Chapter 1-4

HISTORICAL TOPICS AS INDICATORS FOR THE EXISTENCE OF FUNDAMENTALS IN EDUCATIONAL MATHEMATICS

An Intercultural Comparison

Walther L. FISCHER
University Erlangen-Nürnberg, Germany

1. PRELIMINARIES – METHODOLOGICAL PRESUPPOSITIONS FOR A COMPARATIVE STUDY

In a *comparative study* it is not sufficient just to state differences and correspondences between the compared topics. Foremost it is necessary to establish *criteria and norms* under which the comparison is carried out and which in the end enable an evaluation of the differences and correspondences of the stated facts to be made, especially with respect to the realization of certain aims. The constitution of criteria and norms presupposes the existence of a kind of neutral *tertium comparationis*.

2. EVENTS IN HISTORY OF MATHEMATICS AS A TERTIUM COMPARATIONIS – THESES

With respect to the aims of the ICMI-Study-13 our General Thesis is:

> Comparisons of events in the history of mathematics and of mathematics education in different cultures supply a tertium comparationis for the ICMI-Study-13.

In times of rapid developments in all fields of our human life, we need, on the one hand, power enough and ideas to follow up the (dynamical) social and economic changes - and on the other hand, we also need fixed points, in the literary sense of the phrase, or more closely to the reality of the historical process of ideas: we have to determine *cognitive and conceptual invariants* within the flow of time to master the situation of life in the future.

Our Special *Theses* are:

(1) With respect to the ICMI-Study such cognitive and conceptual invariants constitute a tertium comparationis for the Study.

They can be explored by observing the interrelations between history of mathematics and educational mathematics in different cultures, for:

There are anthropological constants and intercultural invariants in the flow of time, and in consequence there are fundamentals in mathematics education. (Fischer 1990; 1991)

(2) Indicators of such invariants and fundamentals in mathematics and mathematics education are to be found in events in the history of mathematics and in the history of mathematics education, for:

The different stages of history of mathematics represent a paradigm of the development of mathematical thinking for essentially they are indicators of universal schemes of the cognitive development in human beings.

Moreover, certain events in the history of mathematics indicate that there are everlasting fundamental educational principles, and that these invariants deliver guidelines for mathematics education in the time to come too.

Our theses are based on the presumption that in the course of time not all of the phenomena are changing. If everything would change and change again, an accumulation of knowledge would not be possible.

3. COMPARISON OF TEXTBOOKS FROM DIFFERENT TIMES AND CULTURES

3.1 Four textbooks from the Far East and the West

In evidence of the assertion of our theses we will refer to certain events in the history of mathematics. *In detail, our theses are substantiated by a* *comparison of four famous mathematical textbooks* from different times, from different cultures, written in different languages: From ancient China *Jiuzhang Suanshu* (ca 150 B.C.) and CHENG Dawei, *Suanfa Tongzong* (1592), from Japan Mitsuyoshi YOSHIDA, *Jinko-ki* (1627) and a German texbook from Adam RIES , *Rechnen auff der Linihen und Federn* (1522).

3.2 China: Jiuzhang Suanshu (about 150 B.C.)

From the Far East we choose from China the *Jiuzhang Suanshu*, the "Nine Chapters on Arithmetical Art", compiled in the early Han-Period (about 150 BC), a work representative of the development of ancient Chinese mathematics from *Zhou* and *Qin* to the Han dynasties. The *Jiuzhang Suanshu* consists of a collection of 246 problems closely concerned with situations of practical life. The problems are worked out as far as they are accompanied by rules which display the method of solution.

3.3 China: CHENG Dawei, Suanfa Tongzong (1592)

Next we draw the attention to CHENG Dawei (1533-1606) *Suanfa Tongzong*, "Systematic Treatise on Arithmetics" from 1592. This book was of great importance for the further development of mathematical ideas and techniques in China and also because of its influence and impact on the development of mathematics in Korea and Japan. We explicitly point to the fact that, although published about 1700 years later, in 9 of the 17 chapters CHENG Dawei almost literally repeats the topics of the 9 chapters of the *Jiuzhang Suanshu.*

