Chapter 4-5

VALUES AND CLASSROOM INTERACTION: STUDENTS' STRUGGLE FOR SENSE MAKING

Reasoning Discourse and Patterns of Student Participation in Classrooms from Germany and Hong Kong

Eva JABLONKA[1] and Christine KEITEL[2]
Freie Universitaet Berlin

1. INTRODUCTION

Values are commonly conceptualised as frames of references or as blueprints for perception and behaviour that operate as a generic background and provide meta-rules and norms (cf. the definitions quoted in the chapter by Chien Chin in this volume; Bishop, 1999). One important aspect of the concept of values is that they are shaped by cultural factors and life in the social realm. Thus they are different from belief systems and personal visions. Values are the principles, standards and qualities explicitly or implicitly considered worthwhile or desirable by the participants of a distinct social practice. This definition refers to the set of values a group of people share. Values are always both product and living condition; they are not *a priori* to interaction.

Values operate at all levels of mathematics education. Bishop (1988) identified values developed in the history of mathematics as a cultural practice. These values refer to ideology, attitude and ownership. They are not fixed, but have changed during history and are sensitive to context influences. Different communities of mathematical practice share different sets of values. Richards (1991) distinguishes at least four linguistic domains with mathematical content that are associated with different cultures concerning the expectations regarding the assumptions, the goals and underlying methodologies (see also Burton, 1999; Ernest, 1999; Love & Pimm, 1996).

In the process of recontextualisation of mathematical practices for pedagogic purposes, the mathematical discourse is transformed. FitzSimons (1999) points out that in classrooms, as a consequence of often conflicting influences from the macro-level (societal, institutional) and teachers' and students' goals, some of the public values of curriculum and pedagogy might be suppressed, whereas others might be more openly admitted. Political arguments about curriculum and pedagogy exemplify Bernstein's (1990) view that these are arguments between different conceptions of the social order and thus are profoundly value-driven. Bernstein (1996) shows that pedagogic discourse cannot be identified with any of the discourses it has recontextualised. Consequently imaginary subjects are created (p.46 ff.). However, these subjects might resemble different variations of the recontextualised practice.

In acknowledging the socio-cultural nature of mathematics and mathematics education, learning mathematics has to be conceived as initiation in mathematical culture (van Oers, 2002) or as the process of becoming mathematical, that is of being able to think and speak mathematics (Lerman, 2002). Teaching is the mediation of institutional culture by local personnel (Lerman & Tsatsaroni, 1998). From this perspective it seems obvious that values operate at the community of practice in the classroom (cf. Seah & Bishop, [ICMI book]). The public values initiate students into what counts as mathematical practice, including discursive and non-discursive elements of it, in that particular classroom. The "mathematical perspective" and its accepted values (e.g. relevance of dealing with numbers, non-contradiction, systematicity, accuracy) emerges neither by itself nor do all students come to share it. In mathematics classrooms values relate to both social behaviour and the learning and teaching of mathematics. When mathematical content is enacted in a classroom, these different sets of values merge.

The values of social behaviour that define what to do and what to refrain from are usually not explicitly warranted, though in an educational practice they are likely to be substantiated when being introduced or contravened. For some situations, giving a reason may be culturally sanctioned; a case in point is excuses. In classrooms, we find many situations in which a teacher feels obliged to offer a reason after a student has violated a norm (such as not to wear a base-cap or a headscarf during a lesson, not to give an answer without being asked etc.). The warrant might refer to an authority or might contain an explanation, depending on the general educational values in operation. In an academic mathematical discourse – if described as an ideal – giving a reason is a value in itself. One shared value linked to mathematical reasoning, is that it should not draw on any authority but the human mind and the stock of scrutinised results. And these results are neither empirical facts nor rules, but logical or theoretical relationships between the

meanings of mathematical objects. The style of the reasoning discourse established in a classroom is most likely to exhibit the values in operation and to show how more general cultural values productively intersect with the local norms of that particular classroom.

The research reported in this paper aims to identify ways in which role-related asymmetries and culturally sanctioned ways of interaction serve as an orientation for the participants in mathematical reasoning discourses in classrooms. Episodes from classroom discourse and student interviews are interpreted in the course of a contrastive analysis. One goal of this ongoing study is to identify links between structure and students' agency.

2. COMMONALITIES AND DIFFERENCES IN MATHEMATICS CLASSROOMS

Germany and Hong Kong are different in structure and policy of their educational systems, but both are providing a highly advanced education in mathematics, focussing on pure rather than applied mathematics, to an increasingly large group of the school population. One of the most significant differences is the fact that Hong Kong schools are examination driven. Several tests have to be passed in order to proceed to higher levels of school and to university. Although teachers are encouraged to adopt student-centred, activity-based and hands-on approaches to organise student learning, the atmosphere of examinations and relatively strong competition suggests that teachers and schools consider the attainment of good exam results as a priority, and students have to be exposed to constant drill on skills in order to secure their abilities in written exams. Mathematics is considered a core subject and is taught every day