Chapter 1-6

PRACTICE MAKES PERFECT: A KEY BELIEF IN CHINA

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1. DIFFERENT VIEWS ABOUT MANIPULATIVE PRACTICE

Mathematics educators in the West usually put emphasis on understanding in mathematics teaching and learning (Commission on Standards for School Mathematics 1989; National Council of Teachers of Mathematics 1980). They consider the creative aspects of mathematics as the most important goal and regard drill and practice as imitatively behavioral manipulation. As Hiebert and Carpenter reviewed and commented comprehensively (1992), when the importance of both conceptual and procedural knowledge is confirmed, the question of whether to be concerned with conceptual relationships or procedural proficiency first is still left. Western educators often stress the need to build meaning for written mathematical symbols and rules before practicing the rules for efficient execution. The inherent reason is that when a procedure is practiced it can become fixed, making it difficult to think of the problem situation in another way. So well-practiced learners are reluctant to connect the rules with other representtations that might give them meaning. For practical teaching their suggestion is that the environment should help students build internal representations of procedures and conceptual networks before encouraging the repeated practice of procedure (cf. Hiebert & T. Carpenter, 1992, p. 78-79).

In China, as well as East Asian countries, routine or manipulative practice is an important mathematics learning style. *Practice Makes Perfect* is the underlying belief. Many mathematics teachers and also students believe it and consider it a general principle for mathematics teaching and learning. Through imitation and practice again and again, people will become highly skilled. Students from East Asia often top the list in international assessments of mathematics education and mathematics competitions, for example, IAEP 1992, TIMSS (Mullis, et al. 2000) and IMO. From the perspective of insiders their achievements would be attributed to a large amount of routine practice, problem solving and frequent tests. Based on the point of view of the dual nature of mathematics conception (Sfard, 1991), the analysis shows that the mechanism of routine practice is not simply interpreted as a way in which students only mechanically imitate and memorize rules and skills. Manipulation is the genetic place of mathematical thinking and the foundation of concept formation. It provides students with a necessary condition of concept formation and is the first step of mathematics comprehension (Li, 2000).

2. CULTURAL SOURCES OF THE BELIEF

Many scholars have probed the cultural sources of Chinese beliefs in mathematics education (e.g. Zhang, 1998; Leung, 1998; Wong, 1998). Generally the cultural roots of the belief *Practice Makes Perfect* could be summed up in the following aspects:

- the tradition of mathematics
- the traditional belief about study
- the examination culture
- the basic ideas in the syllabus (or curriculum standard)

China has a long historical tradition of mathematics, for example the masterpiece of mathematics《九章算术》(Nine Chapters of Arithmetic). It differs greatly from ancient Greek mathematics in ideas and principles. In *Nine Chapters of Arithmetic* there are basically no theorems concerning the topics of number theory, arithmetic or geometry. It tends towards solving a collection of different practical problems while Euclidean geometry from Greece is a way of deduction and generalization, which is more similar to rational thinking in contemporary mathematics. 算学 ("Subject of Computation") has been the synonym of mathematics for a long time in China. Under the influence of this tradition even today not a few Chinese adults still call mathematics "arithmetic". Although more and more teachers think that rules and formulas should be considered as useful tools or paths to the formation of important mathematical ideas and methods, there are many mathematics teachers who see mathematics as nothing but a set of rules and skills for solving all kinds of problems. Mastering the rules and skills is the first important task in general.

In accordance with Chinese culture, study itself is not an easy and light work but an arduous struggle. This idea could even be traced to Chinese pictographic characters of 教育 (education). 子 in the lower-left part of 教 indicates young people. The meaning of upper-left part is hard burden. The whole character of 教 means take hard burden to the shoulder of young people. The character of "育" means development. So 教育(education) presents the idea: young people grow and develop under the condition in which they make every endeavor to tackle tough tasks. However Confucius said in his 《论语》 (*Analects*): 学而时习之，不亦悦乎. Translated into English: "It is pleasant to learn and practice time and again". The pleasantness of study must come from best endeavor. As in the West, mathematics is commonly regarded by Chinese as an abstract and difficult course in school. But most Chinese students, teachers and even parents believe that mathematics is a subject every one could learn well since it is not intelligence but diligence that is essential for success. Although not every student is so clever to learn mathematics easily, most of them could learn it by working hard for success. One of the reasons people believe *Practice makes perfect* is that *diligence could remedy mediocracy* (Wong, N.Y., in press). Students are often encouraged to dedicate greater efforts to mathematics than to other courses. The ordinary way to engage them in learning is doing mathematics – solving problems according to rules and examples. As the Chinese idiom *Slow bird should fly earlier* points out, the greater efforts students devote to mathematics, the greater progress they will make. Diligent practice makes them grasp basic abilities to solve routine problems successfully.