3.4 Japan: Mitsuyoshi YOSHIDA, Jinko-ki (1627)

With respect to Japan we refer to the most famous textbook in the Edo-Period (1603-1867), to Mitsuyoshu YOSHIDA (1598-1672) *Jinko-ki*, "The Book of the Large and Small Numbers" from 1627. Though we know that YOSHIDA got lots of his knowledge from CHENG Dawei's book, it would be wrong and unjustified to state that he just repeated CHENG's book in a shortened form. YOSHIDA took in the contents and methods of the *Suanfa Tongzong* and made it his own and also dropped some of the more difficult topics of the Chinese work. However, he rearranged the remaining problems and self-reliantly adapted the questions to the contemporary situations and to the bare necessities and requirements of life in Japan and added new problems. Of course, via CHENG Dawei there is a historical string leading back even to the *Jiuzhang Suanshu.* (Also see Ueno, 2004)

3.5 Germany: Adam RIES, Rechnen auf den Linien und mit der Feder (1522)

As a typical example of a Western textbook we select a book from the renowned German Adam RIES (1492-1559), the second edition of his "*Rechnen auf den Linien und mit der Feder*" from 1522, ("Calculation on the Lines and with the Quill"). Adam RIES is known as the "German Master of Calculation". In this book calculation using a kind of abacus ("Lines") and using the "new" Indian-Arabian script (by drawing the numerals with a "Quill") was taught.

4. EARLY MATHEMATICAL TEXTBOOKS I: CONFORMITIES, CORRESPONDENCES – FUNDAMENTALS

4.1 List of Contents – the early texts used in instruction

Already a glimpse at the lists of contents, at the titles of the chapters and the main topics handled in the quoted books shows *many correspondences, conformities and concordances in topics of application and likewise with respect to the problem's reference to mathematical conceptions, to concepts and methods.*

Comparing in detail and side by side just the lists of contents of the chosen exemplary books, we may state that all of the textbooks are *collections and compilations of particular (singular) problems.* They evidently were used as textbooks to support the oral instruction in applied elementary mathematics. The topics refer to *the domains of problems relevant in everyday life in the respective times. Some problems of recreational mathematics complete the works.*

4.2 Style of presentation

Moreover, similarities are obvious with respect to the style of the books, displaying the different problem-fields in the form of *sequences of problems* and to the *presentation of the particular problems* in detail.

The *style of the problem's presentation* in the chosen texts is in question-answer-form and the arrangement of the problems shows that a kind of inductive method is practised in teaching the different types of problems proceeding from simply to more complex problem situations.

Even the *formulation of the problems* within the texts of the quoted books is rather stereotyped. It follows a 5-step scheme: *Problem Situation, Statement of the Question, Answer, Solution-Rule, Probation.* The *wording of the problem-settings* mirrors and represents thus the basic elements and the basic type of mastering of any problem situation whatever and wherever. In this respect, it is *a fundamental element* of mathematical thinking and mathematical instruction.

4.3 The solution-rules as fundamentals

The (solution-) rules in the Eastern and Western textbooks describe step sequences of solution-procedures to establish in the end the wanted result of the particular problem, the answer to the question. And again a glimpse at the problems, or just at the list of contents, shows that the '*solution-methods*' are largely the same in all of the textbooks. Formally the '*rules*' consist of a sequence of steps in the form of algorithmic routines, and this in most cases in colloquial prose. They can be considered as fundamentals as well.

4.4 Mathematical conceptions and concepts

In consequence of the above, the contents of the textbooks are largely concordant with respect to the *mathematical concepts* and to the tools constituting the *algorithmic solution-methods.*

Seen from the point of mathematical conceptions (subject-matters), these time-honoured textbooks deal with *integers* (natural numbers - the Chinese books already with negative integers), with *fractions*, with *order structures* of the corresponding numbers and *quantities* (*measuring numbers*), with the *linguistic and symbolic representation* of the elements of these number domains – especially already using a decimal system of notation, with the *elementary arithmetical operations* addition, subtraction, multiplication, division, with powers and roots.

A central part in the foundation of the solution-procedures is played by *proportions*, *proportionalities* or as we call it today *linear functions*. In consequence of that, the most frequently used method is the *rule of three* (Regeldetri). The *rule of double false* (Regula falsi) is explained and applied. Problems of measuring and calculating areas and volumes refer to *basic figures and solids of elementary geometry.* They call for methods to extract *square* and *cubic roots.*

There are problems concerning the solution of *systems of linear equations* and of *diophantine equations.*

In some respects the *Jiuzhang Suanshu* is already more advanced than RIES' book in dealing with systems of linear equations, using a kind of *matrix representation* and calculation and already a scheme known in modern mathematics as HORNER's scheme. – A particular chapter is devoted to applications of PYTHAGORAS' theorem.

