5 Nonlinear Dynamics of Induced Compton and Undulator Processes

In this chapter we will consider the interaction of charged particles with superstrong radiation fields of relativistic intensities in induced coherent processes in vacuum where there is no restriction on the field intensity taking place at the induced Cherenkov interaction in dielectriclike media. Those are the induced Compton and undulator processes.

In the presence of a second wave of different frequency, the Compton scattering, as well as spontaneous undulator radiation in the external EM wave field acquire induced character. Because of its coherent nature (as the Cherenkov one) these induced processes have the same peculiarity and, consequently, the nonlinear interaction of charged particles with the mentioned fields leads to analogous threshold phenomena of particle "reflection" and capture by the plane EM waves in vacuum.

On the other hand, it is clear that the second wave in the induced Compton process or the undulator field perform the role of the third body for the real radiation/absorption of photons by the free electrons in vacuum. Hence, irrespective of revelation of new phenomena the consideration of nonlinear dynamics of induced Compton and undulator processes in current superstrong laser fields is of great interest, especially from the point of view of FEL and laser accelerators. Further, the significance of the undulator (wiggler) is great enough as the unique version of the current FEL and expected X-ray laser due to its large coherent length and effective power of the static magnetic field for relativistic particles.

To achieve relatively large coherent lengths in the induced Compton process we will consider the case of counterpropagating waves.

5.1 Interaction of Charged Particles with Superstrong Counterpropagating Waves of Different Frequencies

Consider the classical dynamics of a charged particle at the interaction with two counterpropagating (along the axis OX) plane EM waves having arbitrary electric field strengths $\mathbf{E}_1\left(t-\frac{x}{c}\right)$ and $\mathbf{E}_2\left(t+\frac{x}{c}\right)$ in vacuum. The relativistic equation of motion in components is written as

$$\frac{dp_x}{dt} = \frac{e}{c}\left(\mathbf{vE}_1 - \mathbf{vE}_2\right), \tag{5.1}$$

$$\frac{dp_y}{dt} = e\left(1 - \frac{\mathrm{v}_x}{c}\right) E_{1y} + e\left(1 + \frac{\mathrm{v}_x}{c}\right) E_{2y},$$

$$\frac{dp_z}{dt} = e\left(1 - \frac{\mathrm{v}_x}{c}\right) E_{1z} + e\left(1 + \frac{\mathrm{v}_x}{c}\right) E_{2z}. \tag{5.2}$$

This set of equations allows exact solution when the particle initial velocity is directed along the axis OX and the waves are monochromatic with circular polarization:

$$\mathbf{E}_1\left(x, t\right) = \left\{0, E_1 \cos \omega_1 \left(t - \frac{x}{c}\right), E_1 \sin \omega_1 \left(t - \frac{x}{c}\right)\right\},$$

$$\mathbf{E}_2\left(x, t\right) = \left\{0, E_2 \cos \omega_2 \left(t + \frac{x}{c}\right), E_2 \sin \omega_2 \left(t + \frac{x}{c}\right)\right\}. \tag{5.3}$$

From Eq. (5.2) in the field (5.3) we obtain

$$p_y = \frac{eE_1}{\omega_1} \sin \omega_1 \left(t - \frac{x}{c}\right) + \frac{eE_2}{\omega_2} \sin \omega_2 \left(t + \frac{x}{c}\right),$$

$$p_z = -\frac{eE_1}{\omega_1} \cos \omega_1 \left(t - \frac{x}{c}\right) - \frac{eE_2}{\omega_2} \cos \omega_2 \left(t + \frac{x}{c}\right) \tag{5.4}$$

(the waves are turned on and turned off adiabatically at $t \to \mp\infty$).

For the integration of Eq. (5.1) we will use the equation for the particle energy exchange in the field

$$\frac{d\mathcal{E}}{dt} = e\left(\mathbf{vE}_1 + \mathbf{vE}_2\right). \tag{5.5}$$

Thus, defining the particle transverse velocity in the field by Eqs. (5.4), from Eqs. (5.1) and (5.5) we obtain the following integral of motion in the induced Compton process:

$$\mathcal{E} - c\frac{\omega_1 - \omega_2}{\omega_1 + \omega_2} p_x = \mathrm{const.} \tag{5.6}$$

The latter together with Eq. (5.4) determines the particle energy in the field

$$\mathcal{E} = \frac{\mathcal{E}_0}{n_1^2 - 1}\left\{n_1^2\left(1 - \frac{\mathrm{v}_0}{cn_1}\right) \mp \left[\left(1 - n_1\frac{\mathrm{v}_0}{c}\right)^2 - \left(n_1^2 - 1\right)\left(\frac{mc^2}{\mathcal{E}_0}\right)^2\right.\right.$$

$$\times \left[\xi_1^2 + \xi_2^2 + 2\xi_1\xi_2 \cos\left(\omega_1 - \omega_2\right)\left(t - n_1 \frac{x}{c}\right)\right]\Bigg]^{1/2}\Bigg\}. \tag{5.7}$$

The parameter n_1 included in Eq. (5.7) is

$$n_1 = \frac{\omega_1 + \omega_2}{|\omega_1 - \omega_2|}, \tag{5.8}$$

and the parameters $\xi_{1,2} \equiv eE_{1,2}/mc\omega_{1,2}$.

As is seen from Eq. (5.7) due to the effective interaction of the particle with the counterpropagating waves a slowed traveling wave in vacuum arises. The parameter n_1 denotes the refractive index of this interference wave and since $n_1 > 1$ (see Eq. (5.8)) the phase velocity of the effective traveling wave $\mathrm{v}_{ph} = c/n_1 < c$. Then the expression under the root in Eq. (5.7) evidences the peculiarity in the interaction dynamics like the induced Cherenkov one that causes the analogous threshold phenomena of particle "reflection" and capture by the interference wave in the induced Compton process. Hence, omitting the same procedure related to bypass of the multivalence and complexity of Eq. (5.7), which has been made in detail for the analogous expression in the Cherenkov process, we will present the final results for particle "reflection" and capture by the effective interference wave in the induced Compton process. The threshold value of the "reflection" phenomenon or the critical field for nonlinear Compton resonance is

$$\xi_{cr}\left(\omega_{1,2}\right) \equiv \left(\xi_1 + \xi_2\right)_{cr} = \frac{\mathcal{E}_0}{mc^2} \frac{\left|\omega_1\left(1 - \frac{\mathrm{v}_0}{c}\right) - \omega_2\left(1 + \frac{\mathrm{v}_0}{c}\right)\right|}{2\sqrt{\omega_1\omega_2}}. \tag{5.9}$$

If one knows the longitudinal velocity v_x of the particle in the field, then it is easy to see that $\xi_{cr}\left(\omega_{1,2}\right)$ is the value of the total intensity of counterpropagating waves at which v_x becomes equal to the phase velocity of the effective interference wave: $\mathrm{v}_x = \mathrm{v}_{ph} = c/n_1$ irrespective of the magnitude of particle initial velocity v_0. The latter is the condition of coherency of induced Compton process

$$\omega_1\left(1 - \frac{\mathrm{v}_x}{c}\right) = \omega_2\left(1 + \frac{\mathrm{v}_x}{c}\right). \tag{5.10}$$

Under condition (5.10) the nonlinear resonance in the field of counterpropagating waves of different frequencies occurs and because of induced Compton radiation/absorption the particle velocity becomes smaller or larger than the phase velocity of the interference wave and the particle leaves the slowed effective wave. In the rest frame of the latter the particle swoops on the motionless barrier (if $\xi_1 + \xi_2 > \xi_{cr}\left(\omega_{1,2}\right)$) and the elastic reflection occurs. In the laboratory frame it corresponds to inelastic "reflection" and from Eq. (5.7) for particle energy after the "reflection" ($\xi_{1,2} \to 0$ adiabatically at $t \to +\infty$)

we have

$$\mathcal{E} = \mathcal{E}_0 \frac{\omega_1^2 \left(1 - \frac{\mathrm{v}_0}{c}\right) + \omega_2^2 \left(1 + \frac{\mathrm{v}_0}{c}\right)}{2\omega_1\omega_2}. \tag{5.11}$$

From this equation it follows that the energy of the particle with the initial velocity $\mathrm{v}_0 = c\left|\omega_1 - \omega_2\right| / \left(\omega_1 + \omega_2\right)$ corresponding to the resonance value of the induced Compton process does not change after the interaction ($\mathcal{E} = \mathcal{E}_0$). For such particle $\xi_{cr}\left(\omega_{1,2}\right) = 0$, i.e., it cannot enter the field: $\xi_1 = \xi_2 = 0$. The particle with the initial velocity $\mathrm{v}_0 > c\left|\omega_1 - \omega_2\right| / \left(\omega_1 + \omega_2\right)$ after the "reflection" is decelerated, while at $\mathrm{v}_0 < c\left|\omega_1 - \omega_2\right| / \left(\omega_1 + \omega_2\right)$ it is accelerated because of direct and inverse induced Compton processes. At the acceleration the particle absorbs photons from the wave of frequency ω_1 and coherently radiates into the wave of frequency ω_2 if $\omega_1 > \omega_2$ and at the deceleration the inverse process takes place. Hence, at the particle acceleration the amplification of the wave of a smaller frequency holds, while at the deceleration the wave of a larger frequency is amplified.

