Chapter 1-2

FROM WASAN TO YOZAN

Comparison between Mathematical Education in the Edo Period and the One after the Meiji Restoration

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1. INTRODUCTION

From the middle of the seventeenth century Japanese mathematics developed in its own way and at the end of the Edo period even in a small village there were many farmers interested in mathematics. Development of wasan is different from that of western mathematics. Wasan mathematicians were interested in finding interesting problems of geometry rather than developing a general theory and mathematical concepts. This is partly because in the Edo period Japanese society was stable, and serious mathematics was only used for the calendar and surveying. On the other hand, in the Edo period the soroban (Japanese abacus) was popular. The Soroban was a powerful tool for calculation and was an indispensable tool for commerce.

In the Edo period elementary education was supported by the private sector and many children went to private schools called Terakoya. Literacy and numeracy were very high in the education of Terakoya in the late Edo period. In the first section of the present paper we shall quote the first paragraph of "Teaching of Yoshishige" written by an old farmer, which will tell of the education system of the Edo period and the popularity of mathematics.

In Section two we shall discuss briefly about "Jinkoki", the most successful text book for basic mathematics. The first edition was published in 1627. In the East Asian tradition a textbook of mathematics is basically a collection of problems with answers, and also a method of how to solve a problem is given. Usually it does not explain the theory itself behind the

problems. A student is supposed to learn the theory implicitly by solving problems. On the other hand "Jinkoki" contains not only useful problems for daily life but also contains many mathematically interesting problems. Many people were interested in solving such problems and found fun with mathematics. Wasan was born and developed in such an atmosphere.

In Section three we shall discuss Confucianism in the Edo period to understand the philosophical background of wasan and education. In Section four we shall briefly describe the development of wasan. We also discuss characteristics of wasan, part of which has been mentioned above.

After the Meiji restoration the government tried to import western technology and culture eagerly. The government introduced the public education system and for mathematics the government decided to use western mathematics. But soroban was so popular and useful it remained a useful tool for mathematical education. In Section five we shall discuss how western mathematics has dominated Japan. Under the strong influence of Chinese culture, Japan developed an educational method based on solving problems to understand mathematical theory. Wasan followed the system. Even after the Meiji government introduced western mathematics for education, this tradition strongly survived. The Greek tradition of mathematical reasoning has been respected in modern Japanese education but mathematical education has failed to convince students of its importance. This is partly because Japanese society does not respect logical reasoning but rather intuition and neighbours' opinions. Even today many mathematicians and mathematical educators follow the habit. Also many problems which attracted students become old fashioned because of the rapid change of our society. We do not have text books like "Jinkoki". This makes the reform of mathematical education in Japan difficult. This also shows that education heavily depends on the society and its tradition.

2. TEACHING OF YOSHISHIGE

The Edo period dated from 1600 to 1868, the year of the Meiji restoration. In 1600 TOKUGAWA Ieyasu[1] achieved military supremacy in the battle of Sekigahara and in 1603 he received the title of shogun from the emperor. Ieyasu created the political "bakuhan" system under which government functioned through two political mechanisms: bakufu or shogunate and han or daimyo (local lord) domain. There were about 250 han's. The

[1] In the following we follow the East Asian tradition that we first write a family name then a given name.

Tokugawa shogunate introduced a four-class concept: the warrior (samurai), the farmer (hyakusho), the artisan (shokunin) and the merchant (shonin). Most samurai lived in towns and most of them were bureaucrats. Artisans and merchants lived mainly in towns. Farmers lived in villages. Farmers had a long tradition of self-governing to use water properly for their rice fields. Local lords used the system and administration was done by documents so that rich farmers were supposed to read and write well. Moreover, farmers were proud of producing rice, which was a basis of the economy in the Edo period. In the Edo period many agrarian risings broke out. There were secret rules for agrarian risings and they usually asked a local lord to replace his bureaucrats who ill-treated farmers so that they would have better treatment (Hosaka 2002). They used the social consensus of Confucianism in which a local lord should rule well according to Confucian ideals. Such agrarian risings often succeeded. But these facts also tell us that even in villages they had a system of education. In the Edo period education was done mostly privately. Many private elementary schools called Terakoya were built. Teachers were often priests in the villages or jobless samurai in towns. Also there were many women teachers. Many Terakoya accepted boys and girls but they had different desks often separated by a screen.