4.5 Number aspects

With respect to the different aspects of the number concept we state that all of the different aspects occur in the textbooks: the number aspect of *cardinal number*, of *ordinal number*, of *measuring number*, numbers in the function of an *operator* and, of course, numbers as *elements in algorithmic processes*.

As most of the problems deal with situations of daily life, *magnitudes* play the most important role in all the early textbooks, i.e. measuring and the aspect of *measuring number* are foremost. In consequence of that, *transformations of different measuring units into one another using tables of measuring units* are found in all early textbooks.

4.6 Topics and domains of subject-matters of the textbooks in detail – strings of topics

With respect to the *subject-matters*, the *application-modes* and *–topics*, already the list of contents and the titles of the chapters manifests that many, many themes and subjects and methods coincide across and over times and cultures too – in our wording: they are fundamentals. According to the vocational necessities of the clientele of the teachers using the quoted textbooks the problems refer to *problem-fields in the domain of applied mathematics*, i.e. they refer to situations of everyday life, of craft and trade, problems in civil community, of goods exchange, of currency, of conversion of measuring units, of land surveying and engineering. Problems of recreational mathematics complete the stock of subject-matters. Lots of the subject-matters are fundamentals.

A comparison of the textbooks in West and East discloses not only on the large scale an extensive conformity in problem- and application-domains. The same is valid on the small scale. Certain topics almost literally are to be found in different times in West and East. *Strings of special (subject or formal) topics pass through the texts, through times and culture*; they are fundamentals.

In the following we display just one example of such an everlasting problem-type to deliver another evidence of the existence of fundamentals which were and will be of a central importance with respect to mathematics education. The selected example is an evidence for our *thesis why today, and in the time to come, certain classical contents are fundamentals and will be justified as topics in mathematics instruction. (Fischer, 1990)*

As a classical example for such a string of topics we choose the *problem field of motions of bodies* ('Pursuit and Meeting'). The general question reads: "When and where do they meet?"

Some of the first problems of this type occur in China. E.g. Problem 14 in Chapter VI of the *Jiuzhang Suanshu* concerns the '*Pursuit of Dog and Hare*'. The solution is done by a proportion or respectively by using the rule of three. The same problem and solution method is known from India and Arabia. In the West it occurs in different variations under the Latin title "*De cursu canis et fuga leporis*" (ALCUIN, 9th cent.). It is found in the works of FIBONACCI (1202), GEORGIUS VON HUNGARIA (15th cent.), WOLLACK (1467), Ulrich WAGNER (1483), Petrus APIANUS (1527), etc..

Those problems of motion which our students still have to solve today are everlasting problems. For they are and function as

(1) Paradigms for certain Fundamental Mathematical Aspects: Rule of Three, Proportionality, Proportions in Arithmetic and Geometry, Linear Equations, Linear Functions.
(2) They are Expansive Problems: Beyond the particular cases they induce more generalized questions, they point beyond the particular case towards generalizations of certain mathematical structures and to different domains of applications.
(3) They are the very beginning of Physical Kinematics: – They establish a pre-stage of a part of theoretical physics.

Many problems in everyday life, in sciences and techniques even today, are variations of such problems: We think of *traffic situations*, the graphical *time-tables of railways*, the *world-lines* in EINSTEIN-MINKOWSKI Geometry, think of the board-computers and the navigation-systems in our cars; in astronomy there are the motions of the *planets*, of *satellites*; in physics there are problems of motions of sub-atomic particles in accelerators, etc..(Fischer, 2001(1))

Considered from this point of view it is not surprising, that lots of particular problems from the ancient textbooks were treated in the course of times all over the world. Some of them were transmitted almost literally e.g. from China to Europe, are repeated nearly in the same wording in the textbooks of the German masters of calculation and still are found in our mathematical school-books today.[1]

Even problems from the domain of recreational mathematics pass through times and cultures. Just for curiosity we mention:

(1) Counting-off Problems: in Europe the "Josephus Problem"; in Japan's Jinko-ki "Mamako.Date".
(2) Problems of Transfusion: in Europe known from TARTAGLIA (1499 – 1547), in Japan again from YOSHIDA in Jinko-ki III/11).
(3) Magic Squares: known from ancient China as the Luo Shu and the He Tu. Later on they are to be found in CHENG Dawei's book, found in YOSHIDA's Jinko-ki. The Luo Shu is put as a problem in Adam RIES' books and is present even in the textbooks for Primary Schools of today in Germany.
(4) We could continue in our collection: "The problem of the 30 birds", the "Chinese Residue Theorem" and many, many others.

