disciplines dealing with language, literature and reasoning. Whatever may be his special field, every pandit possesses a common stock of knowledge in grammar (*vyākaraṇa*), exegesis (*mīmāṃsā*) and logic (*nyāya*). Thanks to that, he is endowed with a sharp linguistic awareness, equipped with a rich store of potential resources of expression, and has at his disposal a number of heuristic tools. In addition the pandit's education aims at improving the natural faculties of speech and memory.

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In the intellectual history of India there has been a period, namely that of the beginnings of Vedic civilization, during which writing was totally ignored, or was not used, even though its existence was known. No written document pertaining to Vedic or Brahmanic civilization has come down to us from before the 3rd century B.C.[2] The Vedic corpus consists of texts spread out over a very long span of time, perhaps a thousand years, prior to that date. No trace of the use of writing appears in the form of that large corpus of texts. No clue is found to suggest that it may have been committed to writing before the Christian era.

This voluminous mass of texts has been transmitted from generation to generation, without being written. One has to admit that there has been oral transmission and a very efficient means of preservation. The whole corpus, divided in several schools, has survived thanks to the memory of professionals trained for that purpose. We know the techniques of recitation, memorization and conservation, from manuals composed in different periods on that subject and by observing the pandits who still practice this art today.

1. TECHNIQUES OF ORAL TRANSMISSION

From the remotest times the recitation of Vedic hymns was a very important activity. The efficacy of any rite was considered to be dependent on the perfect pronunciation of the accompanying prayer or formula. The Brahmans, who were the repositories of the knowledge of the religion and who had to transmit it to later generations, concentrated their attention and efforts on the conservation of the Vedic texts. Developing techniques of accurate recitation was initially a necessity. In later times, even when writing became a common practice, it was not forgotten, nor neglected. It is still practiced in modern times. It is the profession of a few Brahmans who retain its antic form and still refuse to use the help of writing and other tools offered by modern technology. It is a very elaborate art. It includes eleven modes of recitation of the same text. One purpose of this multiplication is the conservation of the text: if a mistake is committed in one recitation, without being repeated in another, comparison between the two recitations helps to detect and to correct it.

One mode of recitation is taken as basic. It presents the text in the form it takes when words are bound together according to the rules of phonetic euphonic combination (*saṃdhi*) of Sanskrit. This recitation is called *saṃhitā-pāṭha* "continuous recitation". It is followed by a recitation where a pause is marked after each word, even after some grammatical components inside the word. This suppresses the euphonic combinations and restores the original form of the words. It is called *pada-pāṭha* "recitation word by word". In the next recitation, the executant groups words by pairs and proceeds word after word: ab, bc, cd . . . It is called *krama-pāṭha* "recitation step by step". The phonetic combinations are carried out inside each pair. As each word appears twice, successively at the end of a pair and at the beginning of the next one, it comes with its two forms, combined and free. This recitation is the sum of the previous two. The fixation of the recitation word by word is ascribed to Gārgya and Śākalya, the recitation pair by pair to Bābhravya and Gālava. These names are known to Pāṇini. We have also other reasons to think that this fixation was anterior to the famous grammarian, who is tentatively placed in the 5th century B.C.

In their turn these three modes of recitation have been taken as the basis for other modes in which the pairs are repeated with an inversion. For example, the *jaṭā-pāṭha* "recitation in the form of meshes", which may be pre-Pāṇinian, takes a pair of words from the *krama-pāṭha*, repeats it in reverse order, then in the original order, and then goes on to the next pair, up to the end of a stanza: ab, ba, ab; bc, cb, bc; . . . The *dhvaja-pāṭha* "flag-recitation?" joins the first and last pair of the stanza, then the second and the one before last, and so on, until the last and first pair are reached respectively: ab, yz; bc, xy; . . . ; yz, ab. The most complicated formula is the *ghana-pāṭha* "dense recitation" which takes a pair, reverses it, takes it again with the addition of a third word, reverses the sequence of the three words, repeats them in their original order, goes to the next pair, and so on up to the end of the stanza: ab ba abc cba abc; bc cb bcd dcb bcd; . . .

Example drawn from *Ṛgveda* 8.100.11: *saṃhitā*

devī́ṃ vā́cam ajanayanta devā́s tā́ṃ viśvárūpāḥ paśávo vadanti |
sā́ no mandréṣam ū́rjaṃ dúhānā dhenúr vā́g asmā́n úpa súṣṭutaítu ‖

(Gods engendered the goddess Speech. Creatures of all forms speak her. May this amiable Speech, a cow giving her milk of force and vitality, well-praised, come to us.)

Pada:
devī́m | vā́cam | ajanayanta | devā́ḥ | tā́m | viśvá-rūpāḥ | paśávaḥ | vadanti |
sā́ | naḥ | mandrā́ | íṣam | ū́rjam | dúhānā | dhenúḥ | vā́k | asmā́n | úpa | sú-stutā | ā́ | etu ‖

Krama:
devī́ṃ vā́cam | vā́cam ajanayanta | ajanayanta devā́ḥ | devās tā́m | tā́ṃ viśvárūpāḥ | viśvárūpāḥ paśávaḥ | viśvárūpā iti viśvá-rūpāḥ | paśávo vadanti | vadantī́ti vadanti | . . . úpa súṣṭutā | súṣṭutaítu | súṣṭutéti sú-stutā | aítu | etv ity etu |

Jaṭā:
devī́ṃ vā́caṃ vā́caṃ devī́ṃ devī́ṃ vā́cam | vā́cam ajanayantājanayanta vā́caṃ vā́cam ajanayanta | ajanayanta devā́ devā́ ajanayantājanayanta devā́ḥ | devā́s tā́ṃ tā́ṃ devā́ devā́s tā́m | tā́ṃ viśvárūpā viśvárūpās tā́ṃ tā́ṃ viśvárūpāḥ | viśvárūpāḥ paśávaḥ paśávo viśvárūpā viśvárūpāḥ paśávaḥ | viśvárūpā iti viśvá-rūpāḥ | paśávo vadanti vadanti paśávaḥ paśávo vadanti | vadantī́ti vadanti | . . . úpa súṣṭutā súṣṭutópópa súṣṭutā | súṣṭutaítv etv ā́ súṣṭutā súṣṭutaítu | súṣṭutéti sú-stutā | aítu | etv ity etu | . . .

. . .

Ghana:
devī́ṃ vā́caṃ vā́caṃ devī́ṃ devī́ṃ vā́cam ajanayantājanayanta vā́caṃ devī́ṃ devī́ṃ vā́cam ajanayanta | vā́cam ajanayantājanayanta vā́caṃ vā́cam ajanayanta devā́ devā́ ajanayanta vā́caṃ vā́cam ajanayanta devā́ḥ | . . .

These techniques have been applied to the sacred text of Veda only. One can infer their efficiency from the fact that the *Ṛgveda*, which is the most ancient Indian text, has been preserved without variant readings. These techniques have ensured the conservation of the form of the text.