In the case of $\omega_1 = \omega_2 \equiv \omega$ the refractive index of the interference wave $n_1 = \infty$ and nonlinear interaction of the particle with the strong standing wave occurs. It is evident that in this case the process is elastic: $\mathcal{E} = \mathcal{E}_0 = \mathrm{const}$ (see Eq. (5.11)) and for the longitudinal momentum of the particle in the field we have

$$p_x = \pm\sqrt{p_0^2 - m^2c^2\left(\xi_1^2 + \xi_2^2 + 2\xi_1\xi_2\cos\frac{2\omega}{c}x\right)}. \tag{5.12}$$

From this equation it is seen that at $\xi_1 + \xi_2 > \xi_{cr}(\omega) = \left|p_0\right|/mc$ the standing wave becomes a potential barrier for the particle and elastic reflection occurs: the root changes its sign and $p_x = -p_0$ (if $\xi_1 + \xi_2 < \xi_{cr}(\omega)$ we have $p_x = p_0$).

Consider now the nonlinear dynamics of a particle with the arbitrary direction of velocity $\mathbf{v}_0$ initially situated in the field of counterpropagating waves (internal particle). It is clear that at the wave intensities $\xi_1 + \xi_2 > \xi_{cr}\left(\omega_{1,2}\right)$ when the "reflection" of an external particle from the slowed traveling wave holds, an internal particle under the specified conditions may be captured by the such slowed wave. Consequently, one needs to define the conditions for the particle capture by the effective field in the induced Compton process.

Let a particle with velocity $\mathbf{v}_0$ be situated in the initial phases $\phi_{10} = \omega_1(t_0 - x_0/c)$ and $\phi_{20} = \omega_2(t_0 + x_0/c)$ of linearly polarized along the axis OY counterpropagating waves (in Eq. (5.3) $E_{1z} = E_{2z} = 0$, so the coordinate z is free and one can assume $\mathrm{v}_{0z} = 0$). The solution of Eqs. (5.1) and (5.2) under these initial conditions for the particle momentum in the field gives

$$p_x = p_{0x} + \frac{n_1^2}{n_1^2 - 1}\frac{\mathcal{E}_0}{c}\left\{1 - n_1\frac{\mathrm{v}_{0x}}{c} \mp \left[\left(1 - n_1\frac{\mathrm{v}_{0x}}{c}\right)^2\right.\right.$$

$$-\left(n_1^2-1\right)\left(\frac{mc^2}{\mathcal{E}_0}\right)^2\left[\frac{1}{2}\left(\xi_1^2+\xi_2^2\right)+\left(\xi_1\sin\phi_{10}+\xi_2\sin\phi_{20}\right)\left(\xi_1\sin\phi_{10}\right.\right.$$

$$\left.\left.\left.+\xi_2\sin\phi_{20}-2\frac{P_{0y}}{mc}\right)+\xi_1\xi_2\cos\left(\phi_1-\phi_2\right)\right]\right]^{1/2}\Bigg\}, \qquad (5.13)$$

$$p_y=p_{0y}+mc\xi_1\left(\sin\phi_1-\sin\phi_{10}\right)+mc\xi_2\left(\sin\phi_2-\sin\phi_{20}\right), \qquad (5.14)$$

where

$$\phi_1-\phi_2=\left(\omega_1-\omega_2\right)\left(t-n_1\frac{x}{c}\right).$$

In the derivation of Eq. (5.13) the averaging over fast oscillations of separate waves with respect to the interference wave (in the intrinsic frame of which only a static magnetic field acts on the particle) in Eqs. (5.1) and (5.5) has been made. Physically it corresponds to time averaging of noncoherent interaction with separate waves in relation to coherent interaction due to induced Compton resonance. In this approximation the integral of motion (5.6) remains applicable and with Eq. (5.13) it determines the energy of the particle at the coherent interaction with the counterpropagating waves of different frequencies.

The equilibrated phases for the particle capture in this process correspond to extrema of the interference wave and the motion of the particle is stable in the phases

$$\left(\phi_1-\phi_2\right)_s=\left(\omega_1-\omega_2\right)\left(t-n_1\frac{x}{c}\right)_s=\pi\left(2k+1\right); \quad k=0,\pm1,\ldots. \qquad (5.15)$$

Equation (5.15) shows that the particle situated in the equilibrated phases moves with the velocity

$$\mathrm{v}_{xs}=c\left(\omega_1-\omega_2\right)/\left(\omega_1+\omega_2\right).$$

Let the particle initial longitudinal velocity be equilibrated: $\mathrm{v}_{0x}=\mathrm{v}_{xs}$. If $p_{0y}=0$ as well, then the analysis of Eq. (5.13) shows that the capture of such particle is possible at $\xi_1=\xi_2$ ($eE_1/\omega_1=eE_2/\omega_2$, i.e., the waves should transfer to the particle equal momenta) and $\left(\phi_1-\phi_2\right)_0=\pi\left(2k+1\right)=\left(\phi_1-\phi_2\right)_s$. From Eq. (5.14) for equilibrated transverse momentum in this case we have $p_{ys}=p_{0y}=0$. If $\mathrm{v}_{0x}=\mathrm{v}_{xs}+\triangle\mathrm{v}$ and $p_{0y}=0$, then we have the following condition for the particle capture:

$$\left|\triangle\mathrm{v}\right|<\frac{c}{n_1}\frac{mc^2}{\mathcal{E}_0}\xi\sqrt{\left(n_1^2-1\right)\left[2+\left(\sin\phi_{10}+\sin\phi_{20}\right)^2\right]}, \qquad (5.16)$$

from which one can define the tolerance for divergences of initial phases and velocity of a nonequilibrium particle. On the other hand, condition (5.16) defines the threshold value of the wave intensities for the capture of a nonequilibrium particle, which coincides with the critical intensity for the "reflection" of an external particle (5.9) at $\xi_1 = \xi_2 \equiv \xi$ and $\phi_{10} = \phi_{20} = 0$ (coefficient $\sqrt{2}$ arises because of different polarization of the waves).

Now let $\mathrm{v}_{0x} = \mathrm{v}_{xs}$ but $p_{0y} \neq 0$. If at that $(\phi_1 - \phi_2)_0 \neq \pi\,(2k+1)$, then the motion of the particle will be stable at the condition

$$p_{0y}\,(\sin\phi_{10} + \sin\phi_{20}) > 0; \quad \frac{|p_{0y}|}{mc\xi} > 1. \tag{5.17}$$

The condition for the capture in this case is $|p_{0y}|\,/mc\xi < 3/2$, which with the condition of stability (5.17) strictly restricts the transverse momentum of the particle. Meanwhile the conditions of stability and capture in the minimums of the interference wave $(\phi_1 - \phi_2)_0 = \pi\,(2k+1)$ are automatically satisfied. Hence, these phases are equilibrated at the arbitrary transverse momentum of the particle $(p_{0y} = p_{ys})$.

If the particle initial velocity differs from the equilibrated one $(\mathrm{v}_{0x} \neq \mathrm{v}_{xs})$ and $p_{0y} \neq 0$, the tolerance for the capture of a nonequilibrium particle is defined analogously to condition (5.16).

5.2 Interaction of Charged Particles with Superstrong Wave in a Wiggler

Consider the nonlinear dynamics of a charged particle at the interaction with a strong EM wave in a magnetic undulator. Let a particle with an initial velocity $\mathrm{v}_0 = \mathrm{v}_{0x}$ enter into a magnetic undulator with circularly polarized field

$$\mathbf{H}(x) = \left\{0, -H\cos\frac{2\pi}{l}x, H\sin\frac{2\pi}{l}x\right\} \tag{5.18}$$

(l is the space period or step of an undulator) along the axis of which propagates a plane monochromatic EM wave of circular polarization with the electric field strength

$$\mathbf{E}(x,t) = \left\{0, E_0\sin\omega_0(t - \frac{x}{c}), E_0\cos\omega_0(t - \frac{x}{c})\right\}. \tag{5.19}$$

The equation of motion of the particle in the fields (5.18) and (5.19) in components is written as

$$\frac{dp_x}{dt} = \frac{e}{c}E_0\left[\mathrm{v}_y\sin\omega_0(t - \frac{x}{c}) + \mathrm{v}_z\cos\omega_0(t - \frac{x}{c})\right]$$

$$+\frac{e}{c}H\left[\mathrm{v}_y \sin\frac{2\pi}{l}x + \mathrm{v}_z \cos\frac{2\pi}{l}x\right], \tag{5.20}$$

$$\frac{dp_y}{dt} = eE_0\left(1-\frac{\mathrm{v}_x}{c}\right)\sin\omega_0(t-\frac{x}{c}) - e\frac{\mathrm{v}_x}{c}H\sin\frac{2\pi}{l}x,$$

$$\frac{dp_z}{dt} = eE_0\left(1-\frac{\mathrm{v}_x}{c}\right)\cos\omega_0(t-\frac{x}{c}) - e\frac{\mathrm{v}_x}{c}H\cos\frac{2\pi}{l}x. \tag{5.21}$$

