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WHAT IS THE CONTENT OF THIS BOOK? A PLEA FOR DEVELOPING HISTORY OF SCIENCE AND HISTORY OF TEXT CONJOINTLY

ABSTRACT

Based on two examples (one taken from thirteenth-century China and the other from eighteenth-century Europe), this paper discusses why various forms of collaboration between history of science and history of text might prove profitable. Texts are not a historical, transparent forms conveying meanings whose history we would write. Scientific texts as such appear to have taken various forms within space and time, designed as they were through an interaction with local conditions of text production of all kinds. Elaborating a description of these various forms of texts would provide methodological tools to read them, since they can by no means be read without the mediation of a method. Here, the achievements of a history of text would benefit history of science in that it would provide a better grasp of the textual contexts for the production of scientific writing, and it would give a better awareness of the various ways in which texts were meant to mean. On the other hand, the history of scientific text could become a systematic concern in history of science as such: scientists design their texts at the same time as they design concepts and results. This represents a constitutive part of their activity, and the study of the production of texts would bring materials to understand how scientists benefit from the cultural and textual contexts within which they work. It would give us a grasp of how they construct the symbolic tools with which they perform their activities and communicate their results, which in the end are texts. In all these respects, a history of scientific text could then become a specific domain of the history of text.

Why suggest approaching history of science from the perspective of a history of text? In the course of my research, this approach appeared to me a necessity as I found myself repeatedly confronted with the question of how to read the sources I was dealing with. I believe that we would all agree on how not to read them: the limitations of a reading which amounts to a mere reformulation of ancient sources in modern scientific terms have already been denounced frequently enough for me to assume so. Yet, although we all agree on the fact that one should not read in that way, although we are aware that such a reading destroys the networks of concepts in the original, substituting other concepts and later concepts for those expressed in the text, I am of the opinion that we still lack a comprehensive account of what actually happens when one reads that way—what operations our reading performs on the original text, what the reading still tells us about the text.[2] That we should not merely reformulate ancient texts using modern concepts seems to me one of those tenets that everybody grants to be true, even though we do not know precisely how and why they are true.

In this paper, I would like to consider what benefits we might derive from analyzing in a critical way an assumption underlying this mode of reading—an assumption that it indeed shares with other modes of reading: when reading scientific texts in this way,

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one assumes that one can apply our own current modes of reading scientific texts to older sources as if texts as such, except for the emergence of modern symbolism, had no history, as if they were invariant in time and space, as if they had always required the same operations from their readers, as if the same elements always meant the same things. Such an assumption now seems to me highly questionable, regarding not only texts in general, as a history of text might teach us, but also scientific texts in particular. I shall start by presenting some of the evidence which led me to question this premise. This will take us to China, in the year 1248, when Li Ye, who had given up the civil service and was living as a hermit, finished his mathematical masterpiece: *Sea-mirror of the circle measurements*.[3]

1. HOW SHALL WE READ *SEA-MIRROR OF THE CIRCLE MEASUREMENTS*?

1.1 *The posing of the problem*

This seems to be the most ordinary book that one might dream of: after the usual preface and table of contents, it opens with a drawing (see figure 1), ordinary as it seems, on which it is entirely based. Right afterwards, the author proceeds to assign names to some of the segments of the drawing, and then goes on to provide numerical values for all the segments taken into consideration and for all the quantities that he will make with them, within the framework of particular dimensions for the drawing. After these preliminaries come the two main parts of the book: first, about 700 formulas are listed stating relationships between the segments of the drawing —one might not have expected that, for such an ordinary drawing, so many relationships could be found; however, here they are, gathered in a compendium which constitutes the last and biggest subchapter of the first chapter of the book. Second, a total of 170 problems concerning the drawing are presented in the next eleven chapters. This last part is famous in the history of mathematics, since we have here the first systematic occurrence, for the solution of most problems, of polynomial computations in order to establish an equation, "the" root of which is the unknown sought for.[4]

As we have described them so far, the elements composing the book appear to be typical: a drawing, names for some segments, numerical values, formulas, and problems. But one has to figure out that *this is the book*: there are almost no remarks by the author commenting on what he is doing. The main second-order statements concerning the content of the book are the titles of the chapters, or, in the case of the first chapter, the titles of its subchapters or their sections: "Map of the circular town" to introduce the drawing, "Set of the names of the *lü*"[5] to introduce the names of the segments considered, and "Correct quantities for the problems to come" to introduce the numerical values of the segments and of the quantities based on them which Li Ye considers.

Then the text reads as follows:[6]

"Set of the names of the *lü*

From C to T, that makes the hypotenuse of [triangle] (13)
From C to Q, that makes the height of [triangle] (13)
From Q to T, that makes the base of [triangle] (13)."

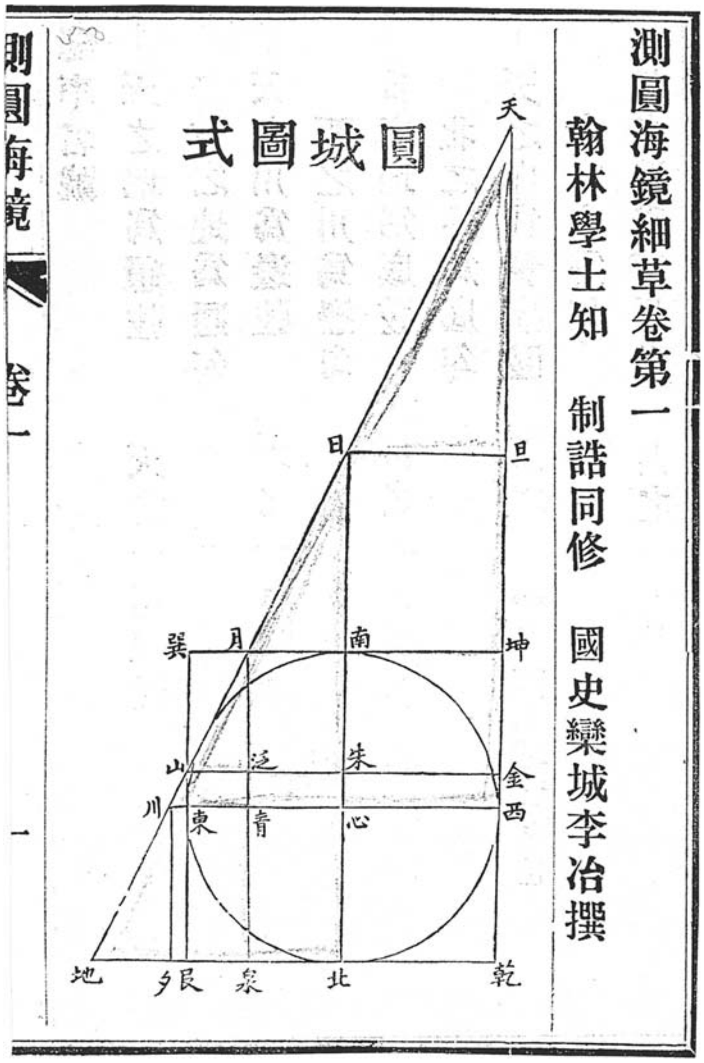


Figure 1. The drawing on which the *Ceyuan haijing* is based, reproduced according to the edition of the Tongwenguan. It is identical to the one contained in the manuscript which is kept in Beijing library (see its attribution in [Mei Rongzhao 1966], p. 111).

In the following, segments are introduced in the same way: each paragraph is devoted to three of them, which form the base, height, and hypotenuse of a triangle. Hence the names refer to a reading of the drawing in terms of triangles: the segments considered are designated according to which function they play in which triangle. As a consequence, the terminology has a tabular structure, the first part of a term in Chinese referring to a triangle, the second to a function.[7]

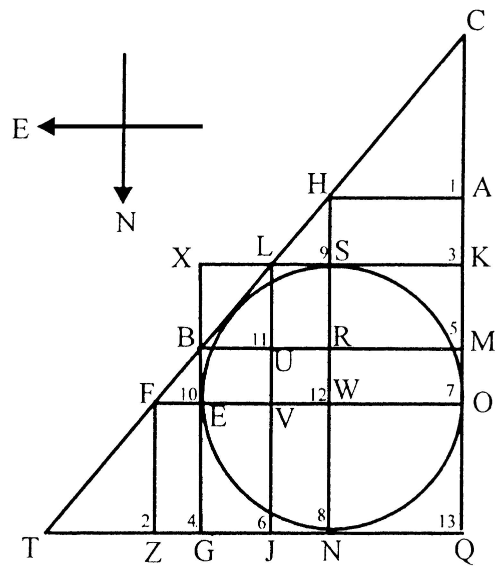


Figure 2. The letters for points and numbers for triangles used in this paper.

The following subchapter has a comparable structure, and contains only one second-order statement:
"Correct quantities for the problems to come

[Triangle] (13)	c	680	a	320	b	600
	$a+b$	920	$b-a$	280		
	$a+c$	1000	$c-a$	360		
	$b+c$	1280	$c-b$	80		
	$c+b-a$	960	$c+a-b$	400		
	$a+b+c$	1600	$(a+b)-c$	240."		

This is the sober way in which the numerical values used in most of the book are introduced, and it is made clear that thirteen quantities based on its three dimensions are attached to any given triangle.[8]

The simplicity of its style is extreme, and the content of the book seems to have always been clear to its modern commentators: about 700 formulas are displayed so as to function as an appropriate repository of geometrical knowledge needed for the second part of the book—the problems which make use of the relationships between segments to perform polynomial computations. A modern book would organize the presentation of knowledge in a similar way: it would first give a compendium and then rely on it to solve the problems.