Similar efforts, perhaps still greater ones, have been made to preserve the knowledge of the meaning. The teacher who makes a disciple memorize a text, explains the text with the purpose of ensuring the transmission of his own way of understanding it. There is a tradition of commentary as old as the tradition of recitation. The teacher's commentary can be memorized in its form. But we observe that less effort has been exerted for preserving the form in the case of commentaries. There is no manipulation of the form of the commentary, no patterns of recitation comparable to those of the commented basic texts. The impression we receive is that the transmission of the ideas here supersedes the transmission of the form. The emphasis is placed rather upon the acquisition by the disciple of the capacity to comment upon the memorized text and to explain it in the same manner as the teacher.

Thus in the course of development of Vedic culture a technique of commentary was evolved; and it may be as old as the technique of recitation. After the liturgical texts of Vedic religion, the later Vedic corpus consists of a group of commentaries and excursus maintaining a close or loose relationship with the sacred texts. Without being subjected to the same refined techniques of recitation, they have been transmitted through oral teaching and memory for centuries. This character explains some aspects of their form.

In the course of their activities, the intellectuals in charge of conservation of form and meaning of the Vedic texts became more and more aware of the means of their work, started a methodological reflexion and tried to establish the state of knowledge of the subjects dealt with. In India scientific literature started in this line of thought. The main subjects dealt with were Vedic poems, ritualistic formulas, rituals and the calendar. Six disciplines were thus born. Being attached to the use of Vedic texts and ritual performance, they are called *Vedāṅga*-s "Ancillaries of the Veda". Four of them deal with language: grammar (*vyākaraṇa*), etymology (*nirukta*), phonetics (*śikṣā*), metrics (*chandas*). Two others are astronomy (*jyotiṣa*) and ritual (*kalpa*). All are techniques with a practical destination, without any interference of any irrational element, as their sole purpose is the conservation of texts by the knowledge of phonetics, metrics, etc., the conservation of meaning by the knowledge of grammar, lexicon and etymology, the correct performance of rites at the correct time with the aid of knowledge of ritual and astronomy.

The principles and codifications worked out in each of these disciplines have been established and entered in numerous texts, several for a given discipline, because of the multiplicity of Vedic schools. None of these texts is dated with certainty. There is only a general agreement on their relative chronology and on the fact that they form the last layer of Vedic literature (approximately 7th-4th century B.C.). They predate to the use of writing in India and thus pertain entirely to the Vedic oral civilization. Their form is very original: the *sūtra*, the mnemonic form *par excellence*. The masterpiece of this genre is the famous grammar of Pāṇini. The term *sūtra* may be used for the entire work or for each individual formula.

2. THE SŪTRA GENRE

A well-known definition of the *sūtra* is:

अल्पाक्षरमसंदिग्धं सारवद्विश्वतोमुखम् ।
अस्तोभमनवद्यं च सूत्रं सूत्रविदो विदुः ॥

alpākṣaram asaṃdigdhaṃ sāravad viśvatomukham
astobham anavadyaṃ ca sūtraṃ sūtravido viduḥ

(The knowers of the *sūtra* know it as having few phonemes, being devoid of ambiguity, containing the essence, facing everything, being without pause and unobjectionable.)

Concise wording is the first obvious character of the *sūtra*. There are several procedures of abbreviations: use of ellipsis extended beyond the tolerance of natural language;

multiplication of technical names to avoid descriptive expressions; abridged lists through mention of only the first and last items; use of markers; use of variables.[3]

The qualification "facing everything" expresses the fact that the formula which is unique by itself, can be applied to a multiplicity of cases. In the traditional method of teaching a distinction is made between two modes of teaching, each one having a technical name. One is the teaching by express mention, in which each concerned object is mentioned for itself; that is technically called *pratipadokta* "mentioned for each word". There is another mode of teaching in which several objects which have a common character are not individually mentioned, but indirectly referred to by the mere mention of their common feature. This character is called *lakṣaṇa* and the same term is technically used for the general rule based upon it. Technical teaching is mainly done through *lakṣaṇa*-s and the *sūtra* is a *lakṣaṇa* in most cases, *i.e.* one rule "facing" multiple objects.

The expression "being without pause" refers to the fact that often the *sūtra* is not self-sufficient, is linked with one or several others. For instance, several *sūtra*-s are to be used successively to execute an algorithm, to construct a geometrical figure etc. There is also a special feature of composition in the formulary considered as a whole. The *sūtra*-s or individual formulas are organized in groups and they are recited in a fixed sequence. This order of recitation is not without purpose. It does not correspond only to a logical order of the contents. It is also determined by considerations of internal composition of the text, allowing ellipses of words which are tacitly redirected from formula to formula, etc. It corresponds also to an order in the practical use of the formulas. There are precise rules of circulation inside the corpus of formulas to help the user quickly to find in his memory those which are useful for his immediate purpose. The individual character of the *sūtra*, as well as the fact that it is an integrated part of an orderly corpus, is indicated in the usage of pandits who employ the word *sūtra* for the individual formula and for the whole formulary.

One should emphasize the fact that the *sūtra* of the Vedic period is an "oral" text, learnt by heart, recited and mentally used. One who imagines it in a written form does not truly appreciate its character and composition. The most complex text in this *genre* is the *sūtra* of Pāṇini, composed of formulas used to build up words and sentences from the roots, suffixes and diverse elements of Sanskrit language. It is composed in a metalanguage with a very advanced degree of formalization and the arrangement of rules is quite complicated. Using this grammar in the form of a printed book, even equipped with well-prepared *indices*, is a difficult task. On the contrary, it is sufficient to observe traditional pandits who have memorized it and who use it mentally with an extreme facility, to understand that it has been composed in the frame of orality and only for mental use. This can be shown through a simple example. Pāṇini presents the rules of phonetic junction at the frontiers of words (*saṃdhi*) in two parts in two different places in his grammar: the beginning of the sixth section and the end of the eighth and last one. The classification of these rules in two groups and their dispersion have no foundation in the contents of the rules. In each of the two groups the classification does not depend on the subject of the rules. One cannot produce a detailed table of contents of these rules, because their sequence does not correspond to their contents and there is no grammatical subtitle which could be given to a sequence of a subgroup. The reason for this classification and dispersion is that Pāṇini gives a particular significance to the location of a rule in his formulary: his convention is that a rule presented in the last portion of the eighth section of his book is considered as non-realized at the time of application of a rule located before. Some rules of *saṃdhi* can be applied only after a series, sometimes quite long, of other necessary operations.

