Chapter 6

MEASUREMENTS OF TRACE GAS FLUXES IN THE ATMOSPHERE USING EDDY COVARIANCE: WPL CORRECTIONS REVISITED

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Abstract This Chapter re-examines theory developed by Webb, Pearman and Leuning (1980, Quarterly Journal of the Royal Meteorological Society, 106, 85-100) to calculate fluxes of trace gas constituents in the atmosphere using the eddy covariance technique. The original theory for one-dimensional flow over homogeneous terrain is extended to three-dimensional flow over inhomogeneous terrain. The equations are relatively simple when concentrations are expressed as mixing ratios per unit of dry air. Advective mass fluxes are written as products of fluxes of dry air and gradients in mixing ratio, while turbulent eddy fluxes requires the covariance of wind speeds and mixing ratios. Theory developed by WPL for one dimensional flows is applicable for the vertical eddy flux.

1 Introduction

The eddy covariance technique is used widely to measure the net exchanges of heat, mass and momentum between the earth's surface and the atmosphere (Baldocchi et al. 2001). Before publication of the paper by Webb et al. (1980) (WPL hereafter), the vertical turbulent flux density of a constituent c was ca $\overline{F}_c = \overline{w'c'_c}$, the covariance between fluctuations in the vertical velocity, w' and the density c'_c showed that this gave incorrect estimates of $\overline{F}_c$ because fluctuations in c_c can result from fluctuations in water vapor density and temperature which are not associated with the net transport of c. These errors are particularly severe for trace constituents such as CO_2. The original WPL theory strictly only applies to steady, one-dimensional flow over

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horizontally homogeneous terrain and hence may not be suitable for the more typical flux measurement installation in inhomogeneous terrain. Further theoretical work is thus warranted.

Recent papers by Kramm et al. (1995), Sun et al. (1995), Paw U et al. (2000), Massman and Lee (2002) and Fuehrer and Friehe (2002) have re-examined the conservation equations used to calculate net exchanges of mass and energy between the earth's surface and the atmosphere for surface boundary layer flows in inhomogeneous terrain. In doing so they revised the theory developed by WPL and introduced extra terms into the equations. This Chapter also examines the theory used to calculate fluxes using the eddy covariance technique and shows that the original WPL theory is still applicable for the vertical component of the eddy fluxes and that the resulting equations are particularly simple when concentrations are expressed as mixing ratios per unit of dry air.

Section 2 develops the conservation equations for the various constituents of moist air to generalize the one-dimensional conservation equation used by WPL; Section 3 utilizes the results for the special case of steady, one-dimensional, horizontally homogeneous flow to derive a key result of WPL; Section 4 considers the general case of non-steady flows in non-homogeneous terrain and discusses the components of the mass balance equation; Section 4 also discusses the case of steady, horizontally homogeneous, on-dimensional flows; Section 5 discusses practical aspects of calculating flux densities using closed- and open-path gas analyzers; and Section 6 draws some conclusions.

2 Conservation Equations for Moist Air and Trace Constituents

Consider a fixed control volume $\mathrm{d}V$ containing moist air with molar concentration $c = c_d + c_v + c_c$ ($\mathrm{mol\,m^{-3}}$), in which c_d, c_v and c_c are the molar concentrations of dry air, water vapor and a trace constituent, c. (Note that while molar quantities are used in this Chapter, all equations can be written in mass units, making suitable allowances for the molecular mass of the various constituents when applying the gas laws.)

The molar conservation equation for all gas components in $\mathrm{d}V$ is

$$\frac{\partial c}{\partial t} + \nabla.\mathbf{F} = S_v + S_c \tag{6.1}$$

where $\partial c/\partial t$ is the rate of change of molar concentration of air in $\mathrm{d}V$, $\mathbf{F}$ is the total flux density vector on the surfaces of the control volume and S_v, S_c ($\mathrm{mol\,m^{-3}s^{-1}}$) are the source/sinks for water vapor and trace constituent within $\mathrm{d}V$. We assume that there is no source or sink of dry air within dV. Equation 6.1 may be written as

$$\frac{\partial c}{\partial t} + \nabla.\mathbf{u}c = S_v + S_c \tag{6.2}$$

where the velocity vector $\mathbf{u}$ has components $\{u, v, w\}$ in the orthogonal directions $\{x, y, z\}$. The velocity vector is *defined* as $\mathbf{u} = \mathbf{F}/c$, i. e. the molar flux density vector for moist air divided by the total molar concentration of moist air. Fluxes in the atmosphere due to molecular diffusion are assumed to be negligible.