Figure 1-2-1. Terakoya[2], EISHOUSA Chouki "Onna Imagawa"

[2] Boys and girls were studying at a Terakoya. Usually they were separated by a screen. But in this picture there is no screen. (Kumon Institute of Children, 2000), p.171.

In Terakoya they taught mainly how to write (calligraphy) and sometimes the soroban (Japanese abacus). Also a writing master was chosen in such a way that children could learn the basics of their future lives as townsmen or farmers.

In the text "Teaching of Yoshishige" (Tamura, 1981) written by TAMURA Yoshishige (1790-1877) in 1873, we can find a vivid description of how education of the Edo period was done in a village. The writing begins as follows.

> I was born on 10th of October 1790. When I became a boy my parents advised me to go to Terakoya to learn how to write. But at that time I hated to practice calligraphy, I did not follow their advice. So my parents tried to teach me at home but I did not practice at all. One day my mother told me that a boy like you who hates learning would find his way into the gutter in the end. My grandmother was next to me and she helped me saying that the boy liked handiwork, so he would be suitable to be a carpenter. But my father was disgusted with the conversation and said that even a carpenter had to write down numbers on lumber but the boy could not do it. Listening to their conversation I felt awkward but I could do nothing. In this way I lived in my youth. I only wrote labels of seeds and agricultural diaries in bad handwriting if it was necessary. But I worked very hard in farming from early in the morning to late in the evening. When I became eighteen years old, a mathematician visited our village to teach mathematics to young men in my village. My grandfather and my uncle advised me to take the mathematical lessons and they kindly offered me all expenses for the lessons. But I rejected their kind offer. I said to them. "I am thankful for your offer but until now I have learned nothing. I am afraid that the study of mathematics for forty days would not give me a good understanding of mathematics. It is shameful for me not to understand mathematics even if I would have a good teacher and worked hard. It is difficult for me not to follow your advice but I hope you will understand me." Then they agreed with me. Thus I only worked very hard in farming without learning calligraphy and mathematics.

Though Yoshishige did not learn calligraphy and mathematics in his youth, he wrote several important books on agriculture. There remain also several beautiful calligraphies of Yoshishige. The above writing shows how the three R's education was done in the Edo period. In this writing Yoshishige also wrote a mental attitude for farmers which is based on Confucianism. There is a phrase describing how a good farmer should keep away from learning mathematics by halves. This also shows that in the late

Edo period mathematics was popular even in villages and many farmers spent much time on learning mathematics.

There remains a record of education of HAGIWARA Nobuyoshi (1828-1909), who belongs to the last generation of wasan mathematicians. At eight years old he went to Terakoya in which a priest of a temple of his village taught him calligraphy until he was thirteen years old. From fourteen to seventeen years old he took private lessons on the Japanese abacus under a wasan mathematician living in his village. From eighteen to twenty years old he studied Chinese classics. Then from twenty three to thirty three years old he studied wasan under SATO Yoshinori and got three licenses of the Seki school out of five. Note that Hagiwara was a middle class farmer and almost all his life he stayed as a farmer. He found the time to study mathematics only at night. He attended Sato's lessons on foot. He had to walk more than fifteen kilometers.

3. JINKOKI

Many young farmers and townsmen were interested in wasan in the Edo period. This is because they had the good textbook "Jinkoki" by YOSHIDA Mitsuyoshi (1598-1672). Its first edition was published in 1627 and widely read though the Edo period. There is an English translation of Jinkoki (Yoshida 2000). A detailed description of Jinkoki can be found in this English translation. Here I only point out two facts. The first is that Jinkoki showed how to use the soroban for multiplication and division and described mathematical subjects necessary for merchants and artisans but also it contains many interesting mathematical problems, which interested many people. The second is that Mitsuyoshi was troubled with an unauthorized publication of Jinkoki and he published several revised versions. With careful study of such revised editions we can find that Mitsuyoshi always tried to improve and rearrange the contents of the book according to his experience of teaching mathematics. Jinkoki covers almost all important mathematics useful for daily professional life. Since in the Edo period the society was stable, there were not so many subjects to change according to the change of the society. Jinkoki was widely read and was one of the best sellers in the Edo period.

