8

Concluding remarks

The problem of asymptotic regulation of the output of a dynamical system, which includes both tracking and disturbance rejection problems, plays an important role in control theory. A particular case, when the reference signals and/or disturbances are generated by an autonomous system of differential equations, is the output regulation problem. This problem has been systematically studied in this book. Our treatment of the output regulation problem is based on the notion of convergent systems. In the development of this approach we have passed several stages.

Convergent systems. We have extended and elaborated the notions of convergent systems originally developed by B.P. Demidovich. We have introduced the notions of the uniform and exponential convergence, the UBSS property, and the input-to-state convergence property. Then we studied various properties of convergent systems. It appears that convergent systems, although nonlinear, have rather simple dynamics and enjoy many stability properties comparable to those of asymptotically stable linear systems. This makes them convenient to deal with. Finally, we have proposed sufficient conditions for various convergence properties for systems with smooth and nonsmooth right-hand sides. These sufficient conditions and properties of convergent systems serve as tools in the subsequent treatment of the output regulation problem. Moreover, they can be used for other control problems as well.

The uniform output regulation problem. Having developed a mathematical apparatus for convergent systems, we have formulated the uniform output regulation problem. This is a new problem formulation for the output regulation problem based on the notion of convergent systems. In this problem formulation one needs to find a controller such that for any input generated by the exosystem, the corresponding closed-loop system has a unique (globally) uniformly asymptotically stable steady-state solution and the regulated output tends to zero along all solutions of the closed-loop system. We have formulated the global, local, as well as robust variants of the uniform output regulation problem. This new problem setting includes as particular cases the output regulation problem for linear systems and the conventional local output regulation problem for nonlinear systems. Moreover, it has several advantages over other existing problem formulations. It allows one to deal with exosystems having complex dynamics, e.g., exosystems with a (chaotic) attractor with an unbounded domain of attraction. Up to now most of the results on the output regulation problem dealt only with exosystems having relatively simple dynamics, for example, with linear harmonic oscillators. Another advantage of this new problem formulation is that it allows one to treat the local and global variants of the uniform output regulation problem in a unified way. As becomes clear from the solvability analysis of the global uniform output regulation problem, many of the known controllers solving the global output regulation problem in some other problem settings in fact solve the global uniform output regulation problem.

Solvability analysis. One of the main advantages of the chosen problem setting for the uniform output regulation problem is that it allows one to obtain relatively simple results on the solvability of the problem. For the global, global robust, and local variants of the uniform output regulation problem, we have provided necessary and sufficient conditions for the solvability of these problems as well as results on the characterization of all controllers solving these problems. These results extend the solvability results for the conventional local output regulation problem, which are based on the center manifold theorem. The solvability analysis of the uniform output regulation problem is based on certain invariant manifold theorems. They serve as nonlocal counterparts of the center manifold theorem. These invariant manifold theorems, although obtained in the scope of the output regulation problem, can be applied in other fields of systems and control theory as well. For example, they can be used for the analysis of synchronization phenomena, the computation of periodic solutions of nonlinear systems excited by harmonic inputs, and for the performance analysis of nonlinear convergent systems.

Controller design. The analysis of the global uniform output regulation problem provides necessary and sufficient conditions under which a controller solves the problem. How to design a controller satisfying these conditions is a separate problem. We have addressed this problem for several classes of nonlinear systems and provided several results on controller design for the global uniform output regulation problem. One of these controller designs is based on the notions of quadratic stabilizability and detectability, which extend the conventional notions of stabilizability and detectability from linear systems theory to the case of nonlinear systems. The controller design based on these notions extends known controllers solving the linear and the local nonlinear output regulation problems to the case of the global uniform output regulation problem for nonlinear systems. For the case of a Lur'e system with a nonlinearity having a bounded derivative and an exosystem being a linear harmonic oscillator, feasibility conditions for such controller design can be easily verified by checking the feasibility of certain LMIs. Moreover, for this class of systems and exosystems we provide a robust controller design, which copes not only with uncertainties in the system parameters, but also with an uncertain nonlinearity from a class of nonlinearities with a given bound on their derivatives. The controller design results obtained at this stage allow us to solve the global uniform output regulation problem for new classes of nonlinear systems.