The conservation equation for the constituent c is

$$\frac{\partial c_c}{\partial t} + \nabla.\mathbf{u}c_c = S_c \tag{6.3}$$

Equation 6.3 may also be written as

$$\frac{\partial c_d \chi_c}{\partial t} + \nabla.(\mathbf{u}c_d\chi_c) = S_c \tag{6.4}$$

where χ_c is the mixing ratio of c relative to dry air $\chi_c = c_c/c_d$.

We next use Reynolds decomposition to separate quantities into mean and fluctuating components, and then take the time-average, represented by the overbar, to give

$$\overline{\frac{\partial(\overline{c}_d + c'_d)(\overline{\chi}_c + \chi'_c)}{\partial t}} + \nabla.[\overline{(\overline{\mathbf{u}} + \mathbf{u}')(\overline{c}_d + c'_d)(\overline{\chi}_c + \chi'_c)}] = \overline{S}_c \tag{6.5}$$

Expanding the terms in this equation yields

$$\overline{c}_d\overline{\frac{\partial \chi}{\partial t}} + \overline{\chi}_c\overline{\frac{\partial c_d}{\partial t}} + \nabla.[\overline{\chi}_c(\overline{\mathbf{u}}\,\overline{c}_d + \overline{\mathbf{u}'c'_d}) + \overline{c}_d\overline{\mathbf{u}'\chi'_c} + \overline{\mathbf{u}}\,\overline{c'_d\chi'_c} + \overline{\mathbf{u}'c'_d\chi'_c}] = \overline{S}_c \tag{6.6}$$

where terms such as $\overline{\overline{w}\,\overline{c}_d\,\chi'} = 0$ by definition.

To proceed, we need to show that the last two terms on the left of Equation 6.6 are small compared to the others. The covariance between c_d and χ_c will be zero when fluctuations in c_d result from fluctuations in temperature and pressure, since these do not alter the mixing ratios of the constituents(i. e. $\overline{\chi'_c P'} = \overline{\chi'_c T'} = 0$ and $\overline{\chi'_c c'_v}$ is small). Fluctuations in moisture content will change both c_d and χ_c but these are expected to have only a small influence on the covariance $\overline{c'_d\chi'_c}$. For similar reasons, the triple moment $\overline{\mathbf{u}'c'_d\chi'_c}$ will also be very small (WPL). With these assumptions and noting that $\overline{\mathbf{u}c_d} = \overline{\mathbf{u}}\,\overline{c}_d + \overline{\mathbf{u}'c'_d}$, Equation 6.6 becomes

$$\overline{c}_d\overline{\frac{\partial \chi_c}{\partial t}} + \overline{\chi}_c[\overline{\frac{\partial c_d}{\partial t}} + \nabla.\overline{\mathbf{u}c_d}] + \overline{\mathbf{u}c_d}.\nabla\overline{\chi}_c + \nabla.(\overline{c}_d\overline{\mathbf{u}'\chi'_c}) = \overline{S}_c \tag{6.7}$$

Finally, there are no sources or sinks of dry air in the control volume and thus

$$\overline{\frac{\partial c_d}{\partial t}} + \nabla.\overline{\mathbf{u}c_d} = 0 \tag{6.8}$$

Substitution of this expression into Equation 6.7 yields

$$\overline{c}_d \overline{\frac{\partial \chi_c}{\partial t}} + \overline{\mathbf{u}c_d}.\nabla\overline{\chi}_c + \nabla.(\overline{c}_d\overline{\mathbf{u}'\chi_c'}) = \overline{S}_c \tag{6.9}$$

Equation 6.9 states that the source/sink for constituent c equals the sum of: 1) the time rate of change of the mixing ratio χ_c in dry air, 2) the dot product of the mean flux of dry air and the gradient of χ_c at each surface of the volume, and 3) the divergence of the turbulent flux of mixing ratio multiplied by the mean density of dry air. Equation 6.9 is the non-steady, three dimensional version of an expression for the eddy flux derived by WPL for steady, one-dimensional, horizontally homogeneous flows. Equation 6.9 can be shown to be a condensed version of Equation B22 in Massman and Lee (2002) provided we assume that $\nabla.\mathbf{u} = 0$. Paw U et al. (2000) also derived a similar form of the conservation equation. Note that contrary to Equation B22 of Massman and Lee (2002) the time-averaging operator applies the time derivative $\partial\chi_c/\partial t$, not just to χ_c.

3 Non-steady, Three Dimensional Flow

Equations 6.1 and 6.9 strictly refer to an infinitesimal control volume dV while in practice we wish to measure the net exchanges of heat, water vapor and trace constituents between the earth's surface and the atmosphere. As detailed in Finnigan et al. (2003), we need to write the conservation equations for a finite control volume representative of a surface patch of area A and height h of the measuring instruments (Figure 6.1).