For instance, the sequence of two words, *dvau* "two" *atra* "here", being given, the following rules must be applied in a fixed sequence. First, application of the substitute *āv* to the diphthong *au* in presence of the initial vowel *a* of the next word, according to the *sūtra* "eco 'yavāyāvaḥ" 6.1.78: *dvāv atra*. Then elision of the final *v* of *āv* in the presence of the initial *a* of *atra*, according to the *sūtra* "lopaḥ śākalyasya" 8.3.19: *dvā atra*. Thus the final *ā* of the first word is now placed just before the initial *a* of the next word. This is the situation where the *sūtra* "akaḥ savarṇe dīrghaḥ" 6.1.101 prescribing a unique substitute *ā* for the hiatus *āa*, can be applied. But that would produce a wrong form: **dvātra*. In fact, this unwanted application does not occur, because the rule of elision of *v* is placed in the final section of the formulary and, consequently is treated as non-realized at the moment of applying the rule of substitution of *ā* to *āa*. The final correct form is: *dvā atra*.

It appears that the order of the rules in Pāṇini's composition depends more on conventions of circulation in the body of the formulary than on the nature of the subject-matter. The formation of a word or a sentence, which is the target of this practical treatise, requires a long series of rules applied in a definite order. And that compels the user to move about constantly in different parts of the book. For a free and easy circulation it is advisable to bear in mind all the formulas and to know their location. The pandit who has been trained in using this grammar since childhood keeps it in his memory and is able to find quickly any rule relevant for his purpose. Seeking a rule from the non-memorized written text can be done only with the help of an index of incipits of formulas in alphabetical order. The table of contents is of little use. The index is useful only when one has already a good knowledge of the incipits and that amounts to learning by heart the whole of the formulary.

There is no doubt that Pāṇini composed his work for memorization and mental use. The conciseness of the formulas has a mnemonic value. There are a few devices of recitation with a mnemonic purpose, commonly used in Sanskrit traditional schools. For example each chapter is divided in groups of twenty *sūtra*-s. At the end of each chapter the last *sūtra* is repeated. By convention the repetition is a marker of the end of a section. Then one recites the incipit of the first *sūtra* of each group of twenty, then the number of groups and the number of *sūtra*-s in the last group when they are less than twenty, then the total number of *sūtra*-s. Finally there is a colophon at the end of each chapter to specify its location in the formulary. All that is considered as part of the text to memorize. It helps in checking whether a *sūtra* has been forgotten and in finding the location of a *sūtra*, its location being eventually relevant to the mechanism of rule applications. But there is no table of contents, no index.

No ancient manuscript of this text has ever been found. There exist recent manuscripts (17th to 20th century), but these are not very numerous. This paucity of the written form is significant, if we consider that it is a text which has been used by pandits in all times, in all their activities. In a 19th century palm-leaf manuscript we have seen, the scribe has actually declared at the end that he had written the text "out of love" for the work. It implies that there was no necessity for him to put it in written form.

The same features—conciseness, emphasis on the essential point, extent of the field of application, links from formula to formula etc.—appear in all ancient Vedāṅga-s or ancillary disciplines of Vedic culture in varying degrees, but Pāṇini's grammar is the text where they are the most conspicuous. In other disciplines there are less constraints. Less sacrifices are made to conciseness, and refined devices of composition are more scarcely used. Still, the basic features are present. The most remarkable is the emphasis on the essential and on the universality of the formula allowing the multiplicity of practical applications. We will, now, give an example in the ritual called *Kalpa-sūtra*, in a section dealing with geometry. It is an interesting passage in which one sees the quest for the efficient formula in the making, as it contains some observations of particular cases leading to general propositions of geometry.

The *Vedāṅga* of rites contains a section on the construction of brick-altars used for the celebration of solemn sacrifices. Vedic religion gives prominence to the performance of oblations of milk or other substances in fire. The offering requires the construction of brick altars called *agnicayana*, literally "piling up (bricks) for fire", or in a shorter form *agni* "fire". The most common altars had a simple geometrical shape, square, trapezoidal or semi-circular. It was made of five layers and to ensure the tightness of the construction, it was a rule that the bricks should not be assembled in the same manner in two consecutive layers. Therefore the even layers were made of bricks with size, shape or arrangement different from those in the odd layers. The priests who built the altars had to face practical problems of geometry, and thus we see the beginnings of Indian abstract geometry in Sanskrit treatises of ritual practice.

3. THE *ŚULBA-SŪTRA*

In several texts of the ritual literature in the *sūtra* form, there is a section called *Śulba-sūtra* "Formulary of the cord" that deals with such problems of geometry. There exist several *Śulba-sūtra*-s, belonging to different schools. One, traditionally ascribed to an author named Baudhāyana, is presumed to be the oldest in the genre. His date is not known, but a most reasonable estimation is that the manual was composed sometime between 700 and 500 B.C.

Let us take one example from this work. The domestic fire-altar has the form of a square and the area of one sq. *vyāyāma* (a unit of measurement equal to 4 *aratni*-s "cubits"). There are 21 bricks in each layer. One method of arranging them consists in drawing the square, dividing one side into three parts and transversely into seven parts. One obtains 21 rectangles. The bricks are made with this shape and size. On the second layer the bricks will be obtained in the same manner but arranged transversely.

This is prescribed by the following formula.[4]

चतुरश्रं सप्तधा विभज्य तिरश्चीं त्रेधा विभजेत् ॥ २.६४ ॥

अपरस्मिन् प्रस्तार उदीचीरुपदधाति ॥ २.६५ ॥

catur-aśraṃ saptadhā vibhajya tiraścíṃ tredhā vibhajet | *II 64*

aparasmin prastāra udīcīr upadadhāti | *II 65*

(II.64. After dividing a quadri-lateral in seven, one should divide the transverse [cord] in three.

II.65. In another layer one places the [bricks] North-pointing).

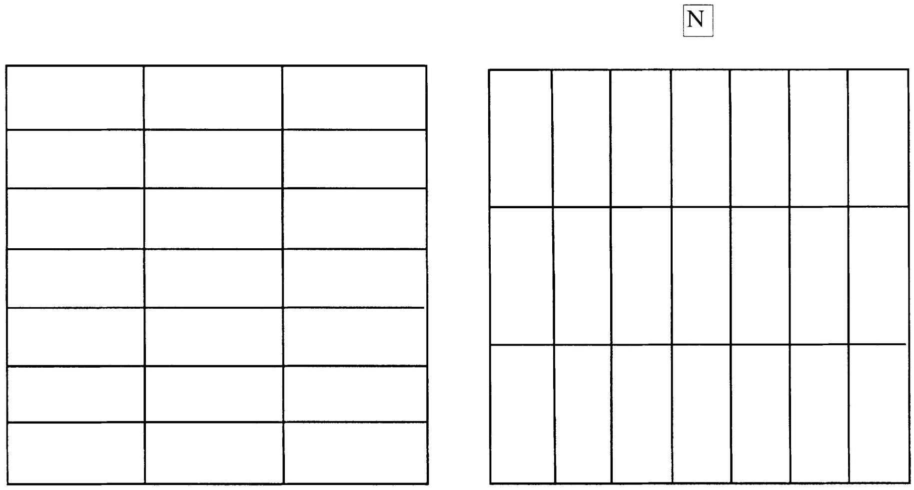


Figure 1. Gārhapatya-citi "altar of the domestic [fire]" II 64–65.