A tradition about teaching in the syllabus of school mathematics in China is that *basic knowledge and basic skill* are emphasized as one of the goals. This phrase began from the beginning of the 1950s after the People's Republic of China was born in 1949. The syllabus has been revised several times in about 50 years, but the *two basics* idea about teaching has been kept until now. Since Chinese mathematics educators strongly believe that a solid base is the prerequisite of a stable skyscraper, it is really a distinctive characteristic of mathematics education in China. In school, the first thing for students is to build a strong, well-knitted mathematics foundation. All aspects and steps of mathematics education including the curriculum, teaching, review, exercise, homework and evaluation serve towards this goal. To put *two basics* into effect, the teaching process is usually deliberately organized to guarantee that teachers and students concentrate on concepts, theories, rules, skills and techniques. For further study, students grasp "two basics" proficiently through practical exercises in classroom and at home.

Frequent quizzes and tests are a driving force too to push students' work for "two basics" harder in manipulation and understanding.

Examination is a cultural tradition too in the long history of China. Tracing back to about 1400 years ago, the Emperor began to choose his officers by examination and a fixed system of officer selection was gradually created from then. All people who could be in the top of the examination list would be appointed as officers in some important positions. It appears equitable in a feudal age with a hereditary system. The principle of equity of the examination is confirmed today though we talk about it now in the area of schooling. Because of the large population in China students are confronted with serious competition for the chance of university admission. Under the pressure of examinations students hope to get higher scores and outperform others. Diligent practice of all learned knowledge and skills is an effective means for this purpose. "Education for exam" has been almost a directional convention for years. Undoubtedly students are driven by exams to engage in learning activities and this is reinforced by a large amount of practice so that their performance could be improved.

Summarizing this as "hardworking + rigid exam", the cultural traditions of Chinese mathematics education lead people to believe that routine practice is the efficient way for mathematics learning.

3. PRACTICE: TWO MEANINGS IN ACTION

Penetrating the behaviors in teaching and learning, *Practice makes perfect,* in Chinese 熟能生巧，is usually understood and interpreted in two ways or is even considered at two levels. The meaning of the character "熟" goes beyond "practice" or "do". It means both *familiarize with* and *be proficient at.* This subtle distinction is differentiated and leads to potentially different ways or even different levels in action. At the first level, it appears as repeating manipulation or real routine. Teachers merely let their students follow some rule mechanically to do many routine exercises such as $3 \times (-8) = -24$, $(-3) \times 8 = -24$ time and again.

It is worth noticing that experienced teachers always arrange their manipulation in a different way. They deliberately work out a set of systematic problems with hierarchy. For example, when we investigated in a school, to grasp the formula: $x^2 - y^2 = (x + y)(x - y)$, the whole domain of the symbols x and y is under consideration and to be exercised steadily by students, for example, $4a^2 - 9b^2$, $1/16s^2 - 25/49t^2$, $0.81x^2 - 0.0036y^2$, $a^2x^4 - b^6y^8$ etc., varied here in coefficients, letters and indexes. So they have different meanings in content.

Another example is in a case study of mathematics teaching. A lesson videotaped in a classroom in Shanghai was analyzed for how the teacher taught the concept of midpoint connector (a segment connecting the midpoints of two sides) of a triangle. We saw that after the concept was introduced, it is natural for the teacher to give students four questions with a system of more and more complicated diagrams (Figure 1): if the thick segment can be seen as the third side of all possible triangles respectively in the four figures, draw out all possible midpoint connectors in these triangles which have the thick segment as the third side. The left figure is the simplest case. Then the next two figures are more complicated. Students may be misled, for example, to draw a segment that is not a midpoint connector of any triangle in the figure. In the right figure three overlapped triangles shear the thick segment as the third side. Often students overlook the small triangle in the middle position and do not find out all possible midpoint connectors. The idea of designing this group of questions is to provide students with variant cases to imagine the midpoint connectors of triangles so that they are prepared to deal with complicated situations when they further their studies.