Convergence region estimates. If a solution to the global uniform output regulation problem cannot be found, it can still be possible to find a controller that solves the corresponding local output regulation problem. The resulting controller solves the output regulation problem for initial conditions of the closed-loop system and the exosystem lying in *some* neighborhood of the origin. In this book we have presented estimation results that, given a controller solving the local output regulation problem, provide estimates of this neighborhood of admissible initial conditions. These results are obtained for both the exact and the approximate local output regulation problem. The proposed estimation results enhance the applicability of the controller design procedures for the nonlinear local output regulation problem. As in the rest of the book, the notion of convergence plays a central role in these estimation results.

Experimental case study. To check the applicability of controllers from the nonlinear output regulation theory in practice, we have performed an experimental case study for the TORA system. For this system we have considered a disturbance rejection problem, which is a particular case of the local output regulation problem. We have designed a controller solving this problem and checked its performance in experiments. To this end, an experimental setup for the TORA system has been built and the proposed controller has been implemented in this setup. Despite the uncertainties and several parasitic effects, such as residual uncompensated friction, backlash, and a residual cogging force present in the system, the proposed controller has demonstrated good performance in the experiments by achieving approximate output regulation (the regulated output converges to small values). The residual regulation error is due to modeling uncertainties, which are inevitable in practice. This is one of the first experimental works in the field of nonlinear output regulation.

The results presented in this book can be extended in several ways. Below we briefly review some possible extensions.

In this book the uniform output regulation problem has been considered for systems modeled by ordinary differential equations (ODEs). At the same time, many practical systems cannot be modeled by ODEs and may require a model in the form of integral equations or partial differential equations (PDEs). The analysis and controller design methods for the output regulation problem for systems given by integral equations or PDEs is a possible step for further research. Another direction of research can be related to the output regulation problem for systems given in the form of differential equations with nonsmooth and discontinuous right-hand sides. Some preliminary results in this direction have been presented in [67].

The problem of controller design for the uniform output regulation problem requires a lot of further research, since the class of nonlinear systems for which the available controller design methods apply is rather limited. One of the main problems is designing controllers that make the corresponding closed-loop system globally uniformly convergent. This interesting problem has connections to different areas of nonlinear control theory. Controller design methods for convergent systems can be beneficial for both tracking and disturbance rejection problems.

In the problem of estimating the set of admissible initial conditions for the local output regulation problem there are several unanswered questions. The estimation procedures presented in this book depend on several parameters, which can be chosen in many ways. How to choose these parameters to obtain the largest (in some sense) estimates is still an open question. Another question is how to choose controller parameters to increase the set of admissible initial conditions.

For a given system and exosystem, the output regulation problem can be solved (if it is solvable) by many controllers. All these controllers achieve the control goal of regulating the output of the system, but the performance of the closed-loop system depends on the particular controller. In this respect, we face the problem of performance analysis for nonlinear systems. This problem has been thoroughly investigated for linear systems, but for nonlinear systems there are more questions than answers, starting with the question of how to quantify the performance of a nonlinear system. A possible approach to tackling the performance analysis problem for nonlinear convergent systems can be based on the invariant manifold theorems presented in this book. These theorems allow to extend the Bode plot defined for linear systems to the case of nonlinear convergent systems. The extended Bode plot can be applied for performance analysis of nonlinear convergent systems. This fact opens an interesting research direction in nonlinear (control) systems theory.

The notion of convergent systems seems to be very useful in many areas of systems and control theory. The research on properties of convergent systems, analysis, and design tools for convergent systems has started relatively recently. At the moment, there is a need for new design and analysis tools for convergent systems.

At the end of the book we can conclude that the approach to the output regulation problem based on the notion of convergent systems appears to be very effective. Moreover, the results presented in this book can be applied not only to the output regulation problem, but to other problems in systems and control theory as well.