Integrating Equation 6.9 horizontally over A and vertically over h we obtain

$$\begin{aligned}
&\int_0^h \int_{x-L}^{x+L} \int_{y-L}^{y+L} \overline{c}_d \overline{\frac{\partial \chi_c}{\partial t}} \mathrm{d}x\mathrm{d}y\mathrm{d}z \\
&+ \int_0^h \int_{x-L}^{x+L} \int_{y-L}^{y+L} \left[\overline{uc_d}\frac{\partial\overline{\chi}_c}{\partial x} + \overline{vc_d}\frac{\partial\overline{\chi}_c}{\partial y} + \overline{wc_d}\frac{\partial\overline{\chi}_c}{\partial z}\right] \mathrm{d}x\mathrm{d}y\mathrm{d}z \\
&+ \int_0^h \int_{x-L}^{x+L} \int_{y-L}^{y+L} \left[\frac{\partial\overline{c}_d\overline{u'\chi_c'}}{\partial x} + \frac{\partial\overline{c}_d\overline{v'\chi_c'}}{\partial y} + \frac{\partial\overline{c}_d\overline{w'\chi_c'}}{\partial z}\right] \mathrm{d}x\mathrm{d}y\mathrm{d}z \\
&= <\overline{S}_c>
\end{aligned} \tag{6.10}$$

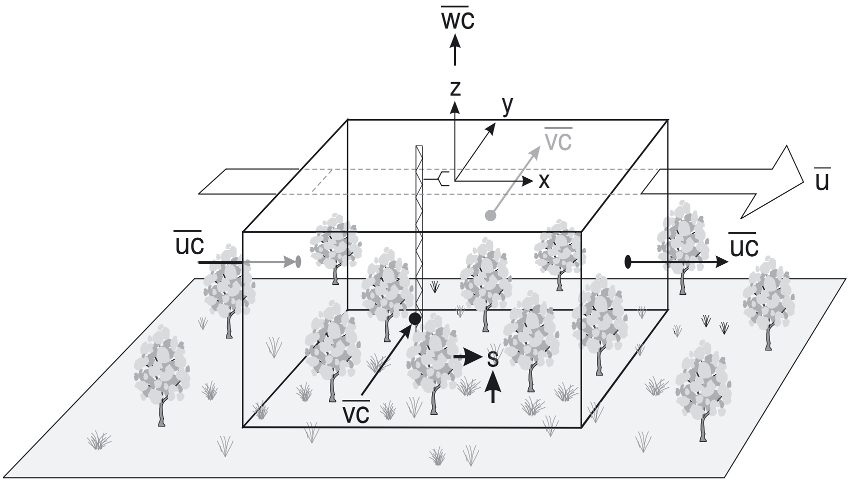

Figure 6.1. Cartesian control volume placed over a vegetated surface.

In writing this equation we have assumed a rectangular Cartesian coordinate frame with the lower boundary of the control volume placed on the ground. The right hand term represents the volume-integral of the source of c between the ground surface and the atmosphere at height h,

$$<\bar{\bar{S}}_c>=\int_0^h\int_{x-L}^{x+L}\int_{y-L}^{y+L}\bar{\bar{S}}_c\mathrm{d}x\mathrm{d}y\mathrm{d}z$$

When measurements are made on a single tower we are unable to measure the spatial averages that appear in Equation 6.10 and we are then obliged to add extra information. The first step usually adopted is to define a coordinate system in which $\bar{v}=\bar{w}=0$ (strictly $\bar{v}\bar{c}_{\bar{d}}=\bar{w}\bar{c}_{\bar{d}}=0$) and where the x-axis is aligned with the mean wind for each averaging period (e. g. McMillen 1988). Methods to define consistent, long-term coordinates have been described by Paw U et al. (2000) and Wilczak et al. (2001) and further discussed by Finnigan (2004) and in Chapter 3. For present purposes, we assume that a suitable coordinate framework has been defined and that it is possible for mean fluxes of dry air to be non-zero through any of the surfaces of the control volume, except at the ground ($\bar{w}\bar{c}_{\bar{d}}|_0=0$). Of course, it is also necessary to satisfy Equation 6.8 as applied to the finite control volume of Figure 6.1. The coordinate system in the subsequent analysis has been aligned with the mean wind direction so that $\bar{v}\bar{c}_{\bar{d}}=0$.