Note how much this reading in terms of our own textual categories enhances the practical value of the compendium. Such a qualification (i.e.: to be practically-oriented) has repeatedly been listed among the "characteristic features" that are sometimes attributed to the so-called "Chinese mathematician." And the usual ways of dealing with the set of numerical values given at the beginning of the book come as a confirmation of the fact that the "Chinese mathematician" is practically oriented. Among the various attitudes towards these numbers adopted by modern commentators, I identify three classes. Some readers choose to forget them—as if they were secondary, or even not present, in the book. Others lament over them: Why did Li Ye give actual values, when he knew that all this was valid with full generality? Still others stress them as, again, an incontrovertible proof of the practical character of the "Chinese mathematician," who would be able to think about things only *in concreto*. Whatever the differences between these attitudes, they all agree in reading the numbers as if they meant the same thing as numbers we would find in a contemporary textbook. This reading turns them into concrete, particular values, and this interpretation is the basis for the three attitudes.

But are we certain that we can read these numbers in that way? Are we certain that we can read the compendium and the problems as we would read their modern counterparts? These are the issues that I would like to address. Let us start with the numbers.

1.2 *The numerical values*

Li Ye's drawing is constituted in such a way that all its triangles are similar to each other. With the numerical values which Li Ye attached to them at the beginning of his book, they appear to be dilatations of the right-angled triangle whose smaller side, a, measures 8, whose height, b, measures 15, and whose hypotenuse, c, is 17. For example, the largest triangle of the figure, (13), whose dimensions we listed above, can be obtained as the result of a dilatation with a factor 40 on the basis of this small triangle with sides 8, 15, and 17.

Now, the dimensions 8, 15, and 17 immediately raise an echo. The resonance comes from a mathematical classic of the Chinese tradition *The nine chapters on mathematical procedures*, a text compiled around the beginning of the common era and which became an official book, or even "the" classic, in mathematics —hence, the first book which would be known to any scholar learned in this topic in China until the Song and Yuan dynasties at least.[9] The ninth chapter of this book, which Li Ye knew,[10] is devoted to the right-angled triangle. The only problem in this chapter that involves a triangle with the dimensions (8, 15, 17) is precisely —is this surprising?— the problem devoted to finding the diameter of a circle inscribed in a right-angled triangle. *The nine chapters* provides a procedure for finding the diameter, and in Liu Hui's commentary the reader can find —Li Ye could find— proofs establishing its correctness. One of these proofs happens to have an intimate connection with Li Ye's drawing. Let us sketch it (see figure 3 and the text in [Qian Baocong 1963], p. 252). Liu Hui draws a parallel to the hypotenuse of the triangle going through the center of the circle and makes the square whose sides are equal to the radius of the circle appear in the right angle of the triangle.[11] Then he relies on two related properties concerning the triangles which, as a consequence, appear within the original

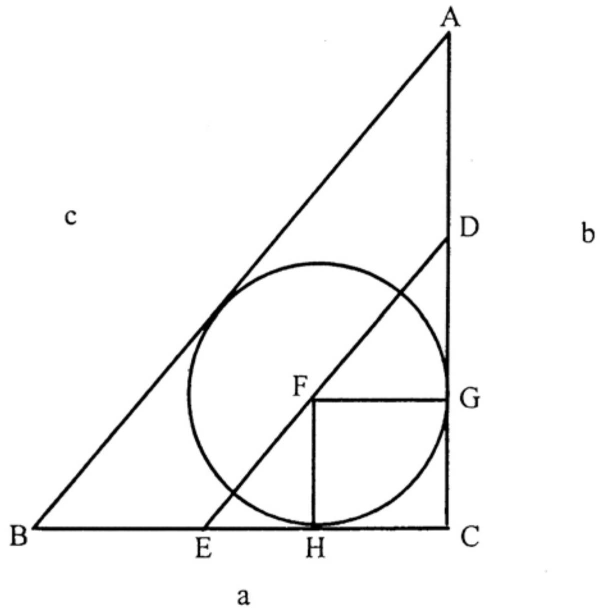


Figure 3. The drawing to which Liu Hui alludes in his commentary on the procedure to find the diameter of the circle inscribed in a right-angled triangle.

triangle: the sum of the lengths of the three sides of triangle DFG is b, whereas the sum of the lengths of the three sides of triangle EFH is a. Therefore, one can compute the coefficients expressing the similarity between the original triangle, on the one hand, and, respectively, triangles DFG and EFH, on the other hand, with respect to the dimensions of the original triangle, which gives, respectively:

$$\frac{b}{a+b+c} \quad \textit{and} \quad \frac{a}{a+b+c}$$

Now, taking into account the fact that the sought-for radius is, respectively, the base of triangle DFG and the height of triangle EFH, one obtains its value by using the previous coefficients of similarity:

$$\frac{b}{a+b+c} \cdot a \quad \textit{or} \quad \frac{a}{a+b+c} \cdot b$$

If, following this incitation, we look back into Li Ye's drawing and consider the sum of the three sides of each of the triangles introduced, as we saw above, by the terminology, we discover a striking fact: the sum of the three sides of each triangle is one of the 13 quantities attached to the largest triangle of the drawing (13). The following table displays the relationship between the triangles and quantities thereby established:

1	b
2	a
3	$b+c-a$
4	$a+c-b$
5	$2b$
6	$2a$
7	$c+b$
8	$a+c$
9	$c-a$
10	$c-b$
11	$a+b-c$
12	c
13	$c+b+a$

Hence, each triangle appearing on Li Ye's drawing has a property similar to the one Liu Hui brought into play in his proof. And we have an almost complete system of such triangles: but for two exceptions, the thirteen triangles prove to be connected precisely with the thirteen quantities attached to a right-angled triangle.[12] A correlation between the triangles and the quantities considered appears: the drawing might have been the fundamental figure attached to the right-angled triangle. Li Ye's reading of the drawing in terms of triangles echoes its relevance for linking triangles and quantities. This stresses the singular nature of the figure, when compared to the drawings considered in Greek geometrical texts such as Euclid's *Elements of geometry*.[13] Drawings seem to have been worked out in different ways in distinct traditions, which can also cause problems in their reading, whether one projects a modern conception of a mathematical drawing onto an ancient one, or a conception elaborated in one tradition onto another. Drawings raise problems of interpretation too.

As a consequence of the property exhibited, each numerical value given by Li Ye, being an integer, can be decomposed into two multiplicative factors: one indicates the triangle on which it is computed, the other the kind of quantity it represents for this triangle.[14] The values Li Ye provides constitute the smallest possible set based on the triangle (8, 15, 17) for which all quantities computed are integers. In addition, they have the property of being the unique set of dimensions for which the multiplicative factors referring each value to a triangle and a quantity correspond exactly to the values computed with respect to (8, 15, 17). For example, the values for the largest triangle, (13), which we gave above, 680, 320, and 600, can be decomposed as 17.(8 + 15 + 17), 8.(8 + 15 + 17), and 15.(8 + 15 + 17), where (8 + 15 + 17) constitutes precisely the one among the thirteen quantities that corresponds to the largest triangle.[15] Note that, on the one hand, numbers have a structure parallel to that of the terminology, and, on the other hand, both manifest the mathematical structure of the situation.[16]

Let us recapitulate the correlations we have obtained. We have correlations of three types between the situation within which Li Ye operates and the problem from *The nine chapters* that it evokes:

— the basic numerical values (8, 15, 17).
— the situation of the circle inscribed in a triangle.

— the property of having triangles inserted in the largest triangle, where the sum of the three sides of the smaller triangles is expressible as a function of the dimensions of the largest triangle.

This accumulation seems to me to rule out the possibility of a mere coincidence.

And, therefore, if we recapitulate in terms of readings, we get the following hypothesis:

— The numbers given by Li Ye at the beginning of his book might not be related in the least to "concrete numbers" or to "particular values," as a projection of contemporary readings would suggest.

— On the contrary, they might function as a quotation from *The nine chapters*, thus placing Li Ye's book in a tradition of research that stems from the classic. They might also indirectly express a property of the drawing, which turns out to be fundamental for the compendium.[17] This property would be stated as such nowhere in the book, but rather disseminated in the compendium.

With this very simple first example, we have arrived in my view at the heart of the problem. Quotations of classics and, thereby, indirect expressions are frequent in ancient Chinese texts.[18] We would just have here a modality of their functioning within mathematics. An echo being raised, its meaning gets unfolded by the reader. When Li Ye writes, he might be expecting such operations from his readers, as they can be expected from any reader in general in this context.

Therefore and more generally, in order to be read by us, mathematical texts need to be conceived as embedded in textual cultures, since they are written in ways showing that they require specific operations of reading —even though these operations might take particular forms in a mathematical context.[19] Only a confrontation between mathematical sources on the one hand, and between them and other kinds of texts on the other hand, can secure the modes of reading to be used. So far, they are just hypothetical. This is a point where a history of text, a history of reading, would be of importance for history of science. But, conversely, scientific sources could in this respect contribute to elaborating the description of the ways in which texts made sense, and in this way prove to be precious material for a history of text.

What is at stake here is clear. If we proceed by projecting our modes of reading onto ancient documents, as if texts had been used everywhere and constantly in the same way, we face the danger of creating, on the basis of our own artifacts, a "practical Chinese mathematician." In response to this danger, our method suggests a remedy.