Compared to Pāṇini's *sūtra* this *Śulba-sūtra* is not formalized to the same extent. Still, it is an "oral text" in the same manner. It has been transmitted for many centuries only through memory and has been used on the basis of memory also. It does not contain anything which could indicate that it has been originally written. It is a compendium of formulas composed to be used by performers of the rites and we do not know any other destination for it than the practical one. Its form and composition indicate that the situation in which it is used is the accomplishment of the task of piling up bricks for sacrificial altars. The tools possessed by the officiant in charge of this function were a cord (*rajju*, f.), pegs (*śaṅku*, m.) and clay to make the bricks (*iṣṭakā*, f.). The formulas were in his memory. That was enough for the accomplishment of the rite.

In the example given above, we note several instances of conciseness. The technical vocabulary derives from practical execution. The sides of the oriented rectangle are named by feminine adjectives. The substantive qualified by them is never mentioned and is understood as being the *rajju* "cord". The only hint to this identification is the feminine ending *ī* for the adjective *tiraścī* "transverse" in II.64. In II.65 the isolated feminine plural adjective *udīcīḥ* "North-pointing" is used in the same manner. The understood substantive is *iṣṭakāḥ* "bricks", and this term is indicated only by the feminine plural ending *īḥ*. One understands the cord and the bricks because they are objects designated by feminine words and they are involved in the practical application of the prescription.

It has not been mentioned in the first *sūtra* that the rectangular bricks had their long side placed on an east-west line. The mention of the north direction in the second *sūtra* implies the transverse position for the previous layer, because of the principle that there should be a different disposition of bricks in each layer. This is conciseness, not only in the wording, but also in the meaning. There is ellipsis of an idea, the east-west direction, in II.64, because it can be inferred from the later mention of the north-south direction.

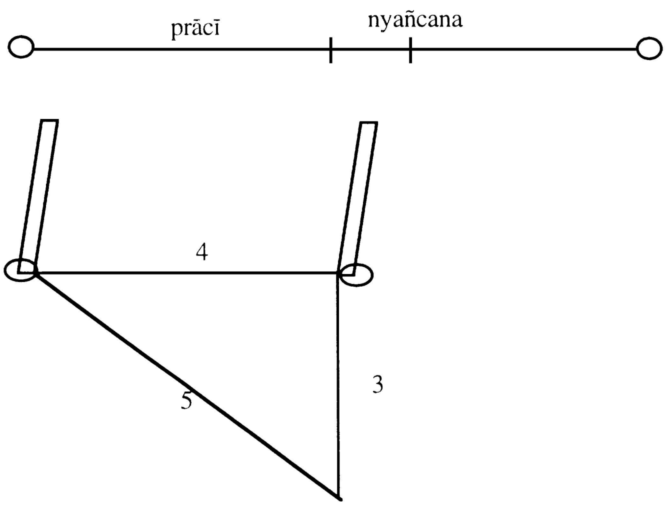


Figure 2. Construction of a square.

Ellipsis is a linguistic device which belongs to natural languages, but to a limited degree. The ellipses found here are clearly beyond the natural limits of usage. This is the result of an effort to formalize the language, and that is explained by the technical destination of the text. The instructions about the "transverse" and the "pointing-to-the-north" can be understood only by the officiant of the sacrifice who has the cord and the bricks in his hands, and whose task it is to arrange the bricks in their respective directions.

The technical vocabulary of the *Śulba-sūtra* is full of imagery. It is always close to practice and derives entirely from the environment in which the officiant works. One does not find here the notions of line or point, but only cords and pegs. Therefore one may raise the question, whether this is a real abstract geometry, different from a collection of pragmatic instructions. Nevertheless, one can recognize here true geometry. In the same text appear formulations of more general geometrical propositions expressed with the same practical vocabulary, as in the following example.

The officiant started his work by molding bricks to a desired shape and size. He was making geometrical constructions with his pegs and cord. In the case of the square, he fixed in the ground two pegs with a distance between them equal to the side of the desired square. He held a cord with a loop at each end and a length equal to two times the distance between both pegs. He put one mark in the middle between the two loops, and another one at a distance from the middle of a quarter of the second half. The second mark had a technical name, *nyañcana*. He attached the loops of the cord on the pegs, then stretched the cord, holding it at the *nyañcana* mark. Thus was obtained a perpendicular to the first side. On that line he marked the length of the side with the help of the middle mark on the cord. By the same process the opposite side was obtained. This procedure is based on the knowledge that the triangle of sides measuring 3, 4, 5 is right-angled.

Baudhāyana's *Śulba-sūtra* prescribes the same procedure of construction for right-angled triangles of sides measuring 5, 12, 13, then 8, 15, 17 etc. Then, after his exposition of such practical procedures for individual cases, he formulates a general rule:

दीर्घचतुरस्रस्याक्ष्णयारज्जुः पार्श्वमानी तिर्यङ्मानी च यत्पृथग्भूते कुरुतस्तदुभयं करोति ॥ १.४८ ॥

dīrgha-catur-asrasya_akṣṇayā-rajjuḥ pārśva-mānī tiryaṅ-mānī ca yat pṛthag-bhūte kurutas tad ubhayaṃ karoti ‖ I.48

(the crosswise-cord of a long-quadri-lateral produces those two which the one-measuring-the-side and the one-measuring-transversely produce).

A cord is said to produce a square, because it is the basic tool to construct a square according to the procedure described above. The long-quadri-lateral is the rectangle. The diagonal is referred to by the word "cord" compounded with an adverb signifying "crosswise". The names of the sides of the rectangle are feminine adjectives qualifying the term "cord", which is understood, and they are compounds of the words *pārśva* "side" and *tiryak* (adverb) "transversely" with the adjective *mānī* "measuring". The word *ubhayam*, literally "both", refers to the two squares which can be constructed on the sides of a right-angled triangle. This proposition amounts to saying that the square constructed on the diagonal of a rectangle is the sum of the squares constructed on the sides. And this is a general proposition, in the sense that there is no more case-wise reference to cords measuring 3, 4, 5 etc., but to cords as sides of a rectangle. The practice-based technical terms are maintained for those cords. But there is also an abstract view of them which leads to the emergence of a general rule from the observed particular cases.

Finally, we wish to emphasize the fact that in this text there are indications only about the gestures and the knowledge of the officiant. The ellipsis, the attribution and choice of technical names are features which belong exclusively to the oral form of language, or to the activity and equipment of the officiant. No writing, no graphical representations are found in it.