Integration of Eqs. (5.21) under the assumed initial conditions (at $t = -\infty$ the particle has only longitudinal velocity, i.e., $p_{0y} = p_{0z} = 0$) gives

$$p_y = -\frac{eE_0}{\omega_0}\cos\omega_0(t-\frac{x}{c}) + \frac{elH}{2\pi c}\cos\frac{2\pi}{l}x,$$

$$p_z = \frac{eE_0}{\omega_0}\sin\omega_0(t-\frac{x}{c}) - \frac{elH}{2\pi c}\sin\frac{2\pi}{l}x. \tag{5.22}$$

The integration of Eq. (5.20) is made analogously to the integration of Eq. (5.1). Using the equation for the particle energy exchange in the field

$$\frac{d\mathcal{E}}{dt} = eE_0\left[\mathrm{v}_y\sin\omega_0(t-\frac{x}{c}) + \mathrm{v}_z\cos\omega_0(t-\frac{x}{c})\right], \tag{5.23}$$

with the help of Eqs. (5.1), (5.22), and (5.23) we obtain the integral of motion in the induced undulator process

$$\mathcal{E} - \frac{c}{1+\frac{\lambda}{l}}p_x = \mathrm{const.} \tag{5.24}$$

Equations (5.22) and (5.24) determine the particle energy

$$\mathcal{E} = \frac{\mathcal{E}_0}{n_2^2-1}\left\{n_2^2\left(1-\frac{\mathrm{v}_0}{cn_2}\right) \mp \left[\left(1-n_2\frac{\mathrm{v}_0}{c}\right)^2 - \left(n_2^2-1\right)\left(\frac{mc^2}{\mathcal{E}_0}\right)^2\right.\right.$$

$$\left.\left.\times\left[\xi_0^2+\xi_H^2-2\xi_0\xi_H\cos\omega_0(t-n_2\frac{x}{c})\right]\right]^{1/2}\right\} \tag{5.25}$$

in the field of a strong EM wave in the magnetic undulator, which is characterized by relativistic parameter

$$\xi_H = \frac{elH}{2\pi mc^2} \tag{5.26}$$

(for large magnitudes of undulator field strength H and space period l when $\xi_H > 1$ such undulator is called a wiggler).

From Eq. (5.25) it follows that at the particle–wave nonlinear resonance interaction in the undulator an effective slowed traveling wave is formed as in the induced Compton process. The parameter

$$n_2 = 1 + \frac{\lambda}{l} \tag{5.27}$$

is the refractive index of this slowed wave, which causes the analogous threshold phenomenon of particle "reflection"% in the induced undulator process. The effective critical field at which the nonlinear resonance and then the particle "reflection" take place in the undulator, is

$$\xi_{cr}\left(\frac{\lambda}{l}\right) \equiv (\xi_0 + \xi_H)_{cr} = \frac{\left|1 - \left(1 + \frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}\right|}{\sqrt{\frac{2\lambda}{l}\left(1 + \frac{\lambda}{2l}\right)}} \frac{\mathcal{E}_0}{mc^2}. \tag{5.28}$$

At this value of the resulting field the longitudinal velocity of the particle v_x reaches the resonant value in the field at which the condition of coherency in the undulator

$$\frac{2\pi}{l}\mathrm{v}_x = \omega_0\left(1 - \frac{\mathrm{v}_x}{c}\right) \tag{5.29}$$

is satisfied. The latter has a simple physical explanation in the intrinsic frame of the particle. In this frame of reference the static magnetic field (5.18) becomes a traveling EM wave with the frequency

$$\omega = \frac{2\pi}{l}\frac{\mathrm{v}_x}{\sqrt{1 - \frac{\mathrm{v}_x^2}{c^2}}}$$

and phase velocity $\mathrm{v}_{ph} = \mathrm{v}_x$. For coherent interaction process this frequency must coincide with the Doppler-shifted frequency of stimulated wave.

The energy of the particle after the "reflection" (in Eq. (5.25) $\xi_0 = \xi_H = 0$ at the sign "+" before the root) is

$$\mathcal{E} = \mathcal{E}_0\left[1 + \frac{1 - \left(1 + \frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}}{\frac{\lambda}{l}\left(1 + \frac{\lambda}{2l}\right)}\right]. \tag{5.30}$$

From this equation it follows that the particle with the initial velocity $\mathrm{v}_0 < c/(1 + \lambda/l)$ after the "reflection" accelerates, while at $\mathrm{v}_0 > c/(1 + \lambda/l)$ it decelerates because of induced undulator radiation.

If a particle is initially situated in the field, under the certain conditions it may be captured by the slowed-in-the-undulator effective wave. We shall define those conditions.

Let a particle with the velocity $\mathbf{v}_0$ be situated in the initial phases $\phi_{10} = \omega_0(t_0 - x_0/c)$ and $\phi_{20} = 2\pi x_0/l$ of a linearly polarized EM wave and undulator field

$$E_y(x,t) = -E_0 \cos\omega_0(t - \frac{x}{c}); \quad H_z(x) = H \cos\frac{2\pi}{l}x. \tag{5.31}$$

The solution of Eqs. (5.1) and (5.2) under these initial conditions for the particle momentum in the field gives

$$p_x = p_{0x} + \frac{n_2}{n_2^2 - 1}\frac{\mathcal{E}_0}{c}\left\{1 - n_2\frac{\mathrm{v}_{0x}}{c} \mp \left[\left(1 - n_2\frac{\mathrm{v}_{0x}}{c}\right)^2\right.\right.$$

$$- \left(n_2^2 - 1\right)\left(\frac{mc^2}{\mathcal{E}_0}\right)^2 \left[\frac{1}{2}\left(\xi_0^2 + \xi_H^2\right) + \left(\xi_0 \sin\phi_{10} + \xi_H \sin\phi_{20}\right)\right.$$

$$\left.\left.\left.\times\left(\xi_0 \sin\phi_{10} + \xi_H \sin\phi_{20} - 2\frac{P_{0y}}{mc}\right) + \xi_0\xi_H \cos\omega_0(t - n_2\frac{x}{c})\right]\right]^{1/2}\right\}, \tag{5.32}$$

$$p_y = p_{0y} + mc\xi_0\left[\sin\omega_0(t - \frac{x}{c}) - \sin\phi_{10}\right]$$

$$+mc\xi_H\left(\sin\frac{2\pi}{l}x - \sin\phi_{20}\right). \tag{5.33}$$

Note that at the derivation of Eq. (5.32) in Eqs. (5.20) and (5.23) the time averaging of noncoherent interaction with respect to coherent interaction has been made. In this approximation the integral of motion (5.24) remains applicable and with Eq. (5.32) determines the energy of the particle at the coherent interaction with the strong EM wave in a wiggler.

The equilibrated phases for the particle capture correspond to extrema of slowed-in-the-undulator effective wave and the motion of the particle is stable in the phases

$$\phi_s = \omega_0\left[t - \left(1 + \frac{\lambda}{l}\right)\frac{x}{c}\right]_s = \pi\left(2k + 1\right); \quad k = 0, \pm1, \ldots. \tag{5.34}$$

From Eq. (5.34) one can define the particle velocity in the equilibrated phase: $\mathrm{v}_{xs} = c/(1 + \lambda/l)$. If the initial velocity of the particle $\mathrm{v}_{0x} = \mathrm{v}_{xs}$ and $p_{0y} = 0$ the capture of such particle is possible at $\xi_0 = \xi_H$ that is $\lambda E_0 = lH$, i.e., the strong wave and wiggler field should transfer to the particle equal momenta and $\phi_{10} - \phi_{20} = \phi_s$ (at that $p_{ys} = 0$). If the initial velocity of the particle differs from the equilibrated one ($\mathrm{v}_{0x} \neq \mathrm{v}_{xs}$) and $p_{0y} = 0$ the tolerance for the capture of nonequilibrium particles is defined analogously to condition (5.16) in the induced Compton process. If $p_{0y} \neq 0$, then as in the case of counterpropagating waves the phases $\phi_0 = \pi\,(2k+1)$ automatically are equilibrated for the arbitrary p_{0y} ($p_{0y} = p_{ys}$). In the other cases the conditions for particle capture by the effective slowed wave in the regime of stable motion in the wiggler are defined as for those in the induced Compton interaction.

The "reflection" phenomenon of charged particles from a plane EM wave, as was shown in the induced Cherenkov process, may be used for monochromatization of the particle beams. Note that the considered vacuum versions of this phenomenon are more preferable for this goal taking into account the influence of negative effects of the multiple scattering and ionization losses in a medium. On the other hand, the refractive index of the effective slowed waves in vacuum n_1 or n_2 in corresponding induced Compton and undulator processes may be varied choosing the appropriate frequencies of counterpropagating waves or wiggler step. In particular, for monochromatization of particle beams with moderate or low energies via the induced Cherenkov process one needs a refractive index of a medium $n_0 - 1 \sim 1$ that corresponds to solid states. Meanwhile, such values of effective refractive index may be reached in the induced Compton process at the frequencies $\omega_1 \sim \omega_2$ of the counterpropagating waves. However, we will not consider here the possibility of particle beam monochromatization on the basis of the vacuum versions of "reflection" phenomenon since the principle of conversion of energetic or angular spreads is the same. To study the subject in more detail we refer the reader to original papers listed in the bibliography of this chapter.