Note that such a reading lets us see that , in what could appear to be as a tedious table, there is a structure the meaning of which has to be unfolded, though we would probably choose to express this meaning discursively. The same phenomenon repeats itself with the compendium, to which we shall now turn briefly.[20]

1.3 *The compendium*

How is the set of 700 formulas that follows the table of numerical values expected to be read? What were the motivations for laying down these formulas? What mathematical information is there made available? Before I deal with these questions, a much more elementary issue needs to be addressed: Why should such questions be raised? The simple explanation, which would see the compendium merely as the repository of geometrical knowledge designed to serve as a foundation for the second main part of the book, and the simple reading of this piece of text as a directory of formulas, have some validity. Indeed, to solve the problems given thereafter, one needs to use relationships between the various segments of the drawing such as those given in the compendium, and the solutions of the problems regularly refer to it explicitly when they use one of its formulas. Why then not keep to this "natural" hypothesis to account for the composition of the compendium and its inclusion in the book?

My reason for finding this explanation insufficient, though it is certainly partly valid, is that this hypothesis cannot explain certain facts related to the compendium. Indeed, if it was composed only with a view to the second part, one would expect that Li Ye would incorporate in the compendium all the formulas he knows concerning the drawing, or at least all the formulas he needs to solve the problems in the second part. However, the formulas used in the problems do not all come from the compendium. This is easy to prove, by comparing the incorrect formulas recorded in the two parts.[21] Some of those used in the course of the solution of one of the problems cannot be found among the incorrect formulas in the compendium. For example, problem 17 of chapter III contains the incorrect formula

The difference between hypotenuse and height (of triangle (13)) added to the height of triangle (7) gives twice the difference between base and height (of triangle (13)),

which is not found in the compendium.[22]

This means that Li Ye has not considered the compendium the only source of formulas for the problems. Nor has he felt it necessary to record there all the formulas that might have to be used in the second part. In any case, once he had finished solving his problems, he did not enter all the formulas he used in the compendium. These details indicate that Li Ye did not conceive the compendium as a place for gathering all relevant geometrical knowledge concerning the drawing.

Moreover, a second point also remains unexplained by the simple hypothesis. The formulas presented in the compendium are not all used in the second part of the book. On the contrary, it is surprising how few formulas actually occur in the problems, compared with the multiplicity of formulas registered in the first part of the book. So there seems to be an interest in formulas themselves that exceeds their being useful for problems and, correlatively, an interest in the compendium that exceeds its practical use. As a consequence, the compendium must also be considered as a piece of text *per se,* devoted to formulas, and not as a directory used for reference only when one needs a specific formula to solve a problem, as the form of the text would incline us to believe. So we again encounter the same warning related to the recognition of a textual unit familiar to the contemporary reader, who is hence tempted to project his accustomed modes of reading onto the text.

How, then, should we read the compendium? And how can we argue that this reading is valid? These questions are not easy to answer, since Li Ye leaves us with a mere juxtaposition of groups of formulas, organized in sections, sub-sections and paragraphs, with no commentary. I shall not develop a full answer to this problem here, but just indicate the direction that, I believe, must be followed, inasmuch as it will again give us interesting material for our purpose.

If something is said in the compendium, if it is to be read as a text on its own, what is said ought to have some connection with the activity that produced it, or at least with the motivations for this activity. And the best entry point for describing the activity that produced the compendium seems to be the incorrect formulas it contains. If a correct formula tends to be mute concerning the reasoning which yielded it, an incorrect formula by contrast is full of information for the historian of science. In that respect, a couple of the incorrect formulas are extremely interesting, since besides the fact that each is incorrect, they follow each other in the compendium and are phrased as grammatically parallel statements:[23]

> "Twice the height of triangle (1) and the difference between the base and height of triangle (11) taken together generate once the difference between the height (of triangle (13)) and the diameter of the circle.
>
> Twice the base of triangle (2) and the difference between the base and height of triangle (11) taken together generate once the difference between the base (of triangle (13)) and the diameter of the circle."

To make clear what I mean by parallel statements here, let me illustrate this textual structure on the basis of two lines of a poem written by the famous Tang writer Wang Wei:[24]

> "The bright moon shines among the pines
> The clear source flows on the rocks"

The two verses have the same syntactical structure and correspond character by character to each other —an adjective to an adjective, a noun to a noun, light to flowing water— building a relationship between the two facts as well as between the elements composing them. Such parallel sentences are extremely frequent not only in Chinese poetry, but also in Chinese philosophical, strategic, political texts, and so on, as well as in mathematical texts[25] like our compendium. And one can take for granted that the Chinese reader reads not only each of the two sentences, but the relationship between them as well—that is to say the correspondence that it establishes between the two facts.

Now, if our reader reads our pair of incorrect formulas as parallel, he is likely to note a mathematical transformation that enables one to obtain one formula from the other. And from this fact, we can deduce much information.

1. Here the parallelism between sentences has a mathematical meaning, and even though each formula is incorrect, the transformation itself is valid. It refers to a symmetry in the drawing, which is invariant with respect to the exchange between base and height.

2. This meaning is related to Li Ye's mathematical activity: since both formulas are incorrect in the same way, Li Ye did not obtain them independently, but used this mathematical transformation. This is the only way to account for such a pair of mistakes. Hence the parallelism —the production of a textual phenomenon— is correlated with a transformation the author used —a mathematical act. Note that the terminology has been selected in such a way that the result of the transformation can be obtained by a mere translation, character by character, applied to the statement of a formula: linguistic operations parallel mathematical operations.

3. Therefore some of Li Ye's mathematical work is expressed by the parallelism: Li Ye uses mathematical transformations to produce new formulas, and he records all the formulas so produced. They become parallel formulas that Li Ye might expect his reader to read as such. His mathematical activity is correlated with what, we can assume, he has given us to read. Hence the conclusion: Li Ye expresses the transformation *via* the parallel formulas that it links.

In fact, this kind of parallelism between formulas pervades and structures the compendium. Hence we are led to the following hypothesis, the consequences of which will not be developed here: the compendium deals with the transformations that produce formulas and their properties, and the results are expressed by the structure of the text. This content would be invisible if we were reading the text as a mere juxtaposition of formulas. In terms of the text, beyond the statements of particular formulas, the groups within which they are organized—subsections, paragraphs and subparagraphs—would as such be meaningful. We again come to the conclusion drawn above that the structures of the text convey meanings to be unfolded by the reader. This refers also to a kind of reading which we can consider as usual for communities of Chinese scholars.

The same way of reading holds true for the problems organized in chapters parallel to each other. But, as regards the problems, we are now confronted with a much more perplexing problem of reading.

1.4 *The problems*

After the compendium, entirely devoted to formulas, *Sea-mirror of the circle measurements* presents 170 problems, grouped in eleven chapters. Within the framework of the particular dimensions given for the drawing at the beginning of the book, the problems generally aim at determining the length of the diameter of the circle when given two or sometimes three pieces of data: the lengths of some segments of the diagram or simple combinations of such lengths. For this purpose, the so-called "procedure of the celestial element" is used.[26] Let us first describe the structure of most of the solutions given to a problem, using the example of problem 13 in chapter III. Its terms read as follows (see figure 4):

"One asks: it appears that the height of triangle (7) is 480 and the hypotenuse of triangle (9) is 153. The question and the answer are the same as before."[27]

As for most of the problems presented by Li Ye, the unknown is found as "the" root of an equation whose computation is described twice, in two distinct parts of the solution. First comes the part called "method" (*fa*), where Li Ye describes, one after the other, how to compute each coefficient of the equation. To give a flavor of what the "methods" look like, let us translate the description of the computation of the first coefficient, the constant term, replacing the names of the data by the letters A and B:

"Method: The two given quantities being subtracted from each other, again double this and subtract from it A. Again multiply this by A and put above. Moreover, B square being multiplied by the position above, that makes the constant term."

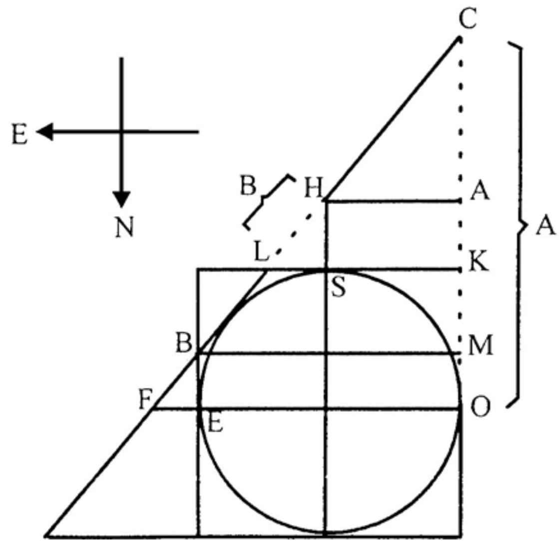


Figure 4. The data of problem III.13.

A procedure is hence provided to compute this term, followed by as many similar procedures as the equation has coefficients. Its description makes use of the names of the data as they are given in the outline of the problem as well as of intermediate quantities designated by the name of the position where they are provisorily placed during the computation. No numerical computation is appended there. We can translate the successive stages in the computation of the constant term as follows:

$A - B$
$2(A - B) - A$
$A[2(A - B) - A]$
$B^2.A[2(A - B) - A]$

Then comes the second part of the solution, called "sketch" (*cao*), where the same equation is produced in an entirely different way. First an unknown is chosen that is linked with the desired diameter in the course of the text, when it happens to be different from this diameter, as is the case here. A geometrical reasoning and polynomial numerical computations follow. The geometrical reasoning makes use only of the unknown, of the two data, and of the quantities that can be obtained on this basis, through reasonings where formulas show their efficiency. Along with these sequences of formulas, polynomials that represent the intermediate quantities are computed numerically, and in the end, the equation is obtained numerically. Let us translate the beginning of the "sketch", again to give a flavor of what the text looks like:

"Sketch: Take one celestial element[28] as the base of triangle (9). If one adds the hypotenuse of triangle (9), one gets

1
153•

as the height of triangle (1). If one subtracts the height of triangle (1) from A, it remains

−1
327•

as hypotenuse of triangle (1)."