4. FROM ORALITY TO WRITING

Vedic civilization gives a remarkable instance of a form of intellectual work operating in the strict limits of orality and still endowed with a striking efficiency. It has been maintained throughout many centuries and some important elements of it, such as recitation and memorization of texts, have been preserved up to the present times. The period around the Christian era witnessed also an evolution of civilization in India in several fields. First, there has been a kind of fixation of the scholarly language. Vedic language had undergone considerable changes and around the Christian era a form of Sanskrit called "classical" became the language of scholars. From that time the grammar has remained practically unchanged until now, even though the vocabulary has been constantly enriched. The name "classical" is often extended to the literature produced in this form of the language. The traditional Sanskrit scholar knows the classical language and often has also memorized one branch of the Vedas. Thus he has the knowledge of both forms of the language and he retains many elements of the old Vedic civilization. It explains the fact that Vedic culture is integrated into the new classical one.

Amongst the innovations of the classical period comes the generalization of the use of writing. With the emergence of writing the already consecrated oral forms, the processes of work and of expression tested for so many centuries of oral and mental practice, were not forgotten. Classical Sanskrit mathematical literature bears both characters. The typical text of mathematics falls into two parts, one composed for memorization with the methods of orality, another one which is the exposition of the same matters or related matters in written form. The duality of the tools, orality and writing, which are at the disposal of the Sanskrit scholar, explains the originality of the texts composed by him.

First, the Indian mathematician owes to writing a new system of numeration. In the beginning, around the Christian era, numbers were expressed by a system of graphic signs which faithfully reproduces the spoken system of numeration. Sanskrit, like all Indo-European languages, has different words for nine units, for ten and several powers of ten. The written notation initially replaced the basic words by as many basic signs, so that it has more than nine signs, without knowing the zero. Where a number is expressed with a sequence of basic words, numeral notation consisted of a parallel sequence of basic signs. The system of writing numbers was, at that time, only a reflection of the linguistic system.

The principle of the place-value and the use of only nine ciphers with a zero are documented from the 3rd century A.D.[5]. This system appears in its final form in usage in Āryabhaṭa's works in the beginning of the 6th century. With the advent of the place-value system with zero the written expression becomes quite different from the oral one. A new tool is born.

With the new written numeration, the Indian mathematicians and astronomers developed the use of another system of metonymic expression of numbers called *bhūta-saṃkhyā* "object-number". It consists in using, in place of a number-noun, the name of an object regularly attached to that number, for instance in saying "void" to mean zero, "moon" for one, "eye" for two, "fire" for three (because there are three fires used in Vedic ritual), "cardinal point" for four or eight if one counts also intermediary directions, or ten by counting also zenith and nadir, "teeth" for thirty-two, "human life duration" for one hundred etc. All synonyms referring to the object can be used, so that there is a great variety of words for one number. This mode of expressing numbers has its roots in Vedic literature. It became more profusely used, when the place-value system entered in common use. And it was adapted to the model of a number written in this system. Generally it is read from right to left, i. e. the reverse of numeral notation. For example:

nanda-adry-ṛtu-śara

(Nanda-mountain-season-arrow) which means: 5679.

Here Nanda, the name of a dynasty of nine kings, means 9, *adri* refers to seven mountains well-known in Indian mythology, *ṛtu* to the six seasons of Indian climate, and *śara* to the five arrows of the god of Love.

This process of expression of numbers appears in the verse portions of texts. The Sanskrit verse, whose form is determined by the number and rhythm of long and short syllables in a line, cannot contain numbers written in cipher: every number must be mentioned in words. It is clear that the mention of high or numerous numbers cannot enter in a metrical sequence, which may be too short or may impose too many constraints, and that the repetition of the same number-nouns thwarts any quest for literary quality. On the contrary the metonymic procedure of expressing numbers offers a rich lexicon of synonyms. Even if Sanskrit allows some freedom in expressing numbers made of many ciphers, even if it offers several types of periphrases, the basic names are limited and have no synonyms. With the mere material offered by the natural language, it would often be impossible to place in a verse the name of a high number. The use of "object-numbers" extends considerably the scope of expression.

It has also a mnemonic value. There is more risk of committing mistakes in memorizing a series of number-names, than in memorizing a figurative series of object-names. If that series is cast in the mould of a verse, it will have a stronger hold in memory. One more advantage of this mode of expression is a better textual conservation. It is clear that a text made of various names arranged in verses, which can be verified by scansion, incurs less risk of corruption than a prose text made of many repetitive number-names, or than one written using ciphers, which is still more exposed to the distraction of copyists[6]. This system has been profusely used, wherever numerical information was intended to be learnt by heart or to be used in practical applications in memorized form. That is the case with the enunciation of algorithms and tables of numerical data. When a text is not destined to memorization, as in the case of an illustrative commentary or of the detailed explanation of an algorithm, the written form is more common and numbers are written in cipher.

Another contribution of writing to Indian mathematics has been the written form of operations. The most original feature of Indian practice was the habit of writing operations on sand. This allows one to strike off an element in the course of the procedure. In fact, we have very scanty documentation on this practice. It is no longer in use nowadays and is known only through short descriptions of procedures in medieval texts. The support was probably a small board on which fine sand was spread. It could also have been just the ground. The term *dhūli-gaṇita* "counting on sand" is used to refer to arithmetics in general. There are also a few mentions of chalk used to write on slate, which is equivalent to writing on sand, as chalk can be easily wiped.

Preservable supports for writing in India have been palm-leaf (*Borassus flabelliformis* or *Coryphœa umbraculifera*), generally engraved with a stylus, birch-bark, used in all periods, and paper, starting from the 11th century, for writing upon with ink. In that case the procedure of the operation is not reproduced completely. No data are wiped, there is no re-writing, and only a few steps are represented. Texts describing operations are often illustrated by examples in commentaries, but they fail to give a detailed account of all that could be done on sand but could not be transcribed on other permanent supports of writing. The most ancient document which has come down to us is the Bakhshālī manuscript, called after the name of the place where it was discovered at the end of 19th century. Its date is a subject of controversy. The best palæographical study so far done has led its author to propose the 12th century[7]. It transmits a collection of problems with answers and thus gives us the most ancient known samples of notation of data for mathematical operations. A remarkable feature is the disposition of numbers in tables with frames[8].

5. THE MODEL OF THE MATHEMATICAL TEXT

The origin of a Sanskrit mathematical text, like any text in any other discipline, is the figure of the traditional scholar called "pandit". We have documents about him, consisting not only of data found in the texts produced by him, but also the living figures whom we can meet with nowadays and whose characteristic features can be, for many of them at least, taken back to former times, because of the perennial quality of intellectual traditions in India. With high probability we can assume the teaching master to have been the most common type among ancient pandits. The typical composition produced for teaching is the *sūtra*, or a composition in the same kind of style, which the master explains orally in his own way. The general rule is that the disciple memorizes the letter of the *sūtra* and remembers the contents, if not the very wording, of the oral explanation. This oral commentary may never have been written but always transmitted orally. Still nowadays, many things told in Sanskrit traditional schools are entrusted only to oral expression and memory. But also the teachings of a master of repute could be couched in writing, by the master himself, by a disciple or by a professional scribe. The last eventuality has probably been the most common, until the recent past. The profession of scribe, active up to the end of 19th century, has disappeared in the 20th century and, now, masters and disciples have to take care of writing themselves, when they accept or wish to write. Even if oral transmission is always appreciated, even if a composition in *sūtra* style and in verse is an aid to memorization, the pandits never refused writing, never neglected the help they could derive from it. And in cases where human memory failed, writing has probably saved a great number of texts.