5.3 Inelastic Diffraction Scattering on a Moving Phase Lattice

Consider now the quantum dynamics of a particle coherent interaction with the counterpropagating waves of different frequencies in the induced Compton process. Neglecting the spin interaction (with the same justification that has been made in the above-considered processes) we will derive from the Klein–Gordon equation in the field of quasimonochromatic waves with the vector potentials $\mathbf{A}_1(t - x/c)$ and $\mathbf{A}_2(t + x/c)$ which is written as

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = \left\{ -\hbar^2 c^2 \bigtriangledown^2 + m^2 c^4 + e^2 \left[\mathbf{A}_1(t - \frac{x}{c}) + \mathbf{A}_2(t + \frac{x}{c}) \right]^2 \right.$$

$$+2ie\hbar c\left[\mathbf{A}_1(t-\frac{x}{c})+\mathbf{A}_2(t+\frac{x}{c})\right]\bigtriangledown\Big\}\Psi. \tag{5.35}$$

As we saw in the classical consideration of the dynamics of the induced Compton process the effective interaction occurs with the slowed interference wave. At the intensities of the waves $\xi_1+\xi_2<\xi_{cr}\,(\omega_{1,2})$ when the particle can penetrate into the interference wave the latter will stand for a phase lattice for the particle (at the satisfaction of the condition of coherency (5.10)) and the coherent scattering will occur as for the diffraction effect on a crystal lattice. However, in contrast to diffraction on a motionless lattice (elastic scattering) the diffraction scattering on the moving phase lattice has inelastic character. To determine this quantum effect we will solve Eq. (5.35) in the eikonal approximation by the particle wave function (3.91) corresponding to multiphoton processes in strong fields. In accordance with the latter the solution of Eq. (5.35) for the waves of linear polarizations (along the axis OY)

$$\mathbf{A}_1(t-x/c)=\mathbf{A}_1(t)\cos\omega_1(t-x/c),$$

$$\mathbf{A}_2(t+x/c)=\mathbf{A}_2(t)\cos\omega_2(t+x/c)$$

we look for in the form Eq. (3.91) and for the slowly varying function $f(x,t)$ (see Eq. (3.92)) we obtain the following equation:

$$\frac{\partial f}{\partial t}+\mathrm{v}_{0x}\frac{\partial f}{\partial x}=\Big\{-\frac{ie^2}{2\hbar\mathcal{E}_0}\Big[A_1^2(t)\cos^2\omega_1(t-\frac{x}{c})+A_2^2(t)\cos^2\omega_2(t+\frac{x}{c})$$

$$+A_1(t)A_2(t)\cos(\omega_1+\omega_2)\left(t-\frac{\omega_1-\omega_2}{\omega_1+\omega_2}\frac{x}{c}\right)$$

$$+A_1(t)A_2(t)\cos(\omega_1-\omega_2)\left(t-\frac{\omega_1+\omega_2}{\omega_1-\omega_2}\frac{x}{c}\right)\Big]$$

$$+\frac{iecp_{0y}}{\hbar\mathcal{E}_0}\left[A_1(t)\cos\omega_1(t-\frac{x}{c})+A_2(t)\cos\omega_2(t+\frac{x}{c})\right]\Big\}f(x,t). \tag{5.36}$$

As is seen from Eq. (5.36) at the interaction with the counterpropagating waves of different frequencies two interference waves are formed — third and fourth terms on the right-hand side — which propagate with the phase velocities

$$\mathrm{v}_{ph}=c\frac{\omega_1+\omega_2}{|\omega_1-\omega_2|}>c$$

and

$$\mathrm{v}_{ph} = c\frac{|\omega_1 - \omega_2|}{\omega_1 + \omega_2} < c,$$

respectively. It is clear that the interaction of the particle with the wave propagating with the phase velocity $\mathrm{v}_{ph} > c$, as well as with the incident separate waves propagating in the vacuum with the phase velocity c (remaining four terms on the right-hand side of Eq. (5.36)), cannot be coherent. These terms correspond to noncoherent scattering of the particle in the separate wave fields which vanish after the interaction. Coherent interaction in this process occurs with the slowed interference wave (fourth term), in accordance with the classical results (see Eqs. (5.8) and (5.10)).

For the integration of Eq. (5.36) we will pass to characteristic coordinates $\tau' = t - x/\mathrm{v}_{0x}$ and $\eta' = t$. Then, if one directs the particle velocity $\mathbf{v}_0$ at the angle ϑ_0 with respect to the waves' propagation axis providing the condition of coherency of the induced Compton process (resonance between the waves' Doppler-shifted frequencies) for the free-particle velocity

$$\mathrm{v}_0 \cos\vartheta_0 = c\frac{|\omega_1 - \omega_2|}{\omega_1 + \omega_2}, \tag{5.37}$$

the traveling interference wave in this frame of coordinates becomes a standing phase lattice over the coordinate τ' and diffraction scattering of the particle occurs. From Eq. (5.36) for the amplitude of the scattered particle wave function we obtain

$$f(\tau') = \exp\left\{-\frac{ie^2}{2\hbar\mathcal{E}_0}\cos(\omega_1 - \omega_2)\tau' \int_{\eta_1}^{\eta_2} A_1(\eta')A_2(\eta')d\eta'\right\}, \tag{5.38}$$

where η_1 and η_2 are the moments of the particle entrance into the field and exit, respectively.

If one expands the exponential (5.38) into a series by Bessel functions and returns again to coordinates x, t with the help of Eq. (3.91) for the total wave function we will have

$$\Psi(\mathbf{r}, t) = \sqrt{\frac{N_0}{2\mathcal{E}_0}} \exp\left[\frac{i}{\hbar}(p_0 \sin\vartheta_0)\, y\right] \sum_{s=-\infty}^{+\infty} (-i)^s J_s(\alpha)$$

$$\times \exp\left\{\frac{i}{\hbar}\left[p_0\cos\vartheta_0 + s\hbar\frac{\omega_1 + \omega_2}{c}\right]x - \frac{i}{\hbar}\left[\mathcal{E}_0 + s\hbar(\omega_1 - \omega_2)\right]t\right\}, \tag{5.39}$$

where the argument of the Bessel function

$$\alpha = \frac{e^2c^2}{2\hbar\mathcal{E}_0\omega_1\omega_2}\int_{t_1}^{t_2} E_1(\eta')E_2(\eta')d\eta' \tag{5.40}$$

(E_1 and E_2 are the amplitudes of the waves' electric field strengths).

Equation (5.39) shows that the diffraction scattering of the particles in the field of counterpropagating waves of different frequencies is inelastic. Due to the induced Compton effect the particle absorbs s photons from the one wave and coherently radiates s photons into the other wave and vice versa (resonance between the Doppler-shifted frequencies in the intrinsic frame of the particle), i.e., the conservation of the number of photons in the induced Compton process takes place in contrast to spontaneous Compton effect in the strong wave field where after the multiphoton absorption a single photon is emitted. However, because of the different photon energies the scattering process is inelastic. From Eq. (5.39) for the change of the particle energy-momentum we have

$$\Delta\mathcal{E} = s\hbar\left(\omega_1 - \omega_2\right);\ \Delta p_x = s\hbar\left(\omega_1 + \omega_2\right)/c\ ;\ \Delta p_y = 0;\ s = 0, \pm 1, \ldots . \tag{5.41}$$

The probability of inelastic diffraction scattering is

$$W_s = J_s^2\left[\frac{e^2c^2}{2\hbar\omega_1\omega_2\mathcal{E}_0}\int_{t_1}^{t_2} E_1(\eta')E_2(\eta')d\eta'\right]. \tag{5.42}$$

According to the condition of eikonal approximation (3.92): $|\Delta p| << p_0$ and $|\Delta\mathcal{E}| << \mathcal{E}_0$ from Eq. (5.41) we have the condition of applicability of the obtained results: $|s|\,\hbar\left(\omega_1 + \omega_2\right)/c << p_0$.

In the case of monochromatic waves

$$W_s = J_s^2\left(\frac{e^2c^2E_1E_2t_0}{2\hbar\mathcal{E}_0\omega_1\omega_2}\right), \tag{5.43}$$

where $t_0 = t_2 - t_1$ is the time duration of the particle motion in the interference wave ($l_c = \mathrm{v}_0 t_0 \cos\vartheta_0$ is the coherent length of the process). For the actual values of the parameters including in Eq. (5.43) the argument of the Bessel function $\alpha >> 1$, consequently the most probable number of absorbed/radiated photons

$$\overline{s} \simeq \frac{1}{2}\xi_1\xi_2\frac{mc^2}{\mathcal{E}_0}\frac{mc^2}{\hbar}t_0. \tag{5.44}$$

The energetic width of the main diffraction maximums

$$\Gamma(\overline{s}) \simeq \overline{s}^{1/3}\hbar\left(\omega_1 - \omega_2\right)$$

and since $\overline{s} >> 1$ then

$$\Gamma(\overline{s}) << \left|\mathcal{E} - \mathcal{E}_0\right|.$$

The scattering angles of s-photon diffraction on the counterpropagating waves are

$$\tan \vartheta_s = \frac{s\hbar\left(\omega_1 + \omega_2\right)\sin\vartheta_0}{cp_0 + s\hbar\left(\omega_1 + \omega_2\right)\cos\vartheta_0} \; ; \qquad s = 0, \pm 1, \ldots . \tag{5.45}$$