4.7 A summary – the reasons for all such conformities

The comparison of historical mathematical textbooks elucidates that certain events in history of mathematics indicate *the existence of fundamentals in mathematics and at the same time of fundamentals in mathematics education* with respect to the topics and to the *basic fundamental mathematical concept structures, the fundamental algorithmic structures.*

If we ask for the *reasons for such conformities* we may state: All these exemplar historic events refer to hitherto unchanged problem situations in practical life. And, as we may complete and state in some keywords: *Similar life situations induce similar tasks, similar problems, similar questions. Similar problem structures induce – culture and language over-lapping – similar conceptual (mathematical) modelling structures (models) and similar solution-techniques (methods) in answering the questions.*

Many topics and ideas and methods remained unchanged, for they refer *to universal basic physical and human situations of everyday life* – yesterday in the ancient times and still today with us – and they depend on the *anthropological constants of the cognitive apparatus of the human mind.* (Fischer, 1997; 2001(2))

[1] The history of such everlasting singular themes was followed up by many authors. We just point to SMITH's "History of Mathematics (1951,1953), to ROUSE-BALL's Recreations (1931), to the books of Martin GARDNER and to many books dedicated to problems of recreational mathematics.

5. EARLY MATHEMATICAL TEXTBOOKS II: ON THE ROUTE TO MATHEMATICS

5.1 The early authors on the route to mathematics

Mathematical texts and especially textbooks in the history of mathematics for a long time and in all cultures[2] were on the whole nothing other than collections of singular (particular) problems completed by rules of solutions or by the solution of particular problems carried out in detail. The rules were nothing other than descriptions of particular algorithmic solution methods adapted to the particular problems. The rules were neither formulas nor theorems but recipes. So, on the one hand *the early textbooks cannot be considered as mathematic books in the narrower sense of the meaning.*

On the other hand, such an assessment of the textbooks and of the achievements of their authors and compilers is just one half of the truth. We should bear in mind that *the conception of the essentials, the objectives and the tasks of mathematics and of mathematical cogitation cannot be characterised uniquely. These conceptions have changed several times in the course of the history of mathematics. More than once the self-conception of mathematics has changed too*[3].

The self-conception of mathematics and of mathematical cognition worldwide developed starting in a *pre-mathematical phase* progressing to *proto-mathematical* and to mathematical phases, stepping onward from *naïve*, to critical (axiomatic) and to *formalistic (non-standard) stages*. In other words, mathematical cogitation firstly started with an *enactive pre-conceptual thinking (reflecting)* by acting on the basis of sense-perceptions, next arrived at the phase of *thinking (reflecting) using iconic proto-concepts* and ended up in the phase of *conceptual thinking* – increasingly on the route *from inductive probabilistic conclusions to deductive inferences and syllogisms*. These steps were accompanied by the formation of certain *types of language* being increasingly "precise" (formalistic structured) and apt to mathematical ideas and procedures. (Fischer, 1997)

If we want to do justice to those authors from long ago, after a more detailed analysis of the books and their history we have to state: *The early texts and their authors were truly already on the route to mathematics, were*

[2] We think of the texts of the ancient Babylonians as well as those of the Egyptians too.

[3] This is true even of the situation of mathematics in the19th and 20th century. We just mention for the last 60 years the name Bourbaki and with respect to educational mathematics the keyword "New Math".

on the route from mere problem-solving to structure- and system-cognition. Therefore, the quoted texts only from our point of view represent the very *beginning of mathematics*. However, from the true historical point of view they are already *certain final points of the scientific development*. The extant texts are compilations of the results and works of many unknown authors before. The solution-rules – before being selected and compiled together with the related problems from already existing materials – had to be elaborated by human beings of an extraordinary kind of mathematical intellect (Fischer, 2001(2), 242). And of course, this development necessarily involved pure mathematical aspects too.