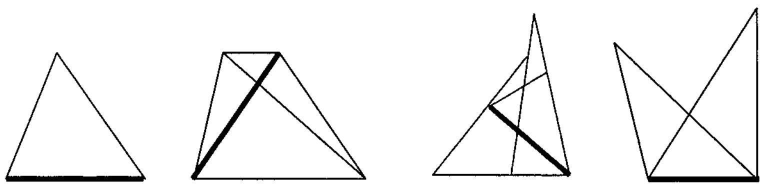

Figure 1

The other kind of change is about the method or strategy. For finding the power of a number, say 98^2, 998^2, 9998^2 ... , we saw experienced teachers begin with the usual way as 98x98 according to the definition of square. It is exactly routine. Then they would turn to the second stage to analyze these numbers and let students recognize the characteristics in these groups. Students were encouraged not to use direct calculation but to explore other ways to cope with these problems of finding the value of a square. Some would find the way to apply the formula $(x - y)^2$ to this context. $98^2 = (100 - 2)^2$, then $98^2 = 100^2 - 2\times100\times2 + 2^2 = 10004 - 400 = 9604$. Furthermore, more ingenious ways might be probed by some students:

$98^2 = 9604$

$998^2 = 996004$.

Then the following answers are conjectured:

$9998^2 = 99960004$

$99998^2 = 9999600004$,

and for the generalization

$999\ldots998^2 = 999\ldots996000\ldots004$

From here it clues students to see a similar way for computing powers such as 103^2, 999^3, 102^3, 104^4 etc.. This is a typical way of manipulation in Chinese classrooms called "一题多解" : solving one problem in variant ways.

Such a way of manipulation is usually called 变式训练, *variant manipulation* or *variation* in English. We call it meaningful manipulation – a transitional stage that lies halfway from simply repeated manipulation to understanding and has its distinctive function. It is *meaningful* because students are provided with a period of time and space to broaden their views to a rule or principle as much as possible. The mechanism within variant manipulation is usually called 举一反三, which means literally "to draw inference from reflection on one example". Students can experience themselves thoroughly in variant (just purely mathematical but not real) situations step by step so as to reflect on and to gain insight into what they practice. It therefore opens a door leading them from proficiency to understanding, i.e. lays a foundation for weaving a concept framework.

4. RESEARCH ON THE CONTROL OF OVERDONE

Undoubtedly manipulation is a sword with two edges since it may bring us a negative by-product too, even when it has a positive function. In the long run, being familiar with processes of mathematics (i.e., can compute, can use formulas and rules) is no more than the initial step. Immoderate practice may delay students' development of object formation (Li, 2000). Unfortunately some teachers may be afraid that their students could not use rules and formulas fluently, so they let them have too much assignment and make a long stay at the process stage. In contrast to the case in the West, a serious problem confronted with in China is that many students are often burdened with too much manipulation and their understanding and creativity are hindered. There is some research that has explored and reflected on the result of this kind of practice. One of the astonishing cases is an investigation into 79 college students who majored in mathematics in a small college. Subjects were asked to answer the following four questions (two items in every one):

1. Among all plane figures with same circumference, which one has biggest area? Why?
2. What is the relation between the hypotenuse and two arms in a right triangle? Why?

3. Can you compute the value of a determinant? Do you know what a determinant means?
4. Can you calculate $d(\sin x) / dx$? Why?

The finding is that all subjects know "how" but few know "why" (Table 1).

Table 1-6-1. The result of the investigation

	Question 1	Question 2	Question 3	Question 4
Know the first	100%	100%	100%	100%
Know both	0%	7.6%	1.2%	6.3%

To treat this phenomenon a project was undertaken by some mathematics educators in Jiangsu Province (Tian, et al, in press). Using both quantitative and qualitative approaches they have developed special assessment scales as guidance for manipulative skills, including polynomial operation, mathematical inference and geometric visualization and construction so far. The idea is that in practical teaching after a period of practical training, teachers could use these scales to evaluate the proficiency of students' performance. If a student or a class of students meets the corresponding criterion, it reminds teachers to stop training for this sort of practice. It means keeping within proper bounds and binding the sack before it is full. We hope such kind of research will be beneficial to the control of overdone manipulation in Chinese classrooms.

As an example, the scale for integral expression skill has 38 items and at least 75 steps (See appendix). There are different requirements in different grades (Table 2).