As in the Cherenkov process at the inelastic diffraction there is an asymmetry in the angular distribution of the scattered particle: $|\vartheta_{-s}| > \vartheta_s$, i.e., the main diffraction maximums are situated at the different angles with respect to the direction of particle initial motion. However, since $|s|\,\hbar\left(\omega_1 + \omega_2\right)/c << p_0$ this asymmetry can be neglected, i.e., $|\vartheta_{-s}| \simeq \vartheta_s$ and the scattering angles of the main diffraction maximums will be determined by the equation

$$\vartheta_{\pm\overline{s}} = \pm\overline{s}\frac{\hbar\left(\omega_1 + \omega_2\right)}{cp_0}\sin\vartheta_0 . \tag{5.46}$$

In the case of counterpropagating waves of equal frequencies ($\omega_1 = \omega_2 \equiv \omega$) the phase velocity of the interference wave $\mathrm{v}_{ph} = 0$ and the coherent scattering on the motionless phase lattice takes place, which is elastic: $\Delta\mathcal{E} = 0$ and $\Delta p_x = 2s\hbar\omega/c$. This is the known Kapitza–Dirac effect for electron diffraction on a standing wave (in the one-photon approximation for the weak waves). As follows from Eq. (5.37) the coherent scattering in this case is possible at the incident angle $\vartheta_0 = \pi/2$, i.e., if the particle velocity is perpendicular to the axis of waves' propagation, to exclude the Doppler shift of waves frequencies because of its counterpropagation (a longitudinal component of the particle velocity will result in different Doppler shifts of equal laboratory frequencies because of different wave vectors $\mathbf{k}$ and $-\mathbf{k}$ of counterpropagating waves and, consequently, will violate the resonance between the waves).

5.4 Inelastic Diffraction Scattering on a Traveling Wave in an Undulator

Charged particles diffraction scattering is also possible on a plane EM wave propagating in vacuum if the interaction proceeds in an undulator. As the diffraction effect is the result of particle coherent interaction with the periodic wave field the effective field in the undulator should be smaller than the threshold value of "reflection" phenomenon: $\xi_0 + \xi_H < \xi_{cr}\left(\lambda/l\right)$ (to prohibit the nonlinear resonance in the field at which the periodic EM field becomes a potential barrier for the particle and coherent interaction with the periodic wave field impossible). Under this condition we will solve the relativistic quantum equation of motion

$$-\hbar^2\frac{\partial^2\Psi}{\partial t^2} = \left\{-\hbar^2c^2\bigtriangledown^2 + m^2c^4 + e^2\left[\mathbf{A}_1(t - \frac{x}{c}) + \mathbf{A}_2(x)\right]^2\right.$$

$$+2ie\hbar c\left[\mathbf{A}_1(t-\frac{x}{c})+\mathbf{A}_2(x)\right]\bigtriangledown\Big\}\Psi, \qquad (5.47)$$

where $\mathbf{A}_1(t-x/c)$ is the vector potential of the quasimonochromatic EM wave and $\mathbf{A}_2(x)$ is the vector potential of the undulator magnetic field. For the linear undulator

$$H_z(x)=H\cos\frac{2\pi}{l}x$$

the vector potential will be described by the equation

$$A_{2y}(x)=\frac{lH}{2\pi}\sin\frac{2\pi}{l}x,$$

and correspondingly the EM wave will be assumed linearly polarized along the axis OY

$$A_{1y}(t-x/c)=A(t)\sin\omega_0(t-x/c).$$

To determine the multiphoton diffraction effect Eq. (5.47) will be solved again in the eikonal approximation. In accordance with the latter we present the solution of Eq. (5.47) in the form of Eq. (3.91). Then taking into account the condition (3.92) for the slowly varying function $f(x,t)$ we obtain the equation

$$\frac{\partial f}{\partial t}+\mathrm{v}_{0x}\frac{\partial f}{\partial x}=\Big\{-\frac{ie^2}{2\hbar\mathcal{E}_0}\Big[A^2(t)\sin^2\omega_0(t-\frac{x}{c})+\frac{l^2H^2}{4\pi^2}\sin^2\frac{2\pi}{l}x$$

$$+\frac{lH}{2\pi}A(t)\cos\omega_0\left(t-\left(1+\frac{\lambda}{l}\right)\frac{x}{c}\right)-\frac{lH}{2\pi}A(t)\cos\omega_0\left(t-\left(1-\frac{\lambda}{l}\right)\frac{x}{c}\right)\Big]$$

$$+\frac{iecp_{0y}}{\hbar\mathcal{E}_0}\left[A(t)\sin\omega_0(t-\frac{x}{c})+\frac{lH}{2\pi}\sin\frac{2\pi}{l}x\right]\Big\}f(x,t). \qquad (5.48)$$

As is seen from Eq. (5.48) under the induced interaction in the undulator, traveling waves propagating with the phase velocities $\mathrm{v}_{ph}=c/\left(1+\lambda/l\right)<c$ and $\mathrm{v}_{ph}=c/\left(1-\lambda/l\right)>c$ arise. We will not repeat here the analogous interpretation of the terms in Eq. (5.48) which correspond to interaction of the particle with the waves propagating with the phase velocities $\mathrm{v}_{ph}\gtrless c$ that has been done for the above-considered induced Compton process. Note only that coherent interaction in this process occurs with the slowed interference wave propagating with the phase velocity $\mathrm{v}_{ph}=c/\left(1+\lambda/l\right)<c$ (third term on the right-hand side of Eq. (5.48)), in accordance with the classical results for the induced interaction in the magnetic undulator (see Eqs. (5.27) and (5.29)).

The integration of Eq. (5.48) is simple if we pass to characteristic coordinates $\tau' = t - x/\mathrm{v}_{0x}$ and $\eta' = t$. Then, if one directs the particle velocity $\mathbf{v}_0$ at the angle ϑ_0 with respect to the wave propagation direction (undulator axis) thus providing the condition of coherency in the undulator for the free-particle velocity

$$\mathrm{v}_0 \cos\vartheta_0 = \frac{c}{1+\frac{\lambda}{l}}, \tag{5.49}$$

the slowed traveling wave in this frame of coordinates becomes a motionless phase lattice (over the coordinate τ') and diffraction scattering of the particle occurs. For the amplitude of the scattered particle wave function we obtain

$$f(\tau') = \exp\left\{-\frac{ie^2 lH}{4\pi\hbar\mathcal{E}_0}\cos\omega_0\tau' \int_{\eta_1}^{\eta_2} A(\eta')d\eta'\right\}, \tag{5.50}$$

where η_1 and η_2 are the moments of the particle entrance into the undulator and exit, respectively.

Expanding the exponential in Eq. (5.50) into a series by Bessel functions with the help of Eq. (3.91) for the final wave function of the scattered particle we will have

$$\Psi(\mathbf{r},t) = \sqrt{\frac{N_0}{2\mathcal{E}_0}}\exp\left[\frac{i}{\hbar}(p_0\sin\vartheta_0)\,y\right]\sum_{s=-\infty}^{+\infty}(-i)^s J_s(\alpha)$$

$$\times\exp\left\{\frac{i}{\hbar}\left[p_0\cos\vartheta_0 + s\hbar\frac{\omega_0}{c}\left(1+\frac{\lambda}{l}\right)\right]x - \frac{i}{\hbar}(\mathcal{E}_0 + s\hbar\omega_0)\,t\right\}, \tag{5.51}$$

where the argument of the Bessel function

$$\alpha = \frac{e^2 lH}{4\pi\hbar\mathcal{E}_0}\int_{t_1}^{t_2} A(\eta')d\eta'. \tag{5.52}$$

The expression for the particle wave function (5.51) shows that the initial plane wave of the free particle as a result of the induced undulator effect is expanded into the envelope of plane waves with all possible numbers of absorbed and emitted photons — the inelastic diffraction scattering occurs. The energy and momentum of the particle after the scattering are

$$\mathcal{E} = \mathcal{E}_0 + s\hbar\omega_0; \quad p_x = p_0\cos\vartheta_0 + \left(1+\frac{\lambda}{l}\right)\frac{s\hbar\omega_0}{c};$$

$$p_y = \mathrm{const}; \quad s = 0, \pm 1, \ldots. \tag{5.53}$$

According to the condition of eikonal approximation (3.92) $s\hbar\omega_0 << \mathcal{E}_0$.

The probability of inelastic diffraction scattering in the undulator is

$$W_s = J_s^2 \left[\frac{e^2 l H}{4\pi\hbar\mathcal{E}_0} \int_{t_1}^{t_2} A(\eta')d\eta' \right] . \tag{5.54}$$

If the incident strong EM wave is monochromatic, the probability of this process is

$$W_s = J_s^2 \left(\frac{e^2 c E_0 l H}{4\pi\hbar\omega_0\mathcal{E}_0} t_0 \right) , \tag{5.55}$$

where $t_0 = t_2 - t_1$ is the time duration of the particle motion in the undulator, and E_0 is the amplitude of the electric field strength of stimulating wave.