Many books named after Jinkoki were published in the Edo period and even in the Meiji period. There is a list of more than 300 books in (Yoshida, 2000). Here I copy two pages of "Dinkoki", which is an introductory book for multiplication and division by soroban but in the front page (back page of the cover) we can find two interesting problems copied from Jinkoki. In this way many people had chances to consider certain interesting problems

and many people were interested in solving mathematical problems. And in the late Edo period they even made up problems by themselves. History of Jinkoki shows that careful choice of mathematical problems is very important in mathematical education, at least in Japan. Today our society is changing rapidly and we need to be aware of the fact that certain problems are universally interesting but other interesting problems depend on the society.

Figure 1-2-2. The first page of Dinkoki. Division by soroban is explained.[3]

4. A FEW WORDS ABOUT CONFUCIANISM IN THE EDO PERIOD

Tokugawa shogunate supported Zhu Xi orthodox (Shushigaku) and also many local lords supported it by inviting teachers to their official school for samurai class. But important Confucian philosophers in the Edo period founded their own interpretation of Confucian classics. Here I mention only two philosophers: ITO Jinsai (1627-1705) and OGYU Sorai (1666-1728).

[3] No textbook of arithmetic of the Edo period explained addition and subtraction by soroban. All the children learned addition and subtraction by soroban from their parents or neighbours.

Jinsai studied Zhu Xi orthodoxy and also Zen Buddhism both of which could not satisfy him. Later he proposed a doctoring "Kogaku" that Confucian classics should be read according to the old meaning when the classics were written. He built a private school "Kogi-do" in Kyoto and taught his doctrine. But his teaching was far advanced. He discussed with his students about interpretations of a phrase in the classics and he chose the best interpretation even if proposed by his student. In the Edo period a teacher's opinion was absolutely correct.

Figure 1-2-3. Dinkoki, The back page of the cover. Mamakodate[4] and Sugizan[5] are explained.

Ogyu Sorai was opposed not only to Zhu Xi orthodoxy but also Jinsai's Kogaku. He pointed out the importance of phonetic study in the study of Confucian classics and he himself studied Chinese. His doctrine was called "Kobunjigaku" and it became popular. He educated many Confucian scho-

[4] In the western literature this problem is known as Josephus' problem.

[5] Sugi is the Japanese name of a cedar. The original problem is counting the numbers of logs of cedar piled up. In almost all the mathematical textbooks of the Edo period they used straw rice bags instead of logs of cedar as in the figure. The problem is to calculate $1+2+\cdots+n$.

lars. Sorai was interested in not only philosophy but also almost everything. For example he criticized wasan. He wrote:

> About mathematics, mathematicians today are only interested in technical matters just searching complicated things and it is useless.

Later we will come back to his statement about wasan. Wasan mathematicians opposed this statement but some of them, for example an important mathematician of the Seki school MATSUNAGA Yoshisuke (1692-1744), agreed partially with Sorai in his letter to KURUSHIMA Yoshihiro (?-1757) and asked him to do serious mathematics. Sorai was an exception and other Confucian philosophers were not interested in wasan.

5. WASAN

Wasan is a mathematics developed in the Edo period under influence of Chinese mathematics in the Song and Yuan dynasties. The mathematic books "Suanxue quimeng" written by Zhu Shijie in 1299 and "Suanfa tongzong" written by Cheng Dawei in 1592 were brought into Japan when TOYOTOMI Hideyoshi invaded Korea. It is shame for Japan that we could not import such books in a peaceful way. (Early modern porcelain makers were also brought to Japan from Korea forcefully and they founded early modern porcelain factories in Japan.)

In the sixteenth century, Japanese merchants had trade with China and they brought the Chinese abacus. Soon, the abacus became popular in Japan and many people started to learn. "Suanfa tongzong" was a textbook for abacus in China. "Suanxue quimeng" is a book on algebra, mainly the theory of equations: Tianyuang Shu. In this book, for the first time Japanese mathematicians learned Tianyuang Shu. But the book is written in such a way that the reader can easily understand Tianyuang Shu. The first Japanese book correctly using Tianyuang Shu is SAWAGUCHI Kazuyuki's "Kokon sanpo-ki" (Old and New Mathematical Methods) published in 1637. In this book he had correctly describe how to express unknowns by letters as was described in "Suanxue quimeng". This method was later completed by SEKI Takakazu (1640?-1708). Seki named his method Boshoho (side writing method) and in wasan any number of unknowns could be used to write down equations. Nevertheless SEKI and all wasan mathematicians had no concept of polynomials. Seki's Boshoho lacked the symbol of equality as with Chinese Tianyuang Shu in which one unknown could be used for higher degree equations.