Table 1-6-2. Norm-referenced table for Integral expression skill

	Average Level		Pass Level		Excellent Level	
	Correct Items	Correct Steps	Correct Items	Correct Steps	Correct Items	Correct Steps
Junior 2	17.90	32.02	≧13	≧22	≧27	≧50
Junior 3	21.84	38.93	≧16	≧26	≧32	≧62
Senior 1	27.67	50.06	≧24	≧41	≧34	≧68
Senior 2	30.06	55.59	≧26	≧45	≧36	≧71

According to the Norm-referenced Table, teachers could choose appropriate level (pass, average or excellent) as a reference to evaluate their students in some grade. For instance, one in junior 3 (grade 9) could pass it when s/he gets more than 16 correct answers of items or 26 correct steps, while one in senior 3 (grade 12) should be correct in 28 items or 49 steps.

This presents a picture of the actual proficiency level of students in Jiangsu Province.

We would note that the test is timed and all items of the scale should be finished independently by students in 10 minutes. This reflects an aspect of point of view on skill proficiency from Chinese mathematics educators.

APPENDIX

Scale for Polynomial Skill

I. Simplifying (1 - 5):

1. $-3x^2y + 5\,x^2y =$
2. $(1/4)ab^2 - 2\,ab^2 =$
3. $(1/3)xy^2\cdot(-6)\,x^2y =$
4. $6\,ab^2c \div (-\,9\,ac) =$
5. $(-3xy^2)^3 =$

Factorizing (6 – 8):

6. $x^{2\,m} - 9 =$
7. $x^2 - 3x + 2 =$
8. $y^2 + y + (1/4) =$

Making perfect square:

9. $x^2 - 3x + 1 =$

II. Simplifying (10 - 14):

10. $-(2/3)\,ab + (3/4)\,ab + ab$

 $=$

 $=$

11. $-y^2 - 2\,x^2 - (-3y^2)$

 $=$

 $=$

12. $3x^2y\cdot(1/2)\,x\cdot(-2\,xy^2)$

 $=$

 $=$

13. $3x^2y^3 \div (1/3)\,xy \div (-\,xy)$

 $=$

 $=$

14. $[(-2n)^2]^3$

 $=$

 $=$

Factorizing (15 - 17):

15. $(a+b)^2 - (x-y)^2 =$
16. $(x+y)^2 + 5(x+y) + 6 =$
17. $(m-n)^2 + 4(m-n) + 4 =$

Making perfect square:

18. $1/2\,x^2 + x + 3/2 =$

III. Simplifying (19 - 20):

19. $4\,x^3 - (-6\,x^3) + 9\,x^3$

 $=$

 $=$

20. $2a^2by\cdot(-1/3\,by) \div a^2by$

 $=$

 $=$

Factorizing (21 - 23):

21. $a^2 - ab + ac - bc$

 $=$

 $=$

22. $m^2 - n^2 + am - an$

 $=$

 $=$

23. $x^2 + 2xy + y^2 - z^2$

=

=

Making perfect square:

24. $x^2 + px + q =$

IV. Simplifying (25 - 30):

25. $-1/2\ ab^2 (b^2 + 3a^2b)$

=

=

26. $(x - 2y^2)\cdot(-2x^2y)$

=

=

27. $(m^3n + mn^2) \div 1/3\ mn$

=

=

28. $(a^3b^4c - 2ab^3c) \div (-2ab^2)$

=

=

29. $(-2ab^2 + a^2b + 3ab^2)^2$

=

=

30. $(-a^2b)^5 \div a^6b^2$

=

=

V. Simplifying (31 - 32):

31. $(4x^2y - 5xy^2) - (3x^2y - 4xy^2)$

=

=

32. $6ab^2\cdot(-1/3)\ ab^4 \div 2a\cdot(-ab^2)$

=

=

VI. Simplifying (33 - 38):

33. $2s^2t + 1/2\ s^3t^2 \div 3/2\ st + 2/3\ s^2t$

=

=

=

34. $ab^2c^2 - ab\cdot 1/2bc^2 - ab^2c^2$

=

=

=

35. $x^4y^2z - xy\cdot x^2y\cdot(-xy)$

=

=

=

36. $-m^5n^3 \div 1/3\ m^2n^2 \div mn + 4m^2$

=

=

=

37. $(21a^2b^2 - 35a^3b^3) \div 7a^2b^2\cdot 2ab$

=

=

=

38. $2x^5y^4 \div 1/2xy^2 + (-3x^2y^2)\cdot 2x^2$

=

=

=

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