For the actual values of the parameters the argument of the Bessel function $\alpha >> 1$, consequently the inelastic diffraction scattering in the undulator is essentially multiphoton as in the Cherenkov and Compton processes. The main diffraction maximums correspond to the most probable number of absorbed/radiated photons

$$\overline{s} \simeq \xi_0 \frac{mc^2}{\mathcal{E}_0} \frac{elH}{4\pi\hbar} t_0 \tag{5.56}$$

with the energetic width $\Gamma(\overline{s}) \simeq \overline{s}^{1/3}\hbar\omega_0$.

The scattering angles of s-photon diffraction in the undulator are

$$\tan\vartheta_s = \frac{s\hbar\omega_0 \left(1 + \frac{\lambda}{l}\right) \sin\vartheta_0}{cp_0 + s\hbar\omega_0 \left(1 + \frac{\lambda}{l}\right) \cos\vartheta_0} \; ; \qquad s = 0, \pm 1, \ldots . \tag{5.57}$$

The main diffraction maximums are situated at the angles (taking into account the condition of applied eikonal approximation)

$$\vartheta_{\pm\overline{s}} = \pm \frac{\left(1 + \frac{\lambda}{l}\right) \overline{s}\hbar\omega_0}{cp_0} \sin\vartheta_0, \tag{5.58}$$

with respect to the direction of the particle initial motion.

5.5 Quantum Modulation of Particle Beam in Induced Compton Process

Consider the effect of a particle beam quantum modulation at the interaction with the counterpropagating waves of different frequencies and intensities smaller than the threshold value for nonlinear Compton resonance or the critical value of the particle “reflection” phenomenon (5.9) (since the quantum modulation of the particle state is the result of coherent interaction with the periodic wave field, while at values larger than the critical one the latter becomes a potential barrier for the particle).

Neglecting the spin interaction the quantum equation of motion (5.35) for the plane waves of circular polarization

$$\mathbf{A}_1 = \{0, A_1 \cos\omega_1 \left(t - \frac{x}{c}\right), A_1 \sin\omega_1 \left(t - \frac{x}{c}\right)\},$$

$$\mathbf{A}_2 = \{0, A_2 \cos\omega_2 \left(t + \frac{x}{c}\right), A_2 \sin\omega_2 \left(t + \frac{x}{c}\right)\}$$

may be presented in the form

$$\hbar^2 c^2 \Delta\Psi - \hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = \left\{ e^2 \left(A_1^2 + A_2^2\right) + m^2 c^4 + 2ie\hbar c \left[\mathbf{A}_1 \left(t - \frac{x}{c}\right) \right.\right.$$

$$\left.\left. + \mathbf{A}_2 \left(t + \frac{x}{c}\right)\right] \nabla + 2e^2 A_1 A_2 \cos\left(\omega_1 - \omega_2\right) \left(t - \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} \frac{x}{c}\right) \right\} \Psi. \tag{5.59}$$

If the initial velocity of the particle is directed along the axis of wave propagation ($\mathbf{p}_{0\perp} = 0$) the noncoherent interaction with the separate waves $\sim A_1$ and A_2 vanishes and we have the equation

$$\hbar^2 c^2 \Delta\Psi - \hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = \left\{ e^2 \left(A_1^2 + A_2^2\right) + m^2 c^4 \right.$$

$$\left. + 2e^2 A_1 A_2 \cos\left(\omega_1 - \omega_2\right) \left(t - \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} \frac{x}{c}\right) \right\} \Psi, \tag{5.60}$$

which describes the coherent interaction with the slowed interference wave of frequency $\omega_1 - \omega_2$ (corresponding to Compton resonance between the counterpropagating waves) and constant renormalization of the particle mass in the field because of the intensity effect of strong waves $\sim A_1^2 + A_2^2$. To determine the effect of quantum modulation at the harmonics of the fundamental frequency $\omega_1 - \omega_2$ the problem will be solved in the approximation of perturbation theory (besides, the wave intensities should be smaller than the critical value in the induced Compton process). It is found this renormalization in the field is rather small and since it vanishes after the interaction as well, we will omit this term. Then one needs to take into account the quantum recoil which has been vanished by consideration of the diffraction effect on the basis of eikonal-type wave function, when the second-order derivatives of the wave function have been neglected. Hence, we will keep the second-order derivatives in Eq. (5.59) and solve it within perturbation theory by the wave function. Then the solution of Eq. (5.60) is sought by the series of harmonics of the fundamental frequency $\omega_1 - \omega_2$:

$$\Psi(\mathbf{r}, t) = \sqrt{\frac{N_0}{2\mathcal{E}_0}} \exp\left[\frac{i}{\hbar}\left(p_0 x - \mathcal{E}_0 t\right)\right]$$

$$\times \sum_{s=-\infty}^{+\infty} \Psi_s \exp\left[is(\omega_1 - \omega_2)\left(t - \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2}\frac{x}{c}\right)\right]. \tag{5.61}$$

(for N_0 particles per unit volume) corresponding to s-photon absorption by the particle from the wave of frequency ω_2 and s-photon coherent radiation into the wave of frequency ω_1 and vice-versa (induced Compton effect with the conservation of the number of interacting photons). Substituting the wave function (5.61) into Eq. (5.60) we obtain the following recurrent equation for the amplitudes Ψ_s:

$$\left[4\hbar^2 s^2 \omega_1 \omega_2 + 2\mathcal{E}_0 s\hbar\left(\omega_1 - \omega_2 - (\omega_1 + \omega_2)\frac{\mathrm{v}_0}{c}\right)\right]\Psi_s$$

$$= -e^2 A_1 A_2 \left[\Psi_{s-1} + \Psi_{s+1}\right]. \tag{5.62}$$

Equation (5.62) will be solved in the approximation of perturbation theory by the wave function:

$$|\Psi_{\pm 1}| << |\Psi_0|; \qquad |\Psi_{\pm 2}| << |\Psi_{\pm 1}|, \ldots.$$

Thus, for the amplitude of the particles' wave function corresponding to absorption of s photons of frequency ω_2 and induced radiation of s photons of frequency ω_1 we obtain

$$\Psi_s = \frac{(-1)^s}{s!}\left(\frac{e^2 A_1 A_2}{2\hbar\mathcal{E}_0}\right)^s \prod_{s_1=1}^{s} \frac{1}{\omega_1 - \omega_2 - (\omega_1 + \omega_2)\frac{\mathrm{v}_0}{c} + 2s_1\frac{\hbar\omega_1\omega_2}{\mathcal{E}_0}}, \tag{5.63}$$

and for the inverse process (absorption of s photons of frequency ω_1 and induced radiation of s photons of frequency ω_2):

$$\Psi_{-s} = \frac{1}{s!}\left(\frac{e^2 A_1 A_2}{2\hbar\mathcal{E}_0}\right)^s \prod_{s_1=1}^{s} \frac{1}{\omega_1 - \omega_2 - (\omega_1 + \omega_2)\frac{\mathrm{v}_0}{c} - 2s_1\frac{\hbar\omega_1\omega_2}{\mathcal{E}_0}}. \tag{5.64}$$

Hence, for the total wave function of the particles after the interaction we have the equation

$$\Psi(\mathbf{r},t) = \sqrt{\frac{N_0}{2\mathcal{E}_0}} \left\{ 1 + \sum_{s=1}^{\infty} \frac{1}{s!} \left(\frac{e^2 A_1 A_2}{2\hbar \mathcal{E}_0} \right)^s \right.$$

$$\times \left[\prod_{s_1=1}^{s} \frac{(-1)^s \exp\left[is(\omega_1 - \omega_2)\left(t - \frac{\omega_1+\omega_2}{\omega_1-\omega_2} \frac{x}{c} \right) \right]}{\omega_1 - \omega_2 - (\omega_1 + \omega_2) \frac{\mathrm{v}_0}{c} + 2s_1 \frac{\hbar\omega_1\omega_2}{\mathcal{E}_0}} \right.$$

$$\left. \left. + \prod_{s_1=1}^{s} \frac{\exp\left[-is(\omega_1 - \omega_2)\left(t - \frac{\omega_1+\omega_2}{\omega_1-\omega_2} \frac{x}{c} \right) \right]}{\omega_1 - \omega_2 - (\omega_1 + \omega_2) \frac{\mathrm{v}_0}{c} - 2s_1 \frac{\hbar\omega_1\omega_2}{\mathcal{E}_0}} \right] \right\} e^{\frac{i}{\hbar}(p_0 x - \mathcal{E}_0 t)}. \tag{5.65}$$

Here the dimensionless parameter of one-photon absorption-radiation is the small parameter of applied perturbation theory

$$\frac{e^2 A_1 A_2}{2\hbar\mathcal{E}_0 \left| \omega_1 - \omega_2 - (\omega_1 + \omega_2) \frac{\mathrm{v}_0}{c} \pm 2 \frac{\hbar\omega_1\omega_2}{\mathcal{E}_0} \right|} << 1. \tag{5.66}$$

The denominators in Eq. (5.65) become zero at the fulfillment of exact resonance (with the quantum recoil $2\hbar\omega_1\omega_2/\ \mathcal{E}_0$) corresponding to the conservation law for the induced Compton process

$$\omega_1 = \omega_2 \frac{1 + \frac{\mathrm{v}_0}{c}}{1 - \frac{\mathrm{v}_0}{c} \pm 2s \frac{\hbar\omega_2}{\mathcal{E}_0}}. \tag{5.67}$$

In this case, perturbation theory is not applicable and consideration must be given to secular perturbation theory.