When the divergences of the horizontal eddy fluxes are small compared to the vertical,

$$\frac{\partial \overline{c}_d \overline{u'\chi_c'}}{\partial x}, \frac{\partial \overline{c}_d \overline{v'\chi_c'}}{\partial y} << \frac{\partial \overline{c}_d \overline{w'\chi_c'}}{\partial z} \tag{6.11}$$

then Equation 6.10 becomes

$$\begin{aligned}
&\int_0^h \int_{x-L}^{x+L} \int_{y-L}^{y+L} \overline{c}_d \overline{\frac{\partial \chi_c}{\partial t}} \mathrm{d}x\mathrm{d}y\mathrm{d}z \\
&+ \int_0^h \int_{x-L}^{x+L} \int_{y-L}^{y+L} \overline{uc_d} \frac{\partial \overline{\chi}_c}{\partial x} \mathrm{d}x\mathrm{d}y\mathrm{d}z \\
\int_{x-L}^{x+L} \int_{y-L}^{y+L} \int_0^h &\left[\overline{wc_d} \frac{\partial \overline{\chi}_c}{\partial z} + \frac{\partial \overline{c}_d \overline{w'\chi_c'}}{\partial z} \right] \mathrm{d}z\mathrm{d}x\mathrm{d}y \\
&= < \overline{S}_c >
\end{aligned} \tag{6.12}$$

Equation 6.12 may be approximated by

$$\begin{aligned}
&\int_0^h \int_{x-L}^{x+L} \int_{y-L}^{y+L} \overline{c}_d \overline{\frac{\partial \chi_c}{\partial t}} \mathrm{d}x\mathrm{d}y\mathrm{d}z \\
&+ \int_0^h \int_{y-L}^{y+L} \overline{uc_d} (\overline{\chi}_c|_{+L} - \overline{\chi}_c|_{-L}) \mathrm{d}y\mathrm{d}z \\
\int_{x-L}^{x+L} \int_{y-L}^{y+L} &\left[\overline{wc_d}|_h (\overline{\chi}_c|_h - < \overline{\chi}_c >) + \overline{c}_d \overline{w'\chi_c'}|_h \right] \mathrm{d}x\mathrm{d}y \\
&= < \overline{S}_c >
\end{aligned} \tag{6.13}$$

in which $< \overline{\chi}_c > h = \int_0^h \overline{\chi}_c \mathrm{d}z$. The vertical advection term was approximated using the product rule of integration and the assumption that $\partial \overline{wc_d}/\partial z \simeq \overline{wc_d}|_h/h$ (Lee 1998, Finnigan 1999). This approximation is unnecessary if the variation of $\overline{wc_d}$ and $\partial \overline{\chi_c}/\partial z$ with height are known.

The mean horizontal mass flux of dry air $\overline{uc_d}$ in Equation 6.13 is not normally measured, but as demonstrated below, it is closely approximated by $\overline{u}\,\overline{c}_d$. The mean streamwise velocity is defined as $\overline{u} = \overline{F}_{t,x}/\overline{c} = (\overline{F}_{d,x} + \overline{F}_{v,x} + \overline{F}_{c,x})/(\overline{c}_d + \overline{c}_v + \overline{c}_c)$, where $\overline{F}_{t,x}$ is the total flux of air in the x direction, and $\overline{c}$ is the total mean concentration. The mean horizontal flux of dry air is $\overline{F}_{d,x} = \overline{uc_d}$. Combining these definitions gives

$$\frac{\overline{c}_d \overline{u}}{\overline{c_d u}} = \frac{\overline{c}_d(\overline{F}_{d,x} + \overline{F}_{v,x})}{\overline{c}\,\overline{F}_{d,x}} = \frac{1 + \overline{F}_{v,x}/\overline{F}_{d,x}}{1 + \overline{\chi}_v} \simeq \frac{1 + \overline{\chi}_v}{1 + \overline{\chi}_v} = 1 \tag{6.14}$$

This derivation assumes that the horizontal flux of the trace constituent c, is small compared to dry air and water vapor, and that the ratio of the advective flux of water vapor to that of dry air is equal to the mixing ratio for water vapor. Thus to a close approximation

$$\overline{c_d u} = \overline{c}_d \overline{u} \tag{6.15}$$

and hence the horizontal eddy flux of dry air is small compared to the total horizontal flux.

The problem of estimating $< \overline{S}_c >$ in the presence of horizontal and vertical advection has been addressed recently by Lee (1998), Finnigan (1999), Paw U et al. (2000), Finnigan (1999) and by Massman and Lee (2002). The horizontal flux divergence terms in Equation 6.10 were assumed by Lee (1998) and by Paw U et al. (2000) to be small compared to those in the vertical, but this assumption was shown to be incorrect by Finnigan (1999). He concluded that partial corrections for advection, using the vertical flux divergence terms but neglecting the horizontal terms, were likely to introduce significant error in the estimate of the net exchange between the surface and the atmosphere. Thus both the vertical and horizontal mean flux divergence terms must be considered when calculating the net exchanges of c for air flow over inhomogeneous terrain. Horizontal advection is introduced by inhomogeneity in the flow ($\partial\overline{c_d u}/\partial x \neq 0$) and/or in the source ($\partial\overline{S}_c/\partial x \neq 0 \Rightarrow \partial\overline{\chi}_c/\partial x \neq 0$). Similar considerations apply to vertical advection.