Such is the way in which the solutions to the problems are presented in general. And this method of presentation immediately raises three questions:

1. Why does Li Ye systematically give two descriptions of each equation in solving a problem?

2. Where does the description in the "method" come from? How does Li Ye know that the procedures he gives actually compute the coefficients of the equation? How did he find them? The means of obtaining them are nowhere stated explicitly. Their description is, in the "method," rhetorical, whereas in the "sketch," the coefficients are produced only numerically.

3. Our translation into modern symbolism underscores that Li Ye is not using the most efficient algorithm to compute the constant term. Why does he first compute $A - B$ and then $2(A - B) - A$, instead of computing directly $A - 2B$? This question is even more intriguing if we consider the second solution that he gives to the same problem III.13, as he regularly does. As above, it is constituted of two parts. Its "method" reads as follows:

"Additional method: From A, subtract twice B. Again multiply this by A. Again multiplying this by B square makes the constant term of an equation of the fourth degree. All the other coefficients are as before."

Thus, in addition to the constant term, the equation given for this other solution is said to be the same as the one obtained in the first solution. But again, if we use modern symbolism to transcribe the successive quantities this new algorithm prescribes to compute the constant term supposed to be different from the previous one, we get:

$A - 2B$
$A(A - 2B)$
$B^2.A(A - 2B)$

Doesn't Li Ye know that this algorithm and the previous are equivalent in yielding the constant terms? Why does he give a cumbersome description first and then this simpler one? Why again does he give all coefficients except the constant term as the same, while giving a new description of this constant term, when it is obvious that even the constant terms are the same in both cases?

These are the questions to which the form of the text seems to lead the reader. We shall now show that they *can all receive answers simultaneously*. To this end, let us perform an experiment on the text. Here is the procedure: let us take the sequences of geometrical formulas presented in the “sketch” and, using the tools of modern algebra, compute symbolically, with respect to the data given in the outline, the successive polynomial expressions that correspond to those Li Ye computes numerically. In the case of the first solution of problem III-13, the following sequence of polynomials occurs in the “sketch”:[29]

$$\begin{array}{llllllll} (1) & 1\,x & (2) & -1\,x & (3) & -2\,x & (4) & -2\,x \\ & B & & A-B & & 2(A-B) & & 2(A-B)-A \end{array}$$

$$\begin{array}{lllll} (5) & -2A\,x & \text{(here the text suggests to put} & (6) & -2B^2.A\,x \\ & A[2(A-B)-A] & \text{this intermediate result “above”)} & & B^2.A[2(A-B)-A] \end{array}$$

It is striking that there is a strong correlation between the sequence of intermediaries Li Ye used in the description of the “method” and the sequence of states of the constant terms appearing now in the successive polynomials, which, in the end, gives the constant term of the equation. Hence the description of the constant term of the equation given in the “method” follows the steps through which this coefficient is progressively computed in the “sketch.” We can even notice that the intermediary term said to be put “above” in the “method” corresponds exactly to the polynomial said to be put “above” in the “sketch”.

The correlation between the two parts of the solution, which then proves to be very strong, is actually even stronger. Let us indeed observe the second “sketch”. It differs from the first one only at the beginning, and if, again, we compute symbolically the successive polynomials that Li Ye computed numerically there, we get the following sequence:

$$\begin{array}{llllllll} (1) & A-B & (2) & -1\,x & (3) & 1\,x & (4) & -1\,x \\ & & & A-B & & B & & A-B-B \end{array}$$

$$\begin{array}{ll} (5) & -2\,x \\ & A-2B \end{array}$$

After this stage, this polynomial that now represents a given geometrical entity will be used in the computations exactly as polynomial (4) above was used in the first solution. From this point onwards the two “sketches” are the same. Hence, first, we again find that the way of describing how to compute the constant term in the second “method” is the same as the way this constant term is actually progressively computed numerically in the second “sketch.” But, second, we can see that the difference between the *descriptions* the two “methods” give to obtain the constant term is *correlated with a difference in the ways through which they are computed in the two “sketches,”* whereas the statement that the other coefficients are “the same” corresponds to the fact that they are, in both “sketches,” computed in exactly the same way. Therefore the correlation between the “sketch” and the “method” appears once more in this second solution, and the comparison of the cases indicates how strong this correlation is: the reason why the constant term is given a new description in the second “method” might be that it is computed in another way, through the second “sketch.” This indicates the direction to follow to solve our third question, since this shows both why the constant term is first given an "awkward" description from our point of view, and why it is then given a second description in the second "method." Moreover this reveals that the lists of computations given in the "method" are symbolical expressions, and have a meaning as such, and do not refer to an actual numerical computation.

I repeated this experiment for each problem where polynomials occur in the course of the solutions, and the same correlation appeared: the same transposition of the numerical computations performed with polynomials in the "sketch" provides the successions of states of the coefficients corresponding to the intermediaries in the description of the equation as given in the "method"; this kind of relation between the two parts of the solutions holds throughout the book.[30] Much can be concluded from this.

First, in terms of history of mathematics, we can tackle the second question we raised, concerning the origin of the "method." It seems to be an inconvertible conclusion that Li Ye deduced the "methods" on the basis of the computations he recorded in the "sketch." But how? To do so, he must have performed the computations not only numerically, as he recorded them, but also, in a way, symbolically. Should we conclude that he has been computing not only with polynomials that have numbers as their coefficients, but also with polynomials having sequences of characters as their coefficients?[31] That would be another perspective from which to consider how unimportant the actual numbers are in Li Ye's book. But that would also assign to the book a content which has until now not been recognized in it and would modify its position within the history of mathematics. Indeed, within the framework of such a conclusion, we come to realize that what we are given in the "methods" appears to be precisely *equations with general coefficients*. In the Chinese mathematical tradition, as opposed to other traditions, an algebraic equation has the symbolic identity of an operation, given by its list of coefficients.[32] Such is the object Li Ye described in the "methods." Moreover, again in the Chinese tradition, what we would now write as formulas or symbolic expressions was expressed by algorithms.[33] Here, in the case of our equations, the coefficients that we would write as symbolic expressions, as I did above, are expressed through algorithms, but this should conceal neither their identity as symbolic expressions of another type, nor their generality.

From this, two consequences can be drawn. First, our approach leads us to recognize, under another form of expression, an object again so far not identified as such in this text: the general expression of an equation the coefficients of which are described through algorithms.[34] Second, the object that we recognize presents interesting correlations with the mathematical work that we are tempted to assign to Li Ye as bridging the gap between the "method" and the "sketch" of each solution, namely computations with polynomials with symbolic coefficients.

This last conclusion has to be taken into account when discussing the methodological problem which our approach raises: we used mathematical knowledge which is not contained explicitly in the text, which is, at least in some aspects, posterior or exterior to the text, to perform an experiment on it and to bring to light some of its underlying structures. The properties of the text so discovered enabled us to solve the second and the third of the problems its reading raises, by providing a hypothesis for the origin of the "method." And I know of no other way to solve this problem and account for the "method" as well as for the descriptions of the constant terms above. Should we then assign to Li Ye precisely this knowledge which we brought into play to work on his book, a knowledge which bridges the gap between the two parts of each solution, and conclude that, computing with polynomials with characters as coefficients, he also obtained general equations as a result, that he conceived of the equation and the polynomial in a general way? To answer this question involves not only methodological problems in the history of mathematics regarding the status of the use of mathematics exterior to a given text to deal with this very text;[35] it also raises issues related to the history of text. And this brings me back to questions 1 and 3 which I formulated above.

In terms of the text, two textual phenomena caught my attention. The first was Li Ye's careful and artificial distinction between the descriptions of the two equal constant terms, while he gives the other equal coefficients as being the same. The second was the division of each solution into two parts describing, in different ways, the same equation. Following these hints, I did an experiment and discovered a mathematical relationship between the two parts of each solution, which explains the origin of the "method" (question 2) and accounts for the strange form it sometimes receives as well (question 3). But, are these mere traces which the historian of mathematics can seize to reconstruct the mathematical work which goes along with the text, as we treated them so far? Or are these conscious indications, modes of expression: shouldn't we form the hypothesis that these textual devices, the structure of the text as well as the structure of the sentences, are precisely the means of expression Li Ye chose to call his reader's attention to a mathematical content which he did not express discursively in his book? So far, I can see only this way of accounting for the textual acts Li Ye performs.