The standard text of mathematics in the classical period is made of verses in a terse style. It differs from the old *sūtra* only because of its metrical form, which is an additional constraint imposed by the author on himself. Often we are under the impression that it aims at competitiveness in terseness and difficulty. Metrical form and brevity render it all the more easy to memorize. Under this form the mathematical text remains an "oral text", which can be transmitted through memory, used mentally, being well-adapted to such functions. This verse-form, precisely the stanza of four verses in formula-style, has received a technical name, *kārikā*. In some cases the name *sūtra* may be used for the verse-form also.

The difficulty and abstract character of the formulas render explanations necessary. The art of commentary was developed in the same time as the verse-*sūtra*-s were becoming more and more sophisticated. We have mentioned the case of the oral explanation. There also developed a type of commentary composed not only for oral transmission, but chiefly to be couched in writing and so adapted to writing resources. Especially in the field of mathematics the commentator has many occasions to resort to the use of graphic devices, the use of the written place-value numeral system, the graphic arrangement of arithmetical operations, drawing geometrical figures etc. In a general manner we can say that the verses have preserved the style of an oral exposition, and the commentary is an expansion of the memorized knowledge using all the facilities provided by writing.

Its primary purpose is to retell the contents of the *sūtra*-s or verses in expanded form, to state precisely the meaning of each word separately and of the sentence as a whole, to express the implications, to emphasize connotations, linguistic and rhetorical devices, to give illustrations of practical applications and, where necessary, to discuss, criticize, answer to criticism, propose corrections, new formulations etc. The methods of comment are well conceived, have technical designations and are systematically used.

Together with the formulary art, an art of commentary was thus developed, with techniques of interpretation obeying as many rules, constraints and terseness. In this brief article it is not possible to give an idea of this art, because of its technicality and magnitude, otherwise than by giving a sample selected for its simple nature.

6. PLACE-VALUE NUMERAL SYSTEM ACCORDING TO ĀRYABHAṬA

We may draw an example from a masterpiece of Indian scientific literature, the *Āryabhaṭīya* of Āryabhaṭa. Āryabhaṭa, born in 476 A.D., composed two works which bear his name, *Āryabhaṭasiddhānta* and *Āryabhaṭīya*. Only the latter is known to us. It deals with mathematics and astronomy. The first chapter, composed of 13 stanzas, sets forth the basic definitions and astronomical tables. The second, of 33 stanzas, deals with a few subjects of geometry and arithmetic. The third chapter, of 25 stanzas, deals with the various units of time and the determination of the true positions of the Sun, Moon and planets. The fourth one, of 50 stanzas, deals with the motion of the luminaries in the celestial sphere. The first thirteen stanzas are in the metre *gīti*, the other 108 are in the metre *āryā*. They are considered as difficult metres imposing a severe constraint upon the composer. The brevity of expression is carried to its extreme. This is the technique of *sūtra* style. The mnemonic quality of this short verse text for the number of subjects dealt with, is obvious. Like the *sūtra*, it is a text to memorize, and once memorized, to be used for practical purposes, especially for astronomical calculations.

The terseness of expression renders Āryabhaṭa's stanzas particularly difficult. The Indian tradition has itself felt this difficulty and numerous commentaries have been written from medieval to modern times. The earliest of those commentaries, which is available to us, is the *Bhāṣya* of Bhāskara I, composed in 629 A.D. about one century after the composition of the original. The designation of *Bhāṣya* implies that with the explanatory gloss and examples there is an examination of the contents in the form of a debate, with objections and answers.

Āryabhaṭa starts his section of mathematics with a prescription of the place-value system of numeration.

एकं च दश च शतं च सहस्रं त्वयुतनियुते तथा प्रयुतम् ।
कोटयर्बुदं च वृन्दं स्थानात्स्थानं दशगुणं स्यात् ।। २ ।।

ekaṃ ca daśa ca śataṃ ca sahasraṃ tv ayuta-niyute tathā prayutam |
koṭy arbudaṃ ca vr̥ndaṃ sthānāt sthānaṃ daśaguṇaṃ syāt ‖ 2

(One, ten, hundred, thousand, myriad, hundred-thousand, million, ten million, hundred-million, milliard: a place should be ten times a place.) (*Gaṇita* 2)

Sthāna, literally "place", is the technical name of the value signified by a place. Each value has a denomination. Sanskrit has a single and common name for each value up to 10^9, and others, of more limited usage, up to yet larger numbers.

The commentator explains the purpose of using the place value system of numeration. It is the economy of signs in the writing of numbers and the resolution of operations. He states the value of the listed number nouns. He completes the abbreviated prescriptive sentence: "a place is ten times the place one has already created", when one writes a number.

Then he shows the validity of the formula, by questioning and answering possible objections. One could say that the clause "a place should be ten times a place" is applicable to other values than those listed in the stanza. Therefore this short list appears to be useless. The formula has the capacity to make all these values known. It was not necessary to repeat them. One could answer that in the stanza there are two parts: the proposition "a place should be ten times a place" is the rule (*lakṣaṇa*); the list of values is the example. At this juncture Bhāskara I calls the author Āryabhaṭa "author of *sūtra*-s" and says that it is not the habit of an author of a *sūtra* to formulate examples with the rules. This shows that he considers the verses of Āryabhaṭa as having the status of *sūtra*-s. Then he says that if examples have no role to play in the stanzas, we must understand the list of values as being a presentation of these numerals as technical names of the prescribed places, not as mere examples of values. These numerals, in fact, do not have here their usual meaning of numbers. They are here technical names of the different place-values created by the rule. The Sanskrit words which mean literally "one, "ten", etc. are the names of the first, the second place, etc. in the writing of numbers. They stand in Sanskrit for the designations "units", "tens", etc. Āryabhaṭa's stanza contains a rule to determine the value of a place and to give it a technical name. Bhāskara adds that the rule allows the determination of many more place values than the listed ten, but that only ten names were given, because above that there is no use of particular names. High numbers can be written and understood with this method. Names are not necessary and not in common use.

Finally, after some remarks on the subject, the commentator indicates how to put down the places in writing. This is what is called *nyāsa*, literally "deposit". The *nyāsa* of the places is a row of ten zero-signs written from right to left:

0000000000

The text of Bhāskara's commentary is known to us through five manuscripts, all recent (18th–19th cent.), in South-Indian scripts and probably derived from one and the same source. If we may accept that the manuscript tradition of that text since the 7th century has always faithfully reproduced the original disposition used by the author, this is a document from which one can reconstruct the practice of writing numbers on a plank covered with sand or on a slate. When Bhāskara I represents the "places" by a line of small circles, he prepares the writing of a number: one writes the places with circles, then one writes ciphers, each in its place, by wiping off the circle and substituting the cipher. In the place where no cipher has been entered, the circle remains, and that represents the fact that this place remains empty.