Zhu Shijie's book "Siyuan Yujian" was not brought to Japan at that time so that wasan mathematicians did not know Zhu Shijie's method to use four

variables. But still there remains a question how SEKI invented or completed his Boshoho and what kind of relation Seki had with Taniguchi's teacher HASHIMOTO Masakazu (?-1683?) and Taniguchi's student TANAKA Yoshizane (1651-1719) who lived in Osaka and Kyoto. Tanaka used Boshoho and there remains evidence that Seki and Tanaka had some contact. Also there is a natural guess that Jesuit missionary gave some influence to wasan. Carlo Spinora (1564-1622), who came to Japan in 1602 and was killed in Japan, taught western mathematics in Kyoto and gave certain influence to the earlier development of wasan. Since Christianity was strictly forbidden in the Edo period, no evidence is left. We found vague evidence that certain mathematicians in the early Edo period seemed Christian. At the moment we cannot find evidence that a mathematician like Hashimoto or Taniguchi had contact with Korean mathematicians and learned Tianyuang Shu from them. Since in the early Edo period many Japanese went to Asian countries for trade, we need to study such a possibility. After Seki's invention of Boshoho, wasan developed rapidly. At the time of Seki's student TAKEBE Katahiro (1664-739) wasan reached its highest peak methodologically speaking. For example, the calculation of π interested many wasan mathematicians. Seki used a regular 2^{17}-gon to calculate π, then he applied the Atkin acceleration, a method found in the twentieth century in numerical analysis, to obtain a correct value of π of 15 places of decimals. Takebe studied it from a slightly different viewpoint and he applied the Richardson acceleration, which was also found in the twentieth century in numerical analysis, to calculate π. From a regular 2^{10}-gon he found a very precise value of π, correct up to 42 places of decimals. But both Seki and Takebe did not explain the reasons why their acceleration methods gave correct answers. This fact already tells us a fundamental character of wasan. For example, wasan mathematicians were ingenious to calculate coefficients of power series expansions, but had very little interest in creating a general theory of power series expansion. So after Seki and Takebe there were many interesting discoveries in wasan, but these results were essentially refinements of earlier investigations. In other words, for wasan mathematics is not a theory but a collection of many interesting results.

Wasan lacked the concept of functions. Though many wasan mathematicians used power series expansion they did not realize that such power series define functions. For wasan, a power series expansion is an algorithm to calculate an area of a configuration or a definite integral. WADA Yasushi (1787-1840) spent much of his life to calculate and make tables of definite integrals $\int_0^1 x^m(1-x)^n dx$. But he never discovered the possibility of changing the interval of the integrations. Thus wasan mathematician could not find the fundamental theorem of calculus.

A lack of strict logical reasoning is another characteristic of wasan. Indifference to logical reasoning is still common in Japan today so that we could not blame wasan mathematicians for lacking serious logical thinking. Of course they had a vague notion of logical thinking. For example in the history of π in wasan we can find a lack of logical reasoning. Jinkoki used 3.16 which is roughly equal to $\sqrt{10}$. Though many mathematicians in the Edo period found that π is 3.14 ···, the value 3.14 was not used in the textbook of elementary mathematics written after Jinkoki (Itakura, Nakamura and Itakura, 1990).

Ogyu Sorai did not accept the calculation of π by using regular polygons. He argued that polygons are polygons even if they have a huge number of sides and they are different from a circle. Sorai argued that calculations by wasan mathematicians were just technical ones and they never gave the real reasoning. Sorai was partially correct in a sense that no wasan mathematician at that time had correct logical reasoning so that they could not show that their results for π were close enough to the real value of π.

On the other hand what Sorai expected was an explanation by a philosophical principle which was also not based on logical reasoning (Itakura, Nakamura and Itakura, 1990). Even today many Japanese do not respect logical reasoning.

But wasan had a different aspect which is important from the view point of education. As I wrote above, in the late Edo period many people were interested in solving problems and making difficult problems. If they could solve difficult problems they often made Sangaku, dedicated it to the Temple or Shrine and hung it so that many people read it (for an example see Figure 4). This became very popular and many mathematicians and amateur mathematicians followed the custom. In this way they could exchange their ideas for solving problems.