5.2 The route from particular problems to paradigms, from heuristics to theorem and proof

To sum up: The *old-aged texts* are not only compilations of problems concerning the practical needs of everyday life, they are really filled in with mathematical ideas and thinking. They introduced and used in a non-formal language many fundamental elements of mathematical thinking (see 4), moreover they already used, for example, the idea of variables, a kind of inductive way of argumentation, being on the way to theorem and proof by heuristic considerations and comments. Especially the grouping of the problems into chapters shows, that they were *on the route from particular problems to idealized problems and to problem-types, from problem-types to paradigms, from heuristics to theorems and proofs*.[4] This general schematic sequence in mathematical cogitation[5] can be partly shown in LIU Hui's (ca 260 AD) commentaries to the *Jiuzhang Suanshu*. (Bai, 1983; Wu, 1982; Li, 1982; Zhong, 1982; Siu 1993; Fischer, 2001(2), 244)

[4] As an Example: Following SWETZ/KAO (1977) we have to distinguish between the person of the Greek philosopher PYTHAGORAS (ca 580-500 BC), the person who first detected the relation between the areas of the squares of the lengths of the legs of special right triangles, the person who first got an insight into the generality of the relation and who formulated the so-called Pythagorean Theorem, and the person who for the first time presented a formal correct proof of the theorem.

[5] in the course of history as well as in mathematics education as well as in the cogitative development of the singular individual.

6. DIFFERENCES BETWEEN EASTERN AND WESTERN TRADITIONS

6.1 Differences within the frame of invariants – differences in educational mathematics

Invariants within a dynamical system constitute a kind of frame within which the phenomena are able to vary their shapes and attributes. This means that *intercultural historical studies at the same time do not only disclose correspondences* via times, languages and cultures, but *also disclose differences with respect to the assessment and self-conception of mathematics and mathematics education.* Of course, these differences are dependent on the state of development in the single historical periods and on the cultural background.

With respect to our theme we may state that *today* on the whole, *mathematics curricula* worldwide are in a certain sense rather uniform concerning the mathematical contents. Differences are to be seen especially in *educational principles, methods, and in instructional practice* which closely depend on the cultural context.

The differences in *educational and instructional methods* derive from the background of the human environment, of social life and its necessities on the one hand, and from the self-comprehension of the human beings within the world, from their self-conception and the assessment of the value placed on their cogitation and way of thinking on the other hand. Especially with respect to mathematics education, differences in the different countries derive from the different assessment of the value placed on mathematics in everyday life. This includes the different accentuation and evaluation of the two poles: *Relation of mathematics and mathematics education to application and intention to an elucidation of concrete and abstract structures*. And all this is closely related to the different estimation and conception of what is meant by *'being', and is closely related to the worldview of the special language of the respective cultural region.*

6.2 Different accentuations of logic in the West and East – different language structures

Every language mirrors a special view of the world and is co-existentially bound with a special type of logic. Again, the concept and role of logic cannot be uniquely determined in the course of time. Roughly speaking Western logic is a 2-valued formal logic coined by truth-values and – since ARISTOTELES (384 – 322 BC) - by syllogisms governed by the principle of contradiction. Eastern logic is a 'logic of ontological polarities' governed by the principle of complementary, harmonizing the different opposite poles.[6] ZHANG Dongsun (CHANG Tungsun) (1952) called this type of logic a "*logic of analogy*", a "*logic of correlations*".[7] The same difference causes distinctions in the use of logical operations (connectives) here and there. While Western logic stresses the logical operator of the exclusive-or (either-or, lat. *aut-aut*), Eastern logic stresses the operator of a non-exclusive-or (lat. *vel*).

Therefore, in spite of certain conformities and concordances between the early Eastern and Western textbooks – caused by the reference of the problems to applications in everyday life – and in spite of the fact that the Eastern texts distinguish between true and false solutions too (i.e. that they followed up in their argumentations implicitly a *2-valued truth-scheme*) the *accentuation of thinking in the Occident was later different from that in the Orient*. Even the application-oriented Western textbooks are thoroughly founded on the basis of ARISTOTELEAN logic, i.e. on the Greek attitude of thinking – let us say: on EUCLID's conception of mathematics. Thinking and reasoning even in inductive stages of oral instruction was (and is) in the end implicitly based on the deductive methodology of EUCLID's "Elements". Especially in the Occident the development of calculation, of calculation techniques and that of pure mathematical ideas and structures in number theory and geometry occurred in front of a background of conceptions and methods oriented in EUCLID's "Elements".