4 Steady, One-dimensional Horizontally Homogeneous Flows

4.1 Fluxes

There is no horizontal advection when the flow is steady and horizontally homogeneous and, because there are no sources of dry air in the control volume, the term $\overline{wc_d}|_h = 0$ in Equation 6.13, i. e. there is no net flux of dry air at height h. This is the key governing constraint used by WPL to develop their theory for correcting eddy covariance measurements for the influence of density fluctuations on trace gas concentration measurements. Under these conditions we can equate the eddy flux density measured at height h to the horizontally averaged source strength, viz.

$$\overline{F}_c =< \overline{c}_d \overline{w'\chi_c'}|_h >=< \overline{S}_c > /A \tag{6.16}$$

where A is the basal area of the control volume. Equation 6.16 shows that the flux density is equal to the product of the mean concentration of dry air and the covariance of vertical velocity and mixing ratio, $\overline{w'\chi_c'}$, measured at height h. Equation 6.16 is identical to that developed by WPL (their Equation 20), except that we have used molar, rather than mass, concentration units to define the mixing ratio. This equation applies to other constituents in the control volume, such as water vapor.

4.2 The vertical velocity of air

Starting from the equation of state $p = cRT$, where p is the total pressure of moist air, R is the ideal gas constant and T is air temperature (°K), we may show that in response to fluctuations in water vapor concentrations, temperature and pressure, fluctuations in the concentration of dry air c_d' are given by

$$c_d' = -c_v' - \overline{c}\left[\frac{T'}{\overline{T}} - \frac{p'}{\overline{p}}\right] \tag{6.17}$$

As discussed above, there is no net flux of dry air through the surfaces of the control volume, and thus in this one-dimensional case

$$\overline{wc_d} = \overline{w}\,\overline{c}_d + \overline{w'c_d'} = 0 \tag{6.18}$$

Combining Equations 6.17 and 6.18, we see that

$$\overline{w} = \frac{1}{\overline{c}_d}\left[\overline{w'c_v'} + \overline{c}(\frac{\overline{w'T'}}{\overline{T}} - \frac{\overline{w'p'}}{\overline{p}})\right] \tag{6.19}$$

where we have only retained terms to first order in the fluctuations. This is a much simplified version of that given by Fuehrer and Freihe (2002). The original derivation by WPL did not include the covariance $\overline{w'p'}$, but using a scale analysis, Sun et al. (1995) argued that the $\overline{w'p'}$ term is unimportant relative to the other terms except when heat fluxes are low and wind speeds are high over aerodynamically rough surfaces. At such times the heat flux itself is small and neglect of $\overline{w'p'}$ introduces only small errors in $\overline{w}$. The covariance $\overline{w'p'}$ is expected to be very small compared to the other two terms when there is no asymmetry in the mean static pressure of upward and downward moving eddies (mean pressure is constant and $\partial p/\partial z \simeq 0$ in the surface boundary layer). This contrasts with the asymmetry in density of air where there are net fluxes of sensible and latent heat. A mean vertical velocity of *moist* air arises whenever there are air density fluctuations induced by non-zero fluxes of water vapor or sensible heat. To a high degree of approximation we may thus write

$$\overline{w} = \frac{1}{\overline{c}_d}\left[\overline{w'c_v'} + \overline{c}\frac{\overline{w'T'}}{\overline{T}}\right] \tag{6.20}$$

WPL also derived an expression for $\overline{w}$ ($\mathrm{m\,s^{-1}}$) in terms of the fluxes of latent heat, $\lambda\overline{E}$, and sensible heat, $\overline{H}$. At typical mid-latitude temperatures and pressures

$$\overline{w} = 10^{-6}(0.54\lambda\overline{E} + 2.80\overline{H}) \tag{6.21}$$

where the energy fluxes are in units of $\mathrm{W\,m^{-2}}$. Under most conditions $\overline{w} < 3\ \mathrm{mm\,s^{-1}}$.

Sun et al. (1995) showed that equating $\rho c_p\overline{w'T'}|_h$ with the H at the surface neglects a component of the H associated with the flux of water vapor that occurs when the temperature of the moisture entering the lower surface of the control volume differs from that leaving the upper surface. This term is generally very small and will be ignored here. Similarly, radiative flux divergence between the surface and the measurement height is also neglected in constructing the energy balance.