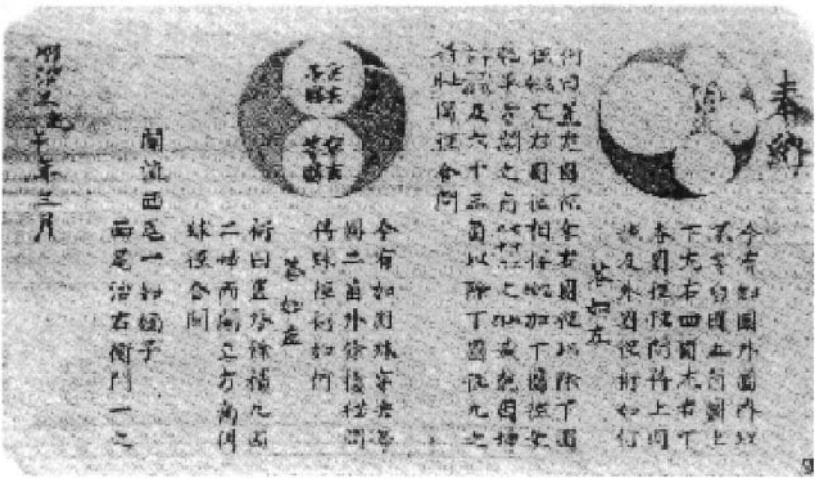

Figure 1-2-4. Sangaku, Yamashita Shrine, Ishikawa, 1870.

Often they say that wasan was a kind of art (Gei), and each wasan school formed a closed society. This is partially correct but partially wrong. Even in the early stage of wasan mathematicians in Edo, Osaka and Kyoto exchanged their ideas. After ARIMA Yoriyuki (1714-1783), who was a local lord and mathematician, published secret results of the Seki school, many advanced mathematical books were published. For example, wasan mathematician NAKASONE Muneyoshi (1824-1906) had copied mathematics books of different schools (Nakasone, 2001). Each school had its own secret results but mathematically speaking they were not so important. Also there were many amateur mathematicians all over Japan and they just enjoyed making and solving mathematical problems. Thus situation of wasan was different from serious Gei like No play or Bugei (martial arts) and wasan schools functioned as an education system. I emphasize the fact that in the Edo period many people enjoyed mathematics and there were many professional mathematicians who lived by teaching mathematics.

6. YOZAN (WESTERN MATHEMATICS)

In the Edo period western mathematics came through Chinese translations. For example, a Chinese translation of Euclid's book on geometry came to Japan in the eighteenth century but no wasan mathematicians accepted its importance. They thought that western mathematics was at a low level, since it only treated simple configurations. But through Chinese

translations wasan mathematicians learned the tables of trigonometric functions and logarithms. INOU Tadataka (1745-1818) surveyed the whole of Japan to make a precise map of Japan. He used the table of logarithms to check his measurements by using trigonometry. The table he used still survives. Surprisingly he corrected certain values of the tables, which tell us that he or his pupil understood how to make such tables. Thus in the Edo period they could use freely the table of logarithms but they did not find the notion of function behind such tables.

Western mathematics also came through Dutch books. Some Japanese people started to learn western mathematics seriously when Tokugawa shogunate opened the naval academy in Nagasaki in 1855. When Tokyo Sugaku Gaisha, which later became the Mathematical Society of Japan, was founded in 1878 certain numbers of former students of the naval academy joined it. The Japanese accepted western mathematics because it was useful to military science. Wasan mathematicians were not interested in military science and they continued to study wasan after the Meiji restoration. Western mathematics became popular through education in the elementary schools. In 1872 the Meiji government introduced the public education system so that in every elementary school western mathematics should be taught. This means that they tried to introduce calculation on paper and abandon calculation by soroban. But immediately they found that it was impossible, since soroban was so common and for daily calculation soroban was very powerful. The education of soroban still remains in school.

Also in the early Meiji period the government invited many western scholars for higher education, in which western mathematics was taught. Gradually wasan mathematicians started to learn western mathematics through Chinese translations. Such an example can be found in the writings of SATO Noriyoshi (1820-1896). Sato Noriyoshi was a mathematician teaching wasan at Seishi-kan, the official school of the domain of Fukuyama from 1846 to 1872. After Seishi-kan was closed he opened a private school of wasan at home. He left many handwritten books and notes on wasan; many of them are copies of books of old masters. Some of his books were donated to the Kyoto University by his grand child. Figure 5 is a copy of a page of his notes.