What kind of research program can we set for ourselves to assess the validity of such a hypothesis? A better knowledge of how texts were written in Li Ye's time might enable us to confirm our conclusion as well as to understand why he used such a device. This is another point where history of science and history of text can collaborate. Until now, I know of no other contemporary Chinese text presenting the same phenomena. Li Ye's text constitutes precious evidence, since its mathematical content enables us to describe its structure rather precisely. It constitutes an invitation to look for other similar writings. In addition to yielding a textual context for the production of such writings, a history of text could help us to determine the sociological context within which they were produced. In particular, we could obtain information to determine whether Li Ye expected that any reader would read such a structure or whether it was meant only for initiates. In any case, we know that some Chinese readers of the *Sea-mirror of the circle measurements* read this structure as such in the text.[36]

At this point, let us recapitulate our findings. We started with some of the most elementary pieces of text that can be found in a scientific book: a set of numerical values, a set of formulas and a set of problems. In each case, it appeared that the kind of reading which would turn them into their modern counterparts might completely miss the way in which they make sense, excising them from the textual context to which they pertain. In consequence, one would fail to grasp what is at stake mathematically. Hence the problem of how to read sources appears crucial for the historian of mathematics and difficult to solve with certainty: elaborating our description of the reading of the *Sea-mirror of the circle measurements* constitutes a fundamental prerequisite for interpreting it, but what arguments can we offer to prove the validity of our answer?

Before we consider this question and related ones with full generality, another point naturally arises: Do we have here a unique case or do such problems recur constantly? We saw that, in many ways, Li Ye made use of the structure of a text to convey mathematical meanings:

- The parallelisms between the formulas, between the paragraphs of his compendium point out, it seems to me, the mathematical transformations that enable one to move from one formula to another, from a group of formulas to another, and from a member of a formula to the other.
- The systematic relationships between the two parts of the solution to any problem, indicated by textual hints, might refer to a computation on polynomials of a nature different from those recorded "discursively" in the text.

If in both cases to overlook the necessity of reading these structures would lead us to miss part of the content, the interpretation of these structures is at least as open as the interpretation of discourse. Now, should we conclude that "the Chinese" or "the Ancients" distinguished themselves by using, for such a distinct topic as mathematics, strangely indirect modes of expression, without commenting on them, a sin which would have been eradicated in modern science? Certainly not. It seems rather that an interest in certain kinds of phenomena regularly led groups of individuals to elaborate specific kinds of texts for the purpose of their research and that these texts, whatever their origin, raise the same problems of reading and of interpretation.

Even though such examples are manifold, I shall deal with only one in what follows. It will lead us to Potsdam, Saint-Petersburg, and Paris during the second half of the eighteenth century and the beginning of the nineteenth century.

2. THE HISTORY OF DUALITY AND THE DESIGN OF TEXTS[37]

2.1 *A similar problem of reading*

In 1753, the *Mémoires de l'Académie des Sciences de Berlin* published a *Mémoire* by Euler, a reasonable product of eighteenth-century Europe, entitled "Principes de la trigonométrie sphérique tirés de la méthode des plus grands et des plus petits." Euler's intention in this *Mémoire* is to derive the whole body of spherical trigonometry from an analytical method that he was largely responsible for developing and that he had placed at the center of a newly established branch of mathematics: the calculus of variations.

Why do so? One perfectly knew at the time how to solve any problem that could be raised in spherical trigonometry: whatever triplet of sides and angles of a triangle drawn on a sphere was given, one knew formulas to find the three other sides or angles. One of Euler's explicit motivations came from geodesy: when dealing in such a way with triangles drawn on the surface of the sphere, one could extend the treatment to triangles drawn on a surface resembling more closely the surface of the earth, a problem which he tackled in a subsequent memoir. But let us keep to the sphere.

Euler starts from the hypothesis that the sides of such triangles are the shortest paths that one can draw between its three vertices, and on this basis, embarks on pages of computations. He has already produced dozens of formulas when suddenly, at a point

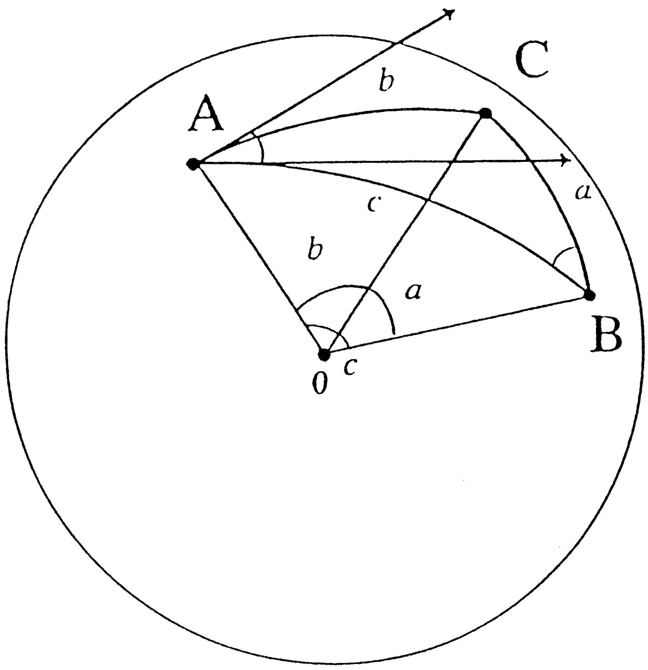


Figure 5. Euler's notations.

when he recapitulates, he interrupts momentarily the course of the text and introduces new notations (see figure 5):

"The previous denominations will be reduced to the present ones in this way:

Previous denominations	a, x, s, y, α, ϕ
Present denominations	c, b, a, A, B, C."[38]

Regarding the introduction of these new notations, here, right in the middle of the paper, he makes no comment whatsoever. Rather he rewrites with them the dozens of formulas obtained so far and proceeds to show how any problem that can arise in spherical trigonometry can be solved by one of these formulas.

When he begins to deal with the problems, his presentation adopts a new form. It is divided into sections that successively tell how, when one is given any triplet of elements out of the three sides and the three angles of a triangle drawn on a sphere, one can obtain the three others. Moreover, within such sections, he sometimes inserts algebraic computations that transform a formula which could be used to do so into another formula with which the actual computations become more convenient. Hence we have, for example, first a section concerning the case when the three sides are given and the three angles are sought. It is immediately followed by a section presenting how to deal with the case when the three angles are given and the three sides sought, and so on. But here we are confronted with a singular phenomenon, which brings us back to the problems discussed in the first part of this paper. When comparing section 1 with section 2, and section 3 with section 4, one discovers that the texts of the two pairs of sections correspond to one another formula by formula, step by step. More precisely, if one takes one of the formulas in one of these sections, and transforms capital letters into small letters and conversely, moreover if one inverts certain signs $+$ (resp. $-$) into $-$ (resp. $+$), one gets the corresponding formula in the other section. In correlation with this, where a sentence in one section says "angle", the corresponding sentence in the other section says "side", and conversely —this, however, in a looser way.

This applies to formulas given to obtain a certain element of the triangle, but it also applies to formulas that mark the successive steps in the transformations producing formulas more convenient for numerical computations. In other words: this applies to formulas as well as to algebraic transformations.[39] Therefore:

- The second part of Euler's memoir is given a precise structure, where whole sections correspond to each other by one and the same transformation applied to the text of their formulas and to the text of their sentences.
- This structure is made possible thanks to the notations Euler introduced, as we saw it, in the middle of his paper. Hence there is a correlation between the notations chosen and the structure of the texts built with them.
- The structure of the text shows that angles and sides play symmetrical parts in the body of spherical trigonometry. For us, readers of the twentieth century, this is mathematically meaningful as being a manifestation of a more general phenomenon called duality. We can bring in later mathematics to read the structure.
- On this structure, Euler is silent like Li Ye. The only thing he gives us to read is the structure of the text, only stressing here and there "comme cy-dessus." But, to qualify him as being "silent" refers to a conception of texts according to which only what is said discursively is said. One may as well consider that such is the mode of expression he chose here.

Now, how can we account for this phenomenon? The whole of the *Mémoire* has a meaning which again exceeds the meaning of its sections. This meaning is not expressed discursively and, correlatively, requires a reading of another kind. Here again is a point where the nature of the text matters methodologically to the historian of science.

But, if this is the case, it is because Euler designed a form of text chosen for the expression of certain meanings. And here one finds another perspective from which to consider the relevance of such questions for history of science and for history of text: the design of texts constitutes a crucial aspect of scientific activity and requires to be described as such. Indeed, the two types of questions are inseparable and should be treated together: it is because scientific activity accompanies the elaboration of types of texts that we meet with the problems we have described regarding their reading and interpretation.

This activity involves, in different times and places, elements that might retrospectively seem to be analogous. However, a critical distance sometimes reveals that similar appearances may conceal deep differences in nature: if the *Sea-mirror of the circle* *measurements* bears on a drawing, we saw that this drawing has little in common with a drawing to which Euclid's *Elements of geometry* might refer. We seem to have rather the 'fundamental drawing of the right-angled triangle' in a tradition of conceiving such drawings that we can trace back to Liu Hui and Zhao Shuang.[40] Here again the problems of how to read a text, of the elaboration by a tradition of textual elements, and of mathematical issues addressed within this tradition merge.

2.2 *The design of texts*

To confirm our conclusion concerning Euler's design of a type of text, one should check whether this fact is not coincidental: Is it by mere chance, in passing, that Euler elaborates such a text? Several facts speak in favor of the contrary:

- The introduction of the notations and the remarks stressing the similarities between parts of the text show that Euler has a concerted interest in the phenomenon.
- A few years earlier, Euler had presented —probably in Berlin even though it was later to be published in Petersburg— a *Mémoire* on polyhedra (Euler 1750). Again, when dealing with these solids delimited by plane faces, vertices and faces can be treated in the same way. And we now know that this is due to the same phenomenon mentioned earlier: duality. And again, whole parts of Euler's *Mémoire* present faces and vertices as having symmetrical properties, and the proofs given are themselves symmetrical to one another. Euler is aware of this fact, since he regularly introduces a development symmetrical to a previous one with the expression "simili modo, . . . " In this case too, the property of the field that it presents a symmetry is expressed only by the structure of the text. Euler does not seek to account for it from a mathematical point of view. Nor does he, in any case, mention the similarities of these phenomena in spherical trigonometry and in the treatment of polyhedra.