5 Practical Considerations

5.1 Fluxes in terms of mixing ratios and concentrations

In developing the above equations it has been assumed that concentrations, mixing ratios and velocities can all be measured as required. When concentrations are measured instead of mixing ratios, the flux of constituent c is written as $\overline{F}_c = \overline{w}\,\overline{c}_c + \overline{w'c'_c}$. Combining this with Equation 6.20 for the mean vertical velocity, WPL obtained

$$\overline{F}_c = \overline{c}_d\overline{w'\chi'_c} = \overline{w'c'_c} + \frac{\overline{c}_c}{\overline{c}_d}\left[\overline{w'c'_v} + \overline{c}\frac{\overline{w'T'}}{\overline{T}}\right] \tag{6.22}$$

The two terms on the right correct the eddy flux for the fluctuations in c due to fluctuations in water vapor concentration and temperature when latent heat or sensible fluxes are non-zero. Note that no such corrections are necessary when mean mixing ratios are used to calculate the eddy flux.

Both forms of Equation 6.22 are useful, depending on whether a close-path or open-path analyzer is used to measure the concentrations of the trace constituent and water vapor. We first examine the use of closed-path analyzers to calculate fluxes and then open-path ones.

5.2 Closed-path analyzers

The mixing ratio form of Equation 6.22 is convenient when closed-path gas analyzers are used, thereby eliminating the need to correct for fluxes of water vapor and sensible heat. Thus while the instrument measures concentrations of water vapor, c_v, and CO_2, c_c, the mixing ratio may be calculated, provided temperature and pressure are also measured simultaneously at the sampling frequency used for water vapor and CO_2 (typically 20 Hz). The mixing ratios for water vapor and CO_2 are given by

$$\chi_w = c_v/(c - c_v), x_c = c_c/(c - c_v) \tag{6.23}$$

where $c = p/RT$ is the total molar concentration in the analyzer chamber at any instant. Pressure fluctuations in the air stream are also taken into account through variations in c.

This approach is attractive since there is no need to assume that all temperature fluctuations have been removed from the signal by the time the air travels from the tubing inlet to the analyzer chamber. It is often assumed that perfect temperature equilibrium is achieved at all frequencies contributing to $\overline{F}_c$, allowing the $\overline{w'T'}$ correction term in Equation 6.22 to be set to zero. However, it is unlikely that all the temperature fluctuations will be eliminated at frequencies $\geq 1/(2\pi t_{av})$, where t_{av} is the averaging period (Leuning and Judd 1996). There will then be some unknown residual covariance between w and T, leading to incorrect estimates of the flux. It is thus recommended that the measured trace gas concentrations be converted to mixing ratio in dry air each instant the gas concentration is measured.

Use of Equation 6.23 assumes that the water vapor and CO_2 concentrations are in phase and that they are attenuated by the same amount as the air travels down the tubing. This assumption is needed, irrespective of the way in which concentrations are expressed and the final eddy flux is calculated. The error in χ_c will be small since both fluctuations and absolute values of $c_v << c$.

Fluctuations in gas concentrations (and hence mixing ratios) are diminished as air flows through the sampling tubing and gas analyzer (Taylor 1954, Philip 1963) and it is thus necessary to apply corrections to the resultant low-pass filtering (Leuning and Moncrieff 1990, Massman 1991, Lenschow and Raupach 1991, Suyker and Verma 1993, Leuning and Judd 1996). The required corrections can be calculated using theory presented in Leuning and Judd (1996, equations 16-19). Further corrections to loss of covariance resulting from the effects of line averaging and spatial separation between the sonic anemometer and the air inlet to can be calculated using the theory presented by Moore (1986), Leuning and Judd (1996), Massman (2000) and Massman and Lee (2002).

Massman in Chapter 7 states that corrections to the calculated flux due to low-pass filtering need to be applied before the WPL corrections are applied. This is true if concentrations and the rightmost form of Equation 6.22 are used to calculate the flux, but only the corrections for low-pass filtering need be applied when the flux is calculated using mixing ratios relative to dry air.

5.3 Open-path gas analyzers

The problem is different for open path systems because we are unable to calculate the mixing ratio point by point as above. We thus have to apply the WPL corrections involving the fluxes of sensible heat and water vapor.

The terms on the right of Equation 6.22 apply when concentrations are measured in situ using an open-path analyzer. In this case the order in which the fluxes are calculated and the WPL corrections are applied is important. The following steps are recommended

- Calculate the sensible heat flux, $\overline{H}$, according to

$$\overline{H} = \overline{\rho} c_p \overline{w'T'} \tag{6.24}$$

  then make corrections for line-averaging along the sonic path length and allow for any separation between the sonic w-axis and the thermometer (e. g., if a separate fine wire or thermocouple is used). Theory presented by Moore (1986), Leuning and Judd (1996) or Massman (2000) may be used to make the required corrections. In Equation 6.24, $\overline{\rho}$ is the mean density of moist air and c_p is the specific heat of air, both in mass units.