Corresponding to wave function (5.65) the current density of the particles after the interaction will be expressed by the equation

$$\mathbf{j}(t,x) = \mathbf{j}_0 \left\{ 1 + 2 \sum_{s=1}^{\infty} \frac{1}{s!} \left(\frac{e^2 A_1 A_2}{2\hbar\mathcal{E}_0} \right)^s \right.$$

$$\times \left[\prod_{s_1=1}^{s} \frac{(-1)^s}{\omega_1 - \omega_2 - (\omega_1 + \omega_2) \frac{\mathrm{v}_0}{c} + 2s_1 \frac{\hbar\omega_1\omega_2}{\mathcal{E}_0}} \right.$$

$$\left. + \prod_{s_1=1}^{s} \frac{1}{\omega_1 - \omega_2 - (\omega_1 + \omega_2) \frac{\mathrm{v}_0}{c} - 2s_1 \frac{\hbar\omega_1\omega_2}{\mathcal{E}_0}} \right]$$

$$\times \cos\left[s(\omega_1 - \omega_2)\left(t - \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2}\frac{x}{c}\right)\right]$$

$$+2\sum_{s=1}^{\infty}\sum_{s'=1}^{\infty}\frac{(-1)^s}{s!s'!}\left(\frac{e^2A_1A_2}{2\hbar\mathcal{E}_0}\right)^{s+s'}\cos\left[(s+s')(\omega_1 - \omega_2)\left(t - \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2}\frac{x}{c}\right)\right]$$

$$\times\prod_{s_1=1}^{s}\prod_{s_2=1}^{s'}\frac{1}{\omega_1 - \omega_2 - (\omega_1 + \omega_2)\frac{\mathrm{v}_0}{c} + 2s_1\frac{\hbar\omega_1\omega_2}{\mathcal{E}_0}}$$

$$\left.\times\frac{1}{\omega_1 - \omega_2 - (\omega_1 + \omega_2)\frac{\mathrm{v}_0}{c} - 2s_2\frac{\hbar\omega_1\omega_2}{\mathcal{E}_0}}\right\}, \tag{5.68}$$

where $\mathbf{j}_0 = \text{const}$ is the initial current density of the particles.

We present in explicit form the expression of modulated current density of the particles for the first three harmonics

$$\mathbf{j}(t,x) = \mathbf{j}_0\left\{1 + B(\omega_{1,2})\cos(\omega_1 - \omega_2)\left(t - \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2}\frac{x}{c}\right)\right.$$

$$+\frac{3}{4}B^2(\omega_{1,2})\cos 2(\omega_1 - \omega_2)\left(t - \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2}\frac{x}{c}\right)$$

$$+\frac{5}{8}B^3(\omega_{1,2})\cos 3(\omega_1 - \omega_2)\left(t - \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2}\frac{x}{c}\right) + \cdots, \tag{5.69}$$

where the modulation depth at the fundamental frequency $\omega_1 - \omega_2$

$$B(\omega_{1,2}) = \frac{\xi_1\xi_2}{\xi_{cr}^2(\omega_{1,2})} \tag{5.70}$$

is represented by the parameter of critical field (5.9) in the induced Compton process. As was mentioned above for quantum modulation of the particle state at the harmonics of interference wave the intensity of the latter should be smaller than the threshold value of nonlinear resonance in the field or the critical value in the induced Compton process. Equation (5.70) shows that this requirement ($\xi_1\xi_2 < \xi_{cr}^2(\omega_{1,2})$) holds in any case since in accordance with perturbation theory (condition (5.66)) $\xi_1\xi_2 << \xi_{cr}^2(\omega_{1,2})$. Note that for the representation of modulation depth in the form of Eq. (5.70) it was assumed that the quantum recoil is smaller than the Compton resonance width because of nonmonochromaticity of actual particle beams.

5.6 Quantum Modulation of Particle Beam in the Undulator

If in the induced Compton process the particles' quantum modulation takes place at the difference of frequencies (and harmonics) of two waves, the induced interaction in the undulator leads to particles' quantum modulation at the stimulating wave frequency and its harmonics. The latter is similar to Cherenkov modulation, but it is important that in this case the modulation takes place in the vacuum.

The quantum equation of motion of the particle (5.47) in the undulator with circular polarization of the magnetic field in the presence of a plane monochromatic EM wave of circular polarization with vector potentials respectively

$$\mathbf{A}_2(x) = \left\{0, -\frac{lH}{2\pi}\cos\frac{2\pi}{l}x, \frac{lH}{2\pi}\sin\frac{2\pi}{l}x\right\},$$

$$\mathbf{A}_1(x,t) = \left\{0, A_0\cos\omega_0(t-\frac{x}{c}), -A_0\sin\omega_0(t-\frac{x}{c})\right\}$$

is written as

$$\hbar^2c^2\Delta\Psi - \hbar^2\frac{\partial^2\Psi}{\partial t^2} = \left\{e^2\left(A_0^2 + \frac{l^2H^2}{4\pi^2}\right) + m^2c^4 + 2ie\hbar c\Big[\mathbf{A}_1\left(t-\frac{x}{c}\right)\right.$$

$$\left. +\mathbf{A}_2\left(x\right)\Big]\nabla - e^2\frac{lH}{\pi}A_0\cos\omega_0\left(t-\left(1+\frac{\lambda}{l}\right)\frac{x}{c}\right)\right\}\Psi. \tag{5.71}$$

The coherent interaction in this process which leads to particles' quantum modulation proceeds with the effective slowed wave $\sim HA_0$ (last term on the right-hand side of Eq. (5.71)). If the free-particle initial velocity is directed along the undulator axis ($\mathbf{p}_{0\perp} = 0$) the noncoherent interaction with the EM wave $\sim A_1$ and magnetic field of the undulator $\sim A_2$ vanishes and we have the equation

$$\hbar^2c^2\Delta\Psi - \hbar^2\frac{\partial^2\Psi}{\partial t^2} = \left\{e^2\left(A_0^2 + \frac{l^2H^2}{4\pi^2}\right) + m^2c^4\right.$$

$$\left. -e^2\frac{lH}{\pi}A_0\cos\omega_0\left(t-\left(1+\frac{\lambda}{l}\right)\frac{x}{c}\right)\right\}\Psi, \tag{5.72}$$

which describes the particle coherent interaction with the effective slowed wave in the undulator and constant renormalization of the particle mass in the field due to the intensity effect of strong wave $\sim A_0^2$ and powerful magnetic field of the wiggler $\sim H^2l^2$. With the same justification made at the solution of this problem in the induced Compton process these constant terms will be neglected and the solution of Eq. (5.72) will be sought in the form

$$\Psi(\mathbf{r},t) = \sqrt{\frac{N_0}{2\mathcal{E}_0}} \exp\left[\frac{i}{\hbar}\left(p_0 x - \mathcal{E}_0 t\right)\right]$$

$$\times \sum_{s=-\infty}^{+\infty} \Psi_s \exp\left[is\omega_0\left(t - \left(1+\frac{\lambda}{l}\right)\frac{x}{c}\right)\right]. \qquad (5.73)$$

Substituting the wave function (5.73) into Eq. (5.72) we obtain the recurrent equation for the amplitudes Ψ_s corresponding to s-photon induced absorption by the particle from the effective slowed wave ($s < 0$) and induced undulator radiation ($s > 0$)

$$\left[\frac{2\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)s^2 + s\left(1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}\right)\right]\Psi_s$$

$$= \frac{e^2 lHA_0}{4\pi\mathcal{E}_0\hbar\omega_0}\left[\Psi_{s-1}+\Psi_{s+1}\right], \qquad (5.74)$$

which will be solved in the approximation of perturbation theory by the wave function:

$$|\Psi_{\pm 1}| << |\Psi_0|; \qquad |\Psi_{\pm 2}| << |\Psi_{\pm 1}|, \ldots .$$

For the amplitude of the particle wave function corresponding to s-photon induced radiation we obtain

$$\Psi_s = \frac{1}{s!}\left(\frac{e^2 lHA_0}{4\pi\mathcal{E}_0\hbar\omega_0}\right)^s \prod_{s_1=1}^{s} \frac{1}{1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c} + 2s_1\frac{\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)}, \qquad (5.75)$$

and for s-photon absorption

$$\Psi_{-s} = \frac{(-1)^s}{s!}\left(\frac{e^2 lHA_0}{4\pi\mathcal{E}_0\hbar\omega_0}\right)^s \prod_{s_1=1}^{s} \frac{1}{1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c} - 2s_1\frac{\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)}. \qquad (5.76)$$

Hence, for total wave function of the particles after the interaction we have

$$\Psi(\mathbf{r},t) = \sqrt{\frac{N_0}{2\mathcal{E}_0}}\left\{1+\sum_{s=1}^{\infty}\frac{1}{s!}\left(\frac{e^2 lHA_0}{4\pi\mathcal{E}_0\hbar\omega_0}\right)^s\right.$$

$$\times \left[\prod_{s_1=1}^{s} \frac{\exp\left[is\omega_0\left(t-\left(1+\frac{\lambda}{l}\right)\frac{x}{c}\right)\right]}{1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}+2s_1\frac{\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)}\right.$$

$$\left.\left. + \prod_{s_1=1}^{s} \frac{(-1)^s\exp\left[-is\omega_0\left(t-\left(1+\frac{\lambda}{l}\right)\frac{x}{c}\right)\right]}{1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}-2s_1\frac{\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)}\right]\right\} e^{\frac{i}{\hbar}(p_0x-\mathcal{E}_0t)}. \tag{5.77}$$

The small parameter of applied perturbation theory (dimensionless parameter of induced one-photon absorption-radiation in the undulator) is

$$\frac{e^2lHA_0}{4\pi\mathcal{E}_0\hbar\omega_0\left|1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}\pm 2\frac{\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)\right|} << 1. \tag{5.78}$$

The denominators in Eq. (5.77) become zero at the fulfillment of exact resonance (with the quantum recoil) between the EM wave and undulator fields

$$\frac{\lambda}{l} = \frac{c}{\mathrm{v}_0} - 1 \pm 2s\frac{\pi\hbar c^2}{l\mathcal{E}_0\mathrm{v}_0}\left(1+\frac{\lambda}{2l}\right), \tag{5.79}$$

for which the perturbation theory is not applicable and the consideration should be made in the scope of secular perturbation theory.