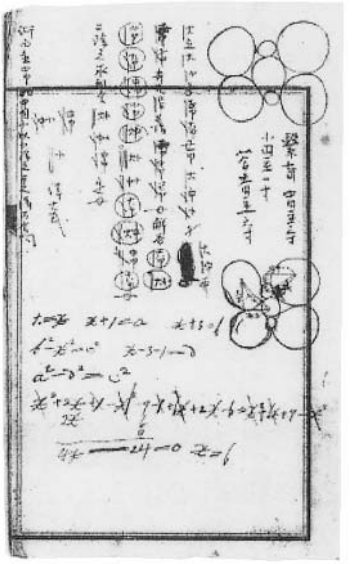

Figure 1-2-5. Sato Yoshinori: Sanpo Senmonshokai, In the upper part the problem is solved by the method of wasan, and the same equations of western mathematics were given in the lower part.

In this page a problem is given and solved by a typical method of wasan and usual western mathematics. The problem is as follows. Three kinds of circles are given. The diameter of the small circle is 1, the one of the middle circle is 3. Two small circles, one middle circle and four big circles are circumscribing as in the picture. What is the diameter of the biggest circle?

In this way, for wasan mathematicians it was not difficult to learn western mathematics up to a certain stage. Moreover, for mathematics in daily life soroban and wasan were enough. When the Meiji government started to survey lands for new tax systems, some wasan mathematicians collaborated in surveying lands. Their knowledge was enough for surveying (see for example [Nakasone, 2001]). In the provinces wasan mathematicians undertook these activities until the end of the Meiji period. Almost all of them believed that wasan was superior to western mathematics, since wasan treated more complicated configurations. Hence wasan mathematicians could not realize the fact that western mathematics had many applications to other sciences and technology. Wasan was never taught in public education. Wasan gradually disappeared and western mathematics took its place.

Also the Meiji government sent young men to Europe to study western sciences and technology. They came back to Japan and started to teach in higher education. In mathematics the most important figure is KIKUCHI

Dairoku (1855-1917) who studied mathematics in England. He became Professor of the Imperial University and later became the Minister of Education. He was a big influence on mathematical education in Japan.

In education in the elementary schools, the first Japanese translations of western textbooks were used. Gradually many Japanese mastered elementary parts of western mathematics and they wrote many textbooks. At the highest peak, all over Japan each province had different textbooks. To control quality of textbooks the government introduced an official examination system. In 1902 a bribery scandal about the official examination was made public and the Minister of Education Kikuchi Dairoku was forced to introduce the state textbooks. This was the end of the tradition of Jinkoki, in which the private sector could choose subjects to teach in mathematics. Of course the basic subjects were determined by the government already in the early Meiji period but still there remained certain freedom. The textbook writers lost their jobs. They tried to publish reference books for examinations in which they could try to convey their ideas of mathematics. This succeeded partly. For example, for reference books to the entrance examinations to universities we can find SEIMIYA Toshio' book (Seimiya, 1968) in which the author explains the methods for finding new theorems in elementary geometry to high school students.

On the other hand mathematical educators gradually lost their interest in having a wide scope of mathematics in education. They just discussed the arrangement of subjects but not the contents of the subjects. Also they have not been sensitive to the rapid change of our society and they forgot to make interesting problems for their students. Also, the old tradition of understanding mathematics by solving many problems has produced the widely spread misunderstanding that mathematics was a subject to memorize formulas and apply them to solve problems. These facts made the Ministry of Education take much control over the content of mathematics in elementary education, which has brought about the present miserable situation of mathematical education in Japan.

7. CONCLUSION

The transition from wasan to yozan (western mathematics) was smoothly done after the Meiji government chose western mathematics as a subject of elementary education. But soroban was very popular until recently. For wasan mathematicians it was not difficult to learn the elementary parts of western mathematics but they missed the importance of logical reasoning of mathematics and they could not develop a general theory. For them mathematics was not a science, but was a collection of many interesting results.

On the other hand, wasan attracted many people and in the late Edo period all over Japan people enjoyed solving problems and making up new problems.

Today in Japan students learn mathematics since it is useful for entrance examinations. But many students find fun in solving mathematical problems. Today, this tradition has almost disappeared, since mathematical educators have forgotten to prepare interesting mathematical problems that fit the modern society. Logical thinking is still missing in Japanese society and this makes mathematical education more difficult.

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