  Hence Euler, confronted roughly at the same time with what we know is the same mathematical phenomenon, expresses it in two cases in the same way: he designs a kind of text the structure of which displays the corresponding symmetry, without further commenting on it. This recurrence makes it all the more difficult for the historian of science to overlook such problems.
- A third fact indicates that Euler's interest in such phenomena is not coincidental. Roughly thirty years later, he comes back again to spherical trigonometry in a *Mémoire* published in 1782 in the *Acta Academiae Scientiarum Imperialis Petropolitanae*. By that time, everything has changed in his treatment of the topic, except that he still wants to give a complete account of spherical trigonometry on the basis of the fewest possible number of principles, that he still uses the same notations as in the *Mémoire* from 1753, and that the symmetry caused by duality is still tacitly central in his presentation of the topic.[41] Moreover, the rule of rewriting which exchanges capital and small letters as well as some signs for computations has become explicit.

But, here, fortunately, if we may say so, Euler makes a mistake which, as in Li Ye's case above, enables us to describe more precisely how he produced this kind of text. In paragraph 22, given as example in figure 6, adding and subtracting 1 to the first and the

Transformatio tertia.

§. 22. Hanc transformationem etiam ex prima forma expedire licet, combinandis his duabus formulis:

$$\text{cof. } a - \text{cof. } b \text{ cof. } c = \text{fin. } b \text{ fin. } c \text{ cof. A},$$
$$\text{cof. } b - \text{cof. } a \text{ cof. } c = \text{fin. } a \text{ fin. } c \text{ cof. B};$$

quarum illa per hanc diuifa praebet,

$$\frac{\text{cof. } a - \text{cof. } b \text{ cof. } c}{\text{cof. } b - \text{cof. } a \text{ cof. } c} = \frac{\text{fin. } b \text{ cof. A}}{\text{fin. } a \text{ cof. B}} = \frac{\text{fin. B cof. A}}{\text{fin. A cof. B}}.$$

Addatur vtrinque vnitas, fietque

$$(\text{cof. } a + \text{cof. } b)(1 - \text{cof. } c) = \frac{\text{fin. (A + B)}}{\text{fin. A cof. B}},$$

fubtrahatur vtrinque vnitas, prodibit

$$(\text{cof. } a - \text{cof. } b)(1 + \text{cof. } c) = \frac{\text{fin. (B - A)}}{\text{fin. A cof. B}},$$

quae aequatio per priorem diuifa dat

$$\frac{\text{cof. } a - \text{cof. } b}{\text{cof. } a + \text{cof. } b} \cdot \text{cot. } \tfrac{1}{2} c^2 = \frac{\text{fin. (B - A)}}{\text{fin. (B + A)}}.$$

Conftat autem effe

$$\frac{\text{cof. } p - \text{cof. } q}{\text{cof. } p + \text{cof. } q} = \text{tang. } \frac{q + p}{2} \text{ tang. } \frac{q - p}{2},$$

vnde colligitur:

$$\text{tang. } \frac{b - a}{2} \cdot \text{tang. } \frac{b + a}{2} \cdot \text{cot. } \tfrac{1}{2} c^2 = \frac{\text{fin. (B - A)}}{\text{fin. (B + A)}}.$$

Transformatio quarta.

§. 24. Haec fimili modo deducitur ex his formulis:

$$\text{cof. A} + \text{cof. B cof. C} = \text{fin. B fin. C cof. } a$$
$$\text{cof. B} + \text{cof. A cof. C} = \text{fin. A fin. C cof. } b$$

quarum illa per hanc diuifa praebet

$$\frac{\text{cof. A} + \text{cof. B cof. C}}{\text{cof. B} + \text{cof. A cof. C}} = \frac{\text{fin. B cof. } a}{\text{fin. A cof. } b} = \frac{\text{fin. } b \text{ cof. } a}{\text{fin. } a \text{ cof. } b}.$$

Vnde vnitatem tam addendo quam fubtrahendo fequentes nouae deriuantur aequationes:

$$(\text{cof. A} + \text{cof. B})(1 + \text{cof. C}) = \frac{\text{fin. } (a + b)}{\text{fin. } a \text{ cof. } b}$$
$$(\text{cof. A} - \text{cof. B})(1 - \text{cof. C}) = \frac{\text{fin. } (b - a)}{\text{fin. } a \text{ cof. } b},$$

et diuidendo illam per hanc nancifcimur:

$$\frac{\text{cof. A} + \text{cof. B}}{\text{cof. A} - \text{cof. B}} \text{ cot. } \tfrac{1}{2} \text{C}^2 = \frac{\text{fin. } (a + b)}{\text{fin. } (b - a)}, \text{ fiue}$$
$$\text{tang. } \frac{\text{B} - \text{A}}{2} \cdot \text{tang. } \frac{\text{B} + \text{A}}{2} = \text{cot. } \tfrac{1}{2} \text{C}^2 \cdot \frac{\text{fin. } (b - a)}{\text{fin. } (b + a)},$$

Figure 6. Two paragraphs symmetrical to each other in [Euler 1781].

third members of

$$\frac{\cos a - \cos b \cos c}{\cos b - \cos a \cos c} = \frac{\sin b \cos A}{\sin a \cos B} = \frac{\sin B \cos A}{\sin A \cos B}$$

he should obtain:

$$\frac{(\cos a + \cos b)(1 - \cos c)}{\cos b - \cos a \cos c} = \frac{\sin (A + B)}{\sin A \cos B}$$

and

$$\frac{(\cos a - \cos b)(1 + \cos c)}{\cos b - \cos a \cos c} = \frac{\sin (B - A)}{\sin A \cos B}$$

But here Euler forgets the denominators of both first members. Then, he divides this equation by the previous one, hence the two identical forgotten denominators compensate for each other, and thus the result is valid.

The key fact for us here is that a symmetrical mistake can be found in § 24 (see figure 6), symmetrical to this one, step-by-step. Indeed, adding and subtracting 1 to the first and the third members of

$$\frac{\cos A + \cos B \cos C}{\cos B + \cos A \cos C} = \frac{\sin B \cos a}{\sin A \cos b} = \frac{\sin b \cos a}{\sin a \cos b}$$

he should again obtain

$$\frac{(\cos A + \cos B)(1 + \cos C)}{\cos B + \cos A \cos C} = \frac{\sin (a + b)}{\sin a \cos b}$$

and

$$\frac{(\cos A - \cos B)(1 - \cos C)}{\cos B + \cos A \cos C} = \frac{\sin (b - a)}{\sin a \cos b}$$

But here, again, Euler forgets the denominator of both first members. As previously, he divides this equation by the previous one; hence, here as before, the two identical denominators he forgot compensate for each other, and thus the result is valid.[42]

Such a pair of mistakes cannot be explained except by assuming that the latter proof has been obtained by a mere translation of the former one, where each step was transformed in accordance with the rule of rewriting involving an exchange between capital and small letters. This implies that Euler used the notations he introduced, which are so appropriate to deal with this symmetry in spherical trigonometry, to translate parts of the text into other parts. Such is the way in which he gave his text this structure. This phenomenon shows, as in Li Ye's case,

- Euler's awareness of the structure of the text.
- his will to produce it, his will to express something in this way.
- how he produced it: mathematical operations are involved in the making of the text.
- that the text supports certain mathematical operations and is meant to do so.

2.3 *A history developing in the structure of the texts*

The structure of the text manifests mathematical knowledge and mathematical activity. This should suffice to prove that we, as historians, need to discuss what is given to be read there when we account for its content. And Euler's case, like Li Ye's, shows that this description is by no means immediate and requires elaboration. However, if we need to consider such questions, this is not only because Euler "meant" his text to have a "tacit" meaning. In this case, where we possess writings concerning spherical trigonometry composed by scientists who read Euler's *Mémoires*, other factors enter into consideration:

- The form of Euler's texts was read as such by subsequent mathematicians. They reproduced his notations or produced new notations presenting the same properties regarding duality. And, in correlation with this, they composed texts with similar forms. A kind of text gets stabilized by the existence of a community of readers, synchronically as well as diachronically.
- The form of the text itself was taken up and improved in its function of displaying symmetries that affect certain parts of mathematics in the same way as spherical trigonometry. This shows that forms of texts have a history too.[43] In this case, the process reached a stable state at the beginning of the nineteenth century in the *Annales de mathématiques pures et appliquées*, whose editor, Gergonne, devoted specific attention to this range of phenomena in mathematics. To this end, he published a series of papers dealing with mathematical topics in which such symmetry occurs, and he designed for them a specific form of presentation in double columns (see figure 7), where not only formulas, but also statements corresponded to each other thanks to a systematic rule of translation. He also rewrote the presentations of these domains of mathematics so as to fit them into this form. Moreover, by asking problems in a symmetrical way in his journal (see figure 8), Gergonne incited readers to work on such symmetries within this way of presentation. Such texts can be found even today in certain mathematical publications.
- Because various domains of mathematics were likely to receive such a formal presentation, the idea that this was due to one and the same mathematical phenomenon came out and was explored with texts of this form. Such was the way in which research on duality, as a property of space, began.[44] Hence the kind of knowledge expressed by such forms has its history as well.