  Sensible heat fluxes are calculated using Equation 6.24 when temperature fluctuations are measured independently of the vertical wind speed. Most installations use the sonic virtual temperature, defined as $T_s = T(1 + 0.32\chi_v)$ (Kaimal and Gaynor 1991). Thus after Reynolds averaging we have to a close approximation

$$\overline{w'T'} = \overline{w'[T_s/(1 + 0.32\chi_v)]'} \tag{6.25}$$

  where the higher order terms in χ_v have been omitted.

- Calculate the flux of water vapor using

$$\overline{E} = (1 + \overline{\chi}_v)[\overline{w'c_v'} + (\overline{c}_v/\overline{T})(\overline{H}/\overline{\rho}c_p \qquad 6.26)$$

  The sensible heat flux has already been corrected for loss of covarian n w and T in step 1, so it is only necessary to correct for loss of covariance between w and c_v. We cannot apply a single correction to both $\overline{w'T'}$ and $\overline{w'c_v'}$ because the geometry will generally differ for the instruments used to measure temperature and water vapor.

- Calculate CO_2 flux. WPL showed that the last two terms on the right of Equation 6.22 may be written in terms of the fluxes of water vapor and sensible heat. Thus for CO_2 we have

$$\overline{F}_c = \overline{w'c_c'} + \overline{c}_c \left[\frac{\overline{E}}{\overline{c}} + \frac{\overline{H}}{\overline{\rho} c_p \overline{T}} \right] \tag{6.27}$$

  Both sensible heat flux and evaporation have been corrected for loss of covariance in the previous steps, so it is only necessary to correct for loss of covariance between w and c_c due to line averaging and spatial separation of instruments.

Careful experimental design will reduce the magnitude of the latter corrections (Leuning and Moncrieff 1990, Suyker and Verma 1993, Leuning and Judd 1996, Massman and Lee 2002). Instruments should be placed as close together as possible while minimizing flow distortion around the sonic anemometer. Instruments should also be placed as high as possible above the zero-plane displacement height while still remaining within the internal boundary layer of the surface being studied. Loss of covariance can also occur if the averaging period is not sufficiently long to capture the low-frequency contributions to the covariance (Finnigan et al. 2003). These contributions are likely to be site-specific and some analysis will be necessary to determine an adequate averaging period for each experimental site.

5.4 Advection

Most of the above has concentrated on the corrections to the eddy flux of a trace constituent needed to account for density fluctuations induced by the fluxes of water vapor and latent heat. As Equation 6.13 shows, the eddy flux is only one component of four needed to estimate the source term, and $\overline{F}_c = < \overline{c}_c \overline{w'\chi_c'}|_h > = < \overline{S}_c > /A$ only under the restrictive conditions of steady, horizontally homogeneous flows. It is the experience of ourselves, and many other researchers, that the eddy flux provides a poor estimate of the source term when the air flow is stably stratified which often occurs at night. The advection terms in Equation 6.13 then dominate and it is necessary to devise new theoretical and experimental approaches to estimating $< \overline{S}_c >$ under these conditions. This represents a major challenge for the micrometeorological community. A more thorough discussion of advection can be found in Chapter 10.

6 Conclusions

The expressions for mass conservation are relatively simple when concentrations are expressed as molar mixing ratios relative to dry air

(Equation 6.13). This contrasts with the more complex expressions which arise when absolute concentrations are used (e.g. Paw U et al. 2000, Massman and Lee 2002, Fuehrer and Friehe 2002). The mass conservation equation expressed horizontal and vertical advection in terms of mass fluxes of dry air and gradients in mixing ratio, and requires the covariance of vertical wind speed and mixing ratios for the vertical turbulent eddy fluxes. Equation 6.22 shows that the theory developed by WPL for 1-D flows is then still applicable. The right hand side of Equation 6.22 should be used to calculate the vertical eddy flux density when concentrations are measured *in situ*, and the left hand side when a closed-path gas analyzer is employed. In the latter case, measured concentrations should be converted to mixing ratio at the sampling frequency used for eddy covariance. Thus water vapor concentration, temperature and pressure within the gas analysis chamber must be measured simultaneously to calculate the mixing ratio χ_c.

7 References

Baldocchi, D., Finnigan, J. J., Wilson, K., Paw U, K. T., and Falge, E.: 2000, 'On measuring net ecosystem carbon exchange over tall vegetation on complex terrain', *Bound.-Layer Meteorol.* **96**, 257-291.