There are further differences, for example between the *character and role of definitions* depending on the different structure of sentences of the Greek and the Chinese languages. In scientific cogitation in the West the definition of a concept is done by abstraction and consists in a formation of an (equivalence-)class combining all attributes (intension) and all objects (extension) being covered by (comprising) the concept. Definitions of this type depend on the linguistic fact that in the Greek language (in contrast to the structure of Chinese language) there is a definite article (Snell, 1948). In the East, definitions are first and foremost 'ostensive', consisting in a presentation (naming) of some typical elements from the set of objects subsumed under (comprised by) the concept.

[6] See the principles of Yang and Yin and the logic of the Yi Jing.

[7] In the archaic period of Greece (before Aristoteles) a fundamental principle in the explanation of nature and the world was the formation of analogies too. See MEHL, 2003.

6.3 The role of the EUCLIDEAN scheme

To specify these general aspects from another view we point to the famous EUCLIDEAN Scheme with its steps: *Analysis, Construction, Proof, Determination* (Fischer 1989). While (even) in the instructional texts in the West the solution procedure largely follows this scheme, in the Eastern texts only some of the steps are realized or realized just in modifications. One early exception in China is LIU Hui (ca 260 AD) in his commentaries to the *Jiuhang Suanshu* (Bai, 1983; Wu, 1982; Fischer, 2001(2)).

In Western texts there is a kind of "*proof*" after having established the solution. The "*answer*" is followed by the "*Probe*" (probation). Reversing the sequence of steps in the solution procedure it is shown that the solution is not only a necessary condition but a sufficient condition too. In this way we "make sure", that the solution really is a solution to the question. The EUCLIDEAN step to "*determination*" (how many solutions are possible? – uniqueness?) is to be found in the old Eastern texts only in a quite unsystematic manner by the presentation of several solution procedures.

The original difference of the logical attitude in the East and West can be specified in detail, for example by the different presentation of the conception and method of what is called today the "*EUCLIDEAN Algorithm*" to determine the greatest common divisor of two numbers. In Jiuzhang Suanshu the solution-rule to Problem I.6. (*yo fen* – cancellation of fractions) just names the step sequence of the procedure 'subsequent mutual subtraction' without any commentaries, explanations or hints on its foundation while in EUCLID, "Elements" Book VII Proposition 1 and 2, the algorithm is deductively proved.

6.4 Notation of numbers

The development and formation of a special mathematical language using special shapes of the numerals, special symbols (token) to designate mathematical relations, operations and functions took place only rather late in the course of history. Thus solution–rules, the step sequences of algorithms in the early texts were represented in terms of colloquial language, i.e. in a non-formalistic manner. The introduction of an adequate mathematical script influenced and accelerated the development of mathematics and mathematics education enormously. This effect may be demonstrated following the introduction of the HINDU-ARABIC shapes of numerals in the West in the 12^{th} century (in some countries like Germany in the 15^{th} century), with a phase transition in Japan after the *Meiji*-Reform at the end of the 19^{th} century.

7. CONCLUSION AND CONSEQUENCES

7.1 The value of classical contents in school mathematics

The comparison of the old textbooks demonstrates that in the course of history there are *common mathematical-formal and subject-oriented strings of themes and topics, which as fundamentals determine the basic knowledge of mathematics education still today.* Those fundamentals, as a kind of invariants, will be valid and have to be observed and realized in educational mathematics of the future too. Especially, *certain "classical" contents, classical formal and applicatory subject-matters of mathematics education will still have a value in the curricula in the time to come.*

7.2 Differences

Besides such conformities there are *differences*, in the past and of course today in educational mathematics and in the daily practice of instruction. Those differences in characterization and estimation of educational principles in the course of the daily classroom work depend on the respective cultural context. The differences vary not only between East and West, they vary sometimes even between single parts of the countries here and there.

7.3 The problem of "problem-solving" – applied and pure aspects

Worldwide in school-mathematics there are *differences in the estimation of applied and pure aspects* of mathematics instruction too – variations quite often from teacher to teacher.

To master the future the younger generation needs a kind of flexibility. Therefore one problem of educational mathematics today is *the realisation of the balance between applied and pure aspects.* Especially with respect to the keyword "*problem-solving*" we may observe that already the ancient texts tended to combine problem-solving and structural-thinking. And that means: within educational mathematics *applied and pure aspects have to be observed and realized likewise.*

7.4 Chances

Conformities between the East and the West enable us to understand each other – differences present us with the chance to enrich and to complete each other.

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