With the help of the wave function (5.77) for the current density of the particles after the interaction we obtain the equation

$$\mathbf{j}(t,x) = \mathbf{j}_0\left\{1+2\sum_{s=1}^{\infty}\frac{1}{s!}\left(\frac{e^2lHA_0}{4\pi\mathcal{E}_0\hbar\omega_0}\right)^s\right.$$

$$\times \left[\prod_{s_1=1}^{s} \frac{1}{1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}+2s_1\frac{\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)}\right.$$

$$\left. + \prod_{s_1=1}^{s} \frac{(-1)^s}{1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}-2s_1\frac{\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)}\right] \times \cos\left[s\omega_0\left(t-\left(1+\frac{\lambda}{l}\right)\frac{x}{c}\right)\right]$$

$$+2\sum_{s=1}^{\infty}\sum_{s'=1}^{\infty}\frac{(-1)^{s'}}{s!s'!}\left(\frac{e^2lHA_0}{4\pi\mathcal{E}_0\hbar\omega_0}\right)^{s+s'}\cos\left[(s+s')\,\omega_0\left(t-\left(1+\frac{\lambda}{l}\right)\frac{x}{c}\right)\right]$$

$$\times \prod_{s_1=1}^{s}\prod_{s_2=1}^{s'} \frac{1}{1-\left(1+\frac{\lambda}{l}\right)\frac{\mathrm{v}_0}{c}+2s_1\frac{\pi c\hbar}{l\mathcal{E}_0}\left(1+\frac{\lambda}{2l}\right)}$$

$$\times \frac{1}{1 - \left(1 + \frac{\lambda}{l}\right) \frac{\mathrm{v}_0}{c} - 2s_2 \frac{\pi c \hbar}{l \mathcal{E}_0} \left(1 + \frac{\lambda}{2l}\right)} \Bigg\} . \tag{5.80}$$

From Eq. (5.80) for the modulation at the fundamental frequency of the stimulating wave we have

$$\mathbf{j}_1(t, x) = \mathbf{j}_0 \left\{ 1 - B(\lambda/l) \cos \omega_0 \left(t - \left(1 + \frac{\lambda}{l} \right) \frac{x}{c} \right) \right\}, \tag{5.81}$$

where the modulation depth

$$B(\lambda/l) = 2\xi_0 \xi_H \left(\frac{mc^2}{\mathcal{E}_0} \right)^2 \frac{\frac{\lambda}{l} \left(1 + \frac{\lambda}{2l}\right)}{\left[1 - \left(1 + \frac{\lambda}{l}\right) \frac{\mathrm{v}_0}{c}\right]^2 - \frac{4\pi^2 c^2 \hbar^2}{l^2 \mathcal{E}_0^2} \left(1 + \frac{\lambda}{2l}\right)^2} . \tag{5.82}$$

The depth of quantum modulation can be represented by the parameter of critical field (5.28) in the induced undulator process. As the resonance width because of nonmonochromaticity of actual particle beams is rather larger than the quantum recoil, then neglecting the latter, for the modulation depth we will have

$$B(\lambda/l) = \frac{\xi_0 \xi_H}{\xi_{cr}^2(\lambda/l)} . \tag{5.83}$$

In accordance with perturbation theory the modulation depth $B(\lambda/l) \ll 1$ (condition (5.78)) and Eq. (5.83) shows that $\xi_0 \xi_H < \xi_{cr}^2(\lambda/l)$, i.e., the effective field in the undulator for the considered regime of coherent interaction holds under the threshold of nonlinear resonance or critical value in the undulator (above which the quantum modulation of particles, as well as the above-considered diffraction scattering, do not proceed).

Bibliography

P.L. Kapitsa, P.A.M. Dirac, Proc. Cambridge Philos. Soc. **29**, 297 (1933)
J.H. Eberly, Phys. Rev. Lett. **15**, 91 (1965)
L.S. Bartell, H.B. Thomson, R.R. Roskos, Phys. Rev. Lett. **143**, 851 (1965)
H. Schwarz, H.A. Tourtellotte, W.W. Gaertner, Phys. Lett. **19**, 202 (1965)
V.S. Letokhov, Usp. Fiz. Nauk **88**, 396 (1966)
M.V. Fedorov, Zh. Éksp. Teor. Fiz. **52**, 1434 (1967)
S. Takede, T.J. Matsui, Phys. Soc. Japan **25**, 1202 (1968)
L.S. Bartell, R.R. Roskos, H.B. Thomson, Phys. Rev. **166**, 1494 (1968)
M.M. Nieto, Am. J. Phys. **37**, 162 (1969)
R. Gush, H.P. Gush, Phys. Rev. D **3**, 1712 (1971)
R.B. Palmer, J. Appl. Phys. **43**, 3014 (1972)

F. Ehlotzky, Opt. Commun. **10**, 175 (1974)
V.M. Haroutunian, H.K. Avetissian, Phys. Lett. A **51**, 320 (1975)
V.M. Haroutunian, H.K. Avetissian, Phys. Lett. A **59**, 115 (1976)
D.F. Alferov, Yu.A. Bashmakov, E.G. Bessonov, Zh. Tekh. Fiz. **46**, 2392 (1976) [in Russian]
H. Al Abawi, F.A. Horf, P. Meystree, Phys. Rev. A **16**, 666 (1977)
H.K. Avetissian, Phys. Lett. A **67**, 101 (1978)
A.A. Kolomensky, A.N. Lebedev, Kvant. Electron. (Moscow) **7**, 1543 (1978) [in Russian]
D.F. Alferov, Yu.A. Bashmakov, E.G. Bessonov, Zh. Tekh. Fiz. **48**, 1592 (1978)
D.F. Alferov, Yu.A. Bashmakov, E.G. Bessonov, Zh. Tekh. Fiz. **48**, 1598 (1978)
H.K. Avetissian, A.A. Jivanian, R.G. Petrossian, Phys. Lett. A **66**, 161 (1978)
H.K. Avetissian, Zh. Tekh. Fiz. **49**, 2118 (1979) [in Russian]
J.K. McIver, M.V. Fedorov, Zh. Éksp. Teor. Fiz. **76**, 1996 (1979)
P.G. Jukov et al., Zh. Éksp. Teor. Fiz. **76**, 2065 (1979)
V.L. Bratman et al., Zh. Éksp. Teor. Fiz. **76**, 930 (1979)
D.F. Alferov, E.G. Bessonov, Zh. Éksp. Teor. Fiz. **49**, 777 (1979)
A.N. Didenko et al., Zh. Éksp. Teor. Fiz. **76**, 1919 (1979)
T.G. Kuper, G.T. Moore, M.O. Scully, Opt. Commun. **34**, 117 (1980)
W.B. Colson, S.B. Segall, J. Appl. Phys. **22**, 219 (1980)
W.B. Colson, S.K. Ride, Phys. Lett. A **76**, 379 (1980)
R. Bonifacio, M.O. Scully, Opt. Commun. **32**, 291 (1980)
A. Bambini, R. Bonifacio, S. Stenholm, Opt. Commun. **2**, 306 (1980)
T. Taguchi, K. Mima, T. Mochizuki, Phys. Rev. Lett. **46**, 824 (1981)
H.K. Avetissian, A.A. Jivanian, R.G. Petrossian, Pis'ma Zh. Éksp. Teor. Fiz. **34**, 561 (1981)
H.K. Avetissian, A.A. Jivanian, R.G. Petrossian, Phys. Lett. A **9**, 449 (1981)
H.K. Avetissian, A.A. Jivanian, R.G. Petrossian, Phys. Lett. A **5**, 263 (1981)
M.V. Fedorov: Electron in a Strong Light Field (Nauka, Moscow 1991) [in Russian]
Y.I. Salamin, C. H. Keitel, J. Phys. B **33**, 5057 (2000)
Y.I. Salamin, C.H. Keitel, F.H.M. Faisal, J. Phys. A **34**, 2819 (2001)
Y. I. Salamin, G. R. Mocken, C. H. Keitel, Phys. Rev. E **67**, 016501 (2003)
H.K. Avetissian, G.F. Mkrtchian: Quantum Signatures in X-ray Compton Laser. 26th International FEL Conference, Trieste, Italy (2004)