Baldocchi, D. D., Falge, E., Gu, L. H., Olson, R., Hollinger, D., Running, S., Anthoni, P., Bernhofer, Ch., Davis, K., Evans, R., Fuentes, J., Goldstein, A., Katul, G., Law, B., Lee, X.-L., Malhi, Y., Meyers, T., Munger, W., Oechel, W., Paw U, K. T., Pilegaard, K., Schmid, H. P., Valentini, R., Verma, S., Vesala, T., Wilson, K., and Wofsy, S.: 2001, 'FLUXNET: A new tool to study the temporal and spatial variability of ecosystem-scale carbon dioxide, water vapor and energy flux densities', *Bull. Am. Meteorol. Soc.* **82**, 2415-2434.

Finnigan, J. J.: 2004, 'A re-evaluation of long-term flux measurement techniques. Part 2: Coordinate systems', *Bound.-Layer Meteorol.*, in review.

Finnigan, J. J.: 1999, 'A comment on the paper by Lee (1988): "On micrometeorological observations of surface-air exchange over tall vegetation"', *Agric. . Meteorol.* **97**, 55-64.

Finnigan, J. J., Clements, R., Malhi, Y., Leuning, R., and Cleugh, H. A.: 2003, 'A re-evaluation of long-term flux measurement techniques. Part 1: Averaging and coordinate rotation', *Bound.-Layer Meteorol.* **107**, 1-48.

Fuerher, P. L. and Friehe, C. A.: 2002, 'Flux corrections revisited', *Bound.-Layer Meteorol.* **102**, 415-457.

Kaimal, J. C. and Gaynor, J. E.: 1991, 'Another look at sonic thermometry', *Bound.-Layer Meteorol.* **56**, 401-410.

Kramm, G., Dlugi, R., and Lenschow, D. H.: 1995, 'A re-evaluation of the Webb correction using density-weighted averages', *J. Hydrol.* **166**, 283-292.

Lee, X.: 1998, 'On micrometeorological observations of surface-air exchanges over tall vegetation', *Agric. For. Meteorol.* **91**, 39-49.

Lenschow, D. H. and Raupach, M. R.: 1991, 'The attenuation of fluctuations in scalar concentrations through sampling tubes', *J. Geophys. Res.* **96**, 5259-5268.

Leuning, R. and Judd, M. J.: 1996, 'The relative merits of open- and closed-path analyzers for measurements of eddy fluxes', *Global Change Biology* **2**, 241-253.

Leuning, R. and Moncrieff, J.: 1990, 'Eddy-covariance CO_2 flux measurements using open-path and closed-path CO_2 analyzers - Corrections for analyzer water vapor sensitivity and damping of fluctuations in air sampling tubes', *Bound.-Layer Meteorol.* **53**, 63-76.

Massman, W. J.: 1991, 'The attenuation of concentration fluctuations in turbulent-flow through a tube', *J. Geophys. Res.* **96**, 5269-5273.

Massman, W. J.: 2000, 'A simple method for estimating frequency response corrections for eddy covariance systems', *Agric. For. Meteorol.* **104**, 185-198.

Massman, W. J. and Lee, X.: 2002, 'Eddy covariance flux corrections and uncertainties in long-term studies of carbon and energy exchanges', *Agric. For. Meteorol.* **113**, 121-144.

McMillen, R. T.: 1988, 'An eddy correlation technique with extended applicability to non-simple terrain', *Bound.-Layer Meteorol.* **43**, 231-245.

Moore, C. J.: 1986, 'Frequency response corrections for eddy correlation systems', *Bound.-Layer Meteorol.* **37**, 17-35.

Paw U, K. T., Baldocchi, D. D., Meyers, T. P., and Wilson, K. B.: 2000, 'Correction of eddy-covariance measurements incorporating both advective effects and density fluxes', *Bound.-Layer Meteorol.* **97**, 487-511.

Philip, J. R.: 1963, 'The damping of a fluctuating concentration by continuous sampling through a tube', *Australian J. Physics* **16**, 454-463.

Sun, J. Esbensen, S. K. and Mahrt, L.: 1995, 'Estimation of surface heat flux', *J. Atmos. Sci.* **52**, 3162-3171.

Suyker, A. E. and Verma, S. B.: 1993, 'Eddy correlation measurement of CO_2 flux using a closed-path sensor: Theory and field tests against open-path sensor', *Bound.-Layer Meteorol.* **64**, 391-407.

Ta

Proc. R. Soc. London **A223**, 446-468.

Webb, E. K., Pearman, G. I., and Leuning, R.: 1980, 'Correction of flux measurements for density effects due to heat and water vapor transfer', *Quart. J. R. Meteorol. Soc.* **106**, 85-100.

Wilczak, J. M., Oncley, S. P., and Stage, S. A.: 2001, 'Sonic anemometer tilt correction algorithms', *Bound.-Layer Meteorol.* **99**, 127-150.