Let me recapitulate what, it seems to me, we can conclude from this case:

- In order to deal with certain phenomena in mathematics, or more generally, types of texts may be created that allow writers to express these phenomena, communicate them, and work on them. Hence a form of text is consciously created for a specific kind of research.
- These meanings are received and reworked under this form. In our case, a whole history develops in the structure of the text. And, in correlation with this, we can see the form of the text and what it is meant to deal with evolve. Indeed, as regards duality, after a time when it was worked out through the structures of texts, it became a topic of discursive treatment.[45] This shows the continuity of nature between content expressed by the form of a text and that expressed discursively.
- Without reading the meanings conveyed by these specific forms of texts, without a critical conception of what a text is, it would be difficult to discuss their content as well as the later evolution of ideas thus displayed. To assign the beginning of the history of duality to its discursive treatment would represent a mistake of the same nature as those which we analyzed concerning Li Ye's *Sea-mirror of the circle measurements*.

Voilà ce qui nous détermine à faire de cette sorte de géométrie *en parties doubles*, s'il est permis de s'exprimer ainsi, le sujet d'un écrit spécial dans lequel, après avoir rendu manifeste le fait philosophique dont il s'agit, dans l'exposé même des premières notions, nous nous en appuyerons, soit pour démontrer quelque théorèmes nouveaux, soit pour donner de quelques théorèmes déjà connus des démonstrations nouvelles, qui les rendent à l'avenir tout à fait indépendans des relations métriques desquelles on a été jusqu'ici dans l'usage de les déduire.

§. I.

Notions préliminaires.

1. Deux points, distincts l'un de l'autre, donnés dans l'espace, déterminent une droite indéfinie qui, lorsque ces deux points sont désignés par A et B, peut être elle-même désignée par AB.	1. Deux plans, non parallèles, donnés dans l'espace, déterminent une droite indéfinie qui, lorsque ces deux plans sont désignés par A et B, peut être elle-même désignée par AB.
2. Trois points donnés dans l'espace, ne se confondant pas deux à deux et n'appartenant pas à une même ligne droite, déterminent un plan indéfini qui, lorsque ces trois points sont respectivement désignés par A, B, C, peut être lui-même désigné par ABC.	2. Trois plans, non parallèles deux à deux dans l'espace, et ne passant pas par une même ligne droite, déterminent un point qui, lorsque ces trois plans sont respectivement désignés par A, B, C, peut être lui-même désigné par ABC.
3. Un plan peut aussi être déterminé dans l'espace par une droite et par un point qui ne	3. Un point peut aussi être déterminé dans l'espace par une droite et par un plan dans le-

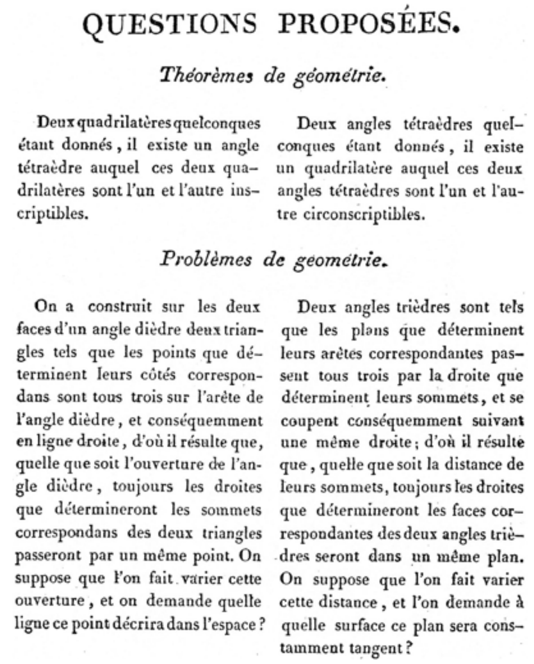
Figure 7. Gergonne's presentation of geometry in double column ([Gergonne 1826], the top paragraph comes from p. 211, and the double column presentation from p. 212).

CONCLUSION

Even if we limit ourselves here to these two examples, the direction of research they open converges with what many other writers have already indicated: various forms of collaboration between history of science and history of text might prove profitable.

Texts are not a historical, transparent forms conveying meanings whose history we would write. Scientific texts as such appear to have taken various forms within space and time, designed as they were through an interaction with local conditions of text production of all kinds: ways of giving names, literary forms, graphics, and writing technology available. These texts have a history which inscribes them in an activity, in a culture, in an environment of readers.

Elaborating a description of these various forms of texts would provide methodological tools to read them, since they can by no means be read without the mediation of a method. Here, the achievements of a history of text would benefit history of science in that it would deliver a better knowledge of the local contexts of text production within



QUESTIONS PROPOSÉES.

Théorèmes de géométrie.

Deux quadrilatères quelconques étant donnés, il existe un angle tétraèdre auquel ces deux quadrilatères sont l'un et l'autre inscriptibles.

Deux angles tétraèdres quelconques étant donnés, il existe un quadrilatère auquel ces deux angles tétraèdres sont l'un et l'autre circonscriptibles.

Problèmes de geométrie.

On a construit sur les deux faces d'un angle dièdre deux triangles tels que les points que déterminent leurs côtés correspondans sont tous trois sur l'arête de l'angle dièdre, et conséquemment en ligne droite, d'où il résulte que, quelle que soit l'ouverture de l'angle dièdre, toujours les droites que détermineront les sommets correspondans des deux triangles passeront par un même point. On suppose que l'on fait varier cette ouverture, et on demande quelle ligne ce point décrira dans l'espace?

Deux angles trièdres sont tels que les plans que déterminent leurs arêtes correspondantes passent tous trois par la droite que déterminent leurs sommets, et se coupent conséquemment suivant une même droite; d'où il résulte que, quelle que soit la distance de leurs sommets, toujours les droites que détermineront les faces correspondantes des deux angles trièdres seront dans un même plan. On suppose que l'on fait varier cette distance, et l'on demande à quelle surface ce plan sera constamment tangent?

Figure 8. Problems set by Gergonne to his readership ([Gergonne 1826], p. 232).

which scientific texts were designed and elaborated. It would describe an environment against which to read our sources.

But, in addition to providing a better grasp of such a textual context for the production of scientific writing, a history of text might give a better awareness of the various ways in which texts were meant to mean and help us discuss how to account for their content. Indeed, the aim of determining with certainty the content of a document —difficult to push aside when such a branch of intellectual history as history of science is conceived as a history of results— appears, in view of the problems previously described, illusory. We presented texts to which it is difficult to ascribe a content, although the stakes are rather high. In particular, we saw that an opposition between form and content, which underlies many ways of reading, would let innumerable aspects of the content escape, even though what escapes is not simple to characterize. This incites us to add another range of issues to those suggested above and support the call for systematically promoting a history of the readings, of the receptions, of scientific texts, which can be developed by examining, when possible, commentaries on, or reactions to, a given writing. The way readers read tells us something about a text, be they in the same textual tradition or not. And a description of the text, including such structures as those described above, could be helpful in characterizing this reading. Such an additional inquiry to complement the quest of assigning results to a writing would inscribe us as readers amidst the actors we observe.

On the other hand, the history of scientific text could become a systematic concern in history of science as such. Indeed, scientists design their texts at the same time as they design concepts and results. This represents a constitutive part of their activity. And this intimate relation can go as far as the fact that concepts and results can be made, as in the case of duality, of texts. Indeed, if texts were not the subject of a specific design, would they vary in form, would their reading call for methodological tools?

How do scientists design these texts? The study of the production of texts would bring materials that would enable us to deal with the question of understanding how scientists benefit from the cultural and textual contexts within which they work.[46] It would give us a grasp of how scientists construct the symbolic tools with which they perform their activities and communicate their results, which in the end are texts. In all these respects, a history of scientific text could then become a specific domain of the history of text.

REHSEIS-CNRS

NOTES

[1] This paper was written for the workshop "History of science, History of text", during my stay at the Wissenschaftskolleg, in Berlin. I gratefully acknowledge the help of the Wissenschaftskolleg, the Otto und Martha Fischbeck Stiftung and the Einstein Forum. It is a pleasure to thank all my colleagues at the Wissenschaftskolleg during this year for the interest they expressed in this project and for the help they gave me in defining it. I am glad to take this opportunity to thank my colleagues Huang Yilong, who helped me to prepare this text, and Fu Daiwie, who invited me to publish it, in a different shape in *Philosophy and the History of Science*, a Taiwanese journal. I owe many thanks to the publisher and the editors of the journal for allowing me to reproduce it in this volume. I am grateful to Chris Fraser and Jeremy Gray for their revision of the English. Needless to say, I am responsible for all the remaining mistakes. The reader can find in (Chemla 1995a) other arguments to support the thesis defended here.

[2] The recent case of the problems raised by the reading of Diophantos' *Arithmetics* with the tools provided by algebraic geometry has brought new material to bear on this question. See (Rashed 1984) and (Chemla, Morelon, Allard 1987), where the reader can find a bibliography on the topic. As to the role played by later mathematics in enabling us to read an ancient source, this case can be compared to Li Ye's, which I present below, as I have argued in "What could be experimentation in the history of mathematics?", VIIIth conference of the International Union for the History and Philosophy of Science, Gand, August 1986 (to appear).

[3] I have discussed this treatise extensively in my Ph. D. thesis (Chemla 1982) and several papers (Chemla 1985, 1988, 1990, 1993a & b), where an updated bibliography can be found.