7. THE MATHEMATICAL PROBLEM

When a stanza deals with a longer algorithm involving several steps, the commentator follows a regular model of detailed exposition of the procedure with examples. He starts with an enunciation of the problem, technically called *uddeśaka*. Very often the *uddeśaka* is a stanza. That indicates it was also an item to be learnt by heart, so that the user has in mind models to imitate in the course of his work. This is followed by a *nyāsa*, i.e. writing down the data of the problem. Then comes the execution of the prescribed procedure; that is called *karaṇa*, literally "execution". The result called *labdha* "obtained" is finally stated.

We give as example the procedure of squaring a fraction, given by Bhāskara I in his commentary on the half-stanza composed by Āryabhaṭa to prescribe the procedure of this operation called *varga*:

वर्गः समचतुरश्रः फलं च सदृशद्वयस्य संवर्गः ।

vargaḥ sama-catur-aśraḥ phalaṃ ca sadr̥śa-dvayasya saṃvargaḥ (*Gaṇita* 3a)
(A *varga* is of-four-equal-sides; and the area is the multiplication of two equals.)

In common Sanskrit *varga* refers to a group of objects of the same class. Bhāskara I declares that *saṃvarga* is one of many synonyms for multiplication in general. He takes *varga* in a more restricted sense, the multiplication of two equal numbers. He takes that as the primary meaning of the word and interprets the above text as a prescription of an additional technical meaning, namely the meaning of a square. *Phala,* literally "fruit", is the technical name of the area of a figure. Āryabhaṭa's stanza means that one calls *varga* the figure with four equal sides, whose area is the multiplication of two equal numbers, i.e. the measures of two equal sides. One may calculate the area of a square by this operation.

Bhāskara I illustrates that by the following problem:

उद्देशकः
षण्णां सचतुर्थानां रूपस्य च पञ्चभागसहितस्य ।
रूपद्वितयस्य च मे ब्रूहि कृतिं नवमहीनस्य ।।

Uddeśakaḥ:
ṣaṇṇāṃ sa-caturthānāṃ rūpasya ca pañca-bhāga-sahitasya
rūpa-dvitayasya ca me brūhi kr̥tiṃ navama-hínasya

(Enunciation [of the problem].

Tell me the square of six with a fourth, of the form with a fifth part, and of a pair of forms minus a ninth.)

The term "form" is a metonymic designation of number "one". The expression "pair of forms" is only a designation of number "two". First, Bhāskara I shows how to write the data of the problem. They are rational numbers which, in Sanskrit, are always expressed by an integer followed by a fraction. When the integer is written on a line, the fraction is placed below it and is itself written on two lines, the numerator called *aṃśa* "part" on the first line, the denominator called *cheda* "divisor" on the second below. If the fraction is written without any particular additional sign, one understands that it is added to the integer above it. If it is marked by a small circle or a cross (the shape of the "plus" sign in the West) placed on its right, one understands that it is subtracted from the integer. Thus Bhāskara I writes:

"न्यासः	६	१	२
	१	१	१ o
	४	५	९
Nyāsaḥ	6	1	2
	1	1	1 o
	4	5	9

(Presentation [of the data] :

6	1	2	
1	1	1 o	
4	5	9	[that is to say 6+1/4; 1+1/5; 2–1/9])

This is followed by the exposition of how the operation is carried out:

करणम् – छेदगुणं सांशमिति २५
४

एतयोः छेदांशयो राश्योः पृथक् पृथग्वर्गराशी १६ ६२५ छेदराशिवर्गेणांशराशिवर्गं हृत्वा
लब्धं ३९
१
१६

एवं शेषयोरपि यथासंख्येन	१	३
	११	४६
	२५	८१

karaṇam — cheda-guṇaṃ sa-aṃśam iti 25
4

etayoḥ cheda-aṃśayo rāśyoḥ pr̥thak pr̥thag varga-rāśī 16 625 *cheda-rāśi-vargeṇa_aṃśa-rāśi-vargaṃ hr̥tvā labdhaṃ* 39
1
16

evaṃ śeṣayor api yathā-saṃkhyena	1	3
	11	46
	25	81

(Realization [of the operation]: [The integer] multiplied by the divisor, with the part, is [$(6 \times 4) + 1$]:

25
4

The squares of these two numbers, divisor and part, are 16 and 625 respectively. After dividing the square of the part by the square of the divisor, one obtains:

39
1
16

In the same way for the other given numbers one obtains:

1	3
11	46
25	81)

In the case of the last operation, one multiplies the integer by the divisor and subtracts the part: $(2 \times 9) - 1 = 17$. The fraction $17/9$ is squared.

At least four commentaries of Āryabhaṭa's work are known. There are many differences of form and interpretation between them. The study of the differences of interpretation goes beyond the limits of this article. It involves many techniques of linguistic and logic analysis of Sanskrit commentators and gives rise to complex problems. Here we will content ourselves with the simple presentation of another example of squaring, given to illustrate the above-quoted formula of Āryabhaṭa by another of his commentators, Sūryadeva Yajvan, born in 1191 in Tamilnadu.

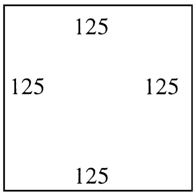
उद्देशकः
बाणार्कसंमिता यस्य चतुरश्रस्य बाहवः ।
त्र्यंशद्वयमिता यस्य तयोः फलमिहोच्यताम् ॥
प्रथमोदाहरणस्य न्यासः

परिलेखः

अस्य सदृशद्वयस्य संवर्गफलम् १५६२५
द्वितीयोदाहरणस्य न्यासः–अत्रांशवर्गः ४ छेदवर्गः ९ अनेनांशवर्गे विभक्ते
लब्धं क्षेत्रफलम् ४
९

uddeśakaḥ—
bāṇa-arka-saṃmitā yasya catur-aśrasya bāhavaḥ |
try-aṃśa-dvaya-mitā yasya tayoḥ phalam iha_ucyatām ‖
prathama-udāharaṇasya nyāsaḥ—

parilekhaḥ



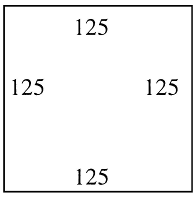
asya sadr̥śa-dvayasya saṃvarga-phalam 15 625
dvitīya-udāharaṇasya nyāsaḥ — atra_aṃśa-vargaḥ 4, cheda-vargaḥ 9, anena_aṃśa-varge vibhakte labdhaṃ kṣetra-phalam 4
9

(Enunciation [of the problem].

The fruit of the square whose arms are measured by Suns and arrows[9] and of the [square whose arms] are measured by a pair of thirds, should be told here.