[4] On the so-called "procedure of the celestial element," see, for instance, (Chemla 1982). Let us recall here that the polynomials in one indeterminate introduced are written down according to a vertical place-value notation:

1
2 stands for $x^2 + 2x + 3$
3•

whereas

1
2• stands for $x + 2 + \frac{3}{x}$
3

What we transcribe as a point going along with the constant term corresponds to a Chinese character. Polynomials are distinguished from equations by means of this mark:

1
2 stands for $x^2 + 2x + 3 = 0$
3

[5] To designate the segments by the term of *lü* states a property and announces a decision concerning the mathematical situation. The concept of *lü* refers to the fact that when all the magnitudes considered are multiplied by the same amount, this does not change the meaning of the whole. This states a property concerning their relationships to each other, and announces the decision to consider the drawing up to a dilatation, and not in the absolute. Concerning the way names are given to objects in some Chinese mathematical texts and the way to read them in consequence, see (Chemla 1993b).

[6] In order to convey an idea of the text conveniently, I associate letters to the points to which Li Ye gives a Chinese name on his drawing, and numbers between brackets to the triangles which he now introduces. See figure 2: the numbers I associate to right-angled triangles are inscribed in their right angle.

[7] In what follows, I shall designate the base by a, the height by b, and the hypotenuse by c.

[8] This constitutes a change since the time when *The ten classics of mathematics* were collated by Li Chunfeng and by the team he headed in the seventh century (see Qian Baocong 1963). There right-angled triangles receive specific attention, but they are not systematically attached to thirteen quantities (see Schrimpf 1963: 216–42). However, in Song-Yuan times, such is the case, and a specific terminology is chosen for these thirteen quantities, even though there are variations from one author to another.

[9] Hereafter I shall abbreviate the title of this book as *The nine chapters*, and, unless otherwise specified, I shall rely on the edition of Qian Baocong (1963). Note that this classic displays knowledge in the form of problems and general algorithms for their solutions. In this respect, it resembles most of the mathematical writings produced in China until the fourteenth century. In using such textual elements as a compendium, Li Ye resorts to a form of presentation examples of which are rare in classical Chinese mathematical literature.

[10] He refers to it and to its commentary by Liu Hui, written in the third century and handed down together with the classic in all surviving editions, in the preface of his two mathematical works.

[11] Elements relevant for the construction of this drawing are discussed in (Chemla 1994a).

[12] Two exceptions appear here:
—Two triangles (5) and (6) are associated with quantities that are not in the list of those attached to a triangle. I discuss this in (Chemla 1993a).
—The quantities $a + b$ and $b - a$, hence the simplest quantities computed with the base and the height of the largest triangle, are missing in the list of quantities obtained as sums of the lengths of the three sides of a triangle. Again, this is discussed in (Chemla 1993a). Yet, it would suffice to add the line drawn by Liu Hui in his proof to Li Ye's drawing to make such triangles appear. And this is precisely what Li Shanlan will do in the nineteenth century when reading the *Ceyuan haijing*, which indicates that he also considers the two texts to refer to each other (See Chemla 1982: 2.55 *sq.*). Is this an additional hint of the connection between the two texts?

[13] Such a figure seems to have been produced by a research program different from some developed on drawings in ancient Greece. In the Chinese texts starting from *The nine chapters* and Liu Hui's commentary onwards, fundamental patterns for figures seem to be sought for. Compare (Chemla 1994a).

[14] I skip the details, which can be found in (Chemla 1993a), if necessary.

[15] His treatment of problem VIII-15 shows that Li Ye is aware of these facts, see (Chemla 1990).

[16] See Chemla 1993b. As we saw above, the names can be read as conveying information about the things they designate. The design of names and, hence, the symbolic operations that a reader performs with them are to be referred to the uses elaborated within certain communities, and not to be taken as invariant. This point confirms the thesis developed in this paper, but I will not elaborate any further here (see Chemla 1993b).

[17] See (Chemla 1990) and (Chemla 1993a).

[18] See Chemla, Martin & Pigeot (eds.) 1995.

[19] As already emphasized, this holds true for names and for drawings, which indicates that two kinds of cultural contexts have to be taken into account: a mathematical one, where such elements as drawings are elaborated in a specific way, and a larger one within which ways of designing names and making quotations present stable features beyond variation. Numbers, names, and the drawing are three kinds of objects for which we suggest here a specific mode of reading. Note that the outcomes draw a coherent picture.

[20] For a more detailed treatment, see Chemla 1990 & 1993a.

[21] This question is treated in (Chemla 1982: 2.39 *sq.*).

[22] This cannot be due to a mere error by a copyist, as one can find a parallel mistake in problem 17 of chapter IV. We shall discuss such mistakes below.

[23] They are the first two formulas of paragraph 49 if one starts numbering the paragraphs at the beginning of the compendium—a paragraph being the textual unit whose first character is printed higher than the others, in the upper margin.

[24] I translate the two verses with which F. Martin illustrates the phenomenon of parallelism in (Martin 1989: 81 *sq*.).

[25] The entire issue 11 of *Extrême-Orient, Extrême-Occident,* (Jullien (ed.)1989), mentioned above is devoted to the question of parallelism in Chinese texts and thought. The two incorrect formulas found in the solutions of problems that I evoked at the beginning of this paragraph are parallel in the same way as those quoted here.

[26] One can find a presentation of this procedure, which amounts to polynomial computations, in any standard introduction to the history of mathematics in China as well as in (Chemla 1982). In order to study Li Ye's text, I shall reformulate parts of his book below and then, contrary to what we find in his text, I shall make use of Arabic numerals.

[27] As indicated on the drawing, we shall designate these unknowns respectively by A and B.

[28] Such is the name of the unknown, on the basis of which polynomials are computed. As announced above, I keep the place-value notation of the polynomials.

[29] I represent the polynomials as Li Ye does, using letters where he uses numerals. Moreover, I use x instead of a character which marks the term of the first degree. I skip here some details that are irrelevant for my purpose. See Chemla 1982, 1985 or 1993a.

[30] See chapter 10 of (Chemla 1982).

[31] I tried in (Chemla 1982), chapter 10, to make use of some details in the *Sea-mirror of the circle measurements* in order to describe the way in which this computation may have been performed.

[32] See Chemla 1995b.

[33] This is very clear from the classical work *The nine chapters* onwards.

[34] This raises the question of the nature of equations in earlier Chinese texts, which I shall not address here.

[35] The difficulties encountered here seem to me comparable to those raised by the reading of Diophantos' *Arithmetics* with the help of algebraic geometry. The tool reveals properties of the text. We would certainly not assign the knowledge of our tool to Diophantos. Still the cause which produces the properties revealed in the text needs to be determined. A discussion of the operations performed on a text when one brings mathematical tools into play to read it should take into account such evidence.

[36] The author of the commentary introduced by *an* "commentary" —until now not identified (see Chemla 1982: 0.3)— states the close relationships linking both parts of the solution and writes (p. 11–2, chapter 2): "The "sketch" is the root of the "method," the "method" is the functioning of the "sketch"." Li Shanlan, in his preface to the new edition, writes (p. 1) : "The "sketch" . . . is the "method" to make the method, the origin of the "method"." Closer to us, Li Yan, when he rewrote Li Ye's "sketches" symbolically, tacitly let the steps in the description Li Ye gave in the "methods" appear (see Chemla 1993a).

[37] For this paragraph I mainly rely on (Chemla 2004), where the reader will find all the details, a more careful treatment, and a bibliography which I do not reproduce here. I am grateful to Serge Pahaut, who called my attention to the history of duality, and we started to work together on the influence of research on spherical trigonometry on the emergence of a specific way of dealing with this notion (see Chemla, Pahaut 1988).

[38] "Les dénominations précédentes se réduiront aux présentes de cette manière:
Dénominations précédentes a, x, s, y, α, ϕ
Dénominations présentes c, b, a, A, B, C." (Euler 1753: 294)

[39] There are exceptions but, in the loose account given here, I shall not comment on them, see Chemla 2004.

[40] See our paper to appear on the question of drawing in the Chinese mathematical tradition.

[41] See figure 6, where two paragraphs symmetrical to each other are reproduced (they come from p. 82 and p. 84 in the original publication).

[42] In his edition of this text, A. Speiser writes symmetrical footnotes to correct both mistakes.

[43] Here we should make precise that so far, for the sake of simplicity, we have simplified the account one should give of the history of such designs of texts as the one found in Euler's *Mémoires* on spherical trigonometry. Indeed, Euler did not initiate a trend of research: there existed before him a whole tradition

of writings about this topic, presenting some symmetries in their composition (one can think of Viète, for instance). Euler himself inherited this form of text, which he modified and extended, thereby introducing changes in the conception of duality itself, and his readers, in turn, carried on this process.

44 See Chemla 1989 & 1994b.

45 In Cavaillès' terms: "thématisé".

46 When he introduces the presentation in double column, Gergonne refers explicitly to bookkeeping. A textual element is found available in the context and transformed so as to serve some specific mathematical research. In the same way, Li Ye can use the grammar of written Chinese of his time to record his formulas. However, he introduces some restrictions on the common use for the sake of mathematics, thanks to which the statements of formulas are made unambiguous. This turns the language he uses into an artificial one —see my paper in collaboration with Alain Peyraube (to appear). This phenomenon is all the more remarkable in that we do not find it either in *The nine chapters on mathematical procedures* or in the writings of a mathematician slightly posterior to Li Ye, Zhu Shijie.

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