Presentation [of the data] of the first example:

figure



For that [square] the fruit which is the multiplication of two equals, is 15 625.

Presentation [of the data] of the second example: [. . .] In this case, the square of the part is 4, the square of the divisor is 9; when the square of the part has been divided by that [square of the divisor], the area of the square 4/9 is obtained.)

The difference between what we have here and the example of Bhāskara I is the emphasis given to the representation of the geometrical square. Bhāskara I has only written the number to be squared. Sūryadeva has given the figure of the square and written the number as the measure of the side. This detail of presentation is not without significance. In the presentation of Bhāskara I only the arithmetic operation is illustrated. In Sūryadeva's exposition the assimilation to geometrical representation is brought to light. In fact both commentators have underlined the fact that Āryabhaṭa identifies the squaring of a number and the finding of the area of a square. In their mind the word *varga* is primarily the technical name of the arithmetical square and secondarily that of the geometrical square. They consider that, when Āryabhaṭa prescribes it, he does a superimposition of the latter on the former. Superimposition (*upacāra*) is a mode of linguistic expression recognized among Indian grammarians and poeticians. The relation between arithmetics and geometry is translated by them in the form of a linguistic fact. In the sophisticated and compelling poetics of Sanskrit pandits, the rule is that any figurative mode of expression should be justified by an intention of the user. Therefore both commentators specify the intended purpose behind the use of such a device. On that point they have the following difference.

For Bhāskara I the purpose of the superimposition is to exclude the rhombus from the present definition of the *varga*. The term *catur-aśrasya* refers to a figure "of four equal sides". That is ambiguous, being common for the square and the rhombus. The identification with the arithmetical square excludes the rhombus, because the latter has equal sides, but its area is not the multiplication of two equal sides. Thus, Bhāskara I interprets that Āryabhaṭa intended to give a technical name to the geometrical square and selected the name of the multiplication of equal numbers, because that created an identity, excluding the case of the rhombus.

Sūryadeva declares that the purpose of Āryabhaṭa was only to show that the multiplication of equal numbers is the essence not only of the area of the square, but also of the arithmetical square. The superimposition done in the form of a linguistic device of expression is justified by the fact it gives the information that a common property is dealt with here, namely the identity of the operation to be carried out in order to calculate the area of the square and the squaring of a number. And that is a fact of reality. But Bhāskara I has clearly investigated something more and argued for the introduction of an additional piece of information, the exclusion of the rhombus, in Āryabhaṭa's proposition, by exploiting the use of the linguistic device of superimposition. A consequence of this difference in the estimation of the intent of Āryabhaṭa's definition, is that Sūryadeva, who had more interest in assimilating the arithmetical operation and the geometrical conception, has emphasized the latter in his example by drawing the figure of the square and writing down the numbers at the same time.

This pattern of exposition is abundantly represented in Sanskrit mathematical literature. For certain famous texts commentaries have been repeatedly composed in the course of history. They differ in the choice of examples and eventual excursus. There are cases of self-commentary. There is also a simplified form of mathematical text where the commentary offered by the author of the formulas is reduced to an illustration by a few examples of problems. The *karaṇa* portion is even suppressed in the presentation of the example, because it can be understood easily from the general rule contained in the basic formula. Sometimes the definition itself is called *karaṇa-sūtra* "formula of procedure". It is followed only by the *uddeśaka* "enunciation of the problem" with its *nyāsa* "presentation of the data" and the result (*labdha* "obtained"). In that case the rule of procedure and the *uddeśaka* are in verse, the *nyāsa* is in prose. This is the pattern followed by the *Līlāvatī* of Bhāskara II (12th century A.D.), a collection of arithmetic and geometry which achieved great fame from its high literary style.

Thus Sanskrit mathematics owe their originality not only to the structure of the Sanskrit language, but also to the habits of exposition of Indian pandits. We have highlighted their propensity to orality, their trust in using memory and in mental activity. One could add also their literary inclinations.[10] Together with the metrical form, ubiquitous in all scientific and technical disciplines in India, devices of metaphorical expression, whose natural home is poetry, find their way in their compositions. We have mentioned a metonymic foundation in the terminology. One could also draw attention to the diversity of expression. If, in the West, one notes a tendency to stabilize terminology, definitions, formulations of theorems etc., in the Indian context one has to reckon with a propensity to the perpetual renewal of modes of expression. The Sanskrit mathematical text is a literary text. It imitates the form and spirit of the poetic text. It always tries to renew, even to surpass itself. Sanskrit poetry is definitely situated in the structures of orality. It is a poetry of sound emitted and heard, close to music, an inner object of meditation. The ideal mathematical text aims at being that same kind of mental object which speech transmits.

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NOTES

1 The bulk of mathematical literature produced in India is in Sanskrit. It has not been exclusively in this language. Some expositions are found in other languages also, Ardhamāgadhī, Prakrit, Dravidian languages. But behind them there are Sanskrit parallels or sources. A Jaina pandit of ancient times may have had more reverence for the language of the founder of his religion, but has followed the model of the Sanskrit pandit. And in the Middle Ages Jainas have often used Sanskrit directly, especially in non-religious subjects.

2 It is a fact that the Indus civilization (3rd and 2nd millennia B.C.) knew writing and that there may have been a contact between its last period and the beginning of Vedic civilization, which occurred in the second part of the 2nd millennium. But no speculation can be useful on this point as long as the Indus script remains undeciphered. It has so far resisted every attempt at decipherment. We do not even know which language it transcribes. No clue has been found allowing us to suspect that it could be the Vedic language.

3 See Pierre-Sylvain Filliozat 1993a, 1993b.

4 We give the most literal translation. Technical names are rendered by their etymology, and, when several English words are used to translate a single Sanskrit word, they are joined with hyphens. Words in square brackets and figures are added by us. We know only very recent manuscripts of this work. When a figure is found in a manuscript, it is not a part of the ancient text, it belongs to a recent commentary. The ancient text was not written and we do not know any ancient graphic representation.

5 In a very acceptable restoration of the text of a verse from *Yavanajātaka* 79.6d; see text vol. I p. 494 and commentary vol. II p. 406, in Pingree's edition, 1978.

[6] See Billard, R. 1971: 21–22.
[7] Kaye, G. R. 1927, Hayashi, Takao 1995.
[8] See P.-S. Filliozat & G. Mazars 1987.
[9] This is an example of metonymic expressions, namely the mention of something connected to a number in order to express that number, described above. In Indian mythology there are twelve Suns and the Love-god has five arrows (*bāṇa*) in the form of flowers. The compound *bāṇa-arka* "Suns-arrows" is equivalent to 5–12 and is read from right to left: 125. "Fruit" is the technical name for the area of a figure. The problem is: find the area of a square with a side of 125, and of another with a side of $2/3$.
[10] See Pierre-Sylvain Filliozat 1997.

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Part IV

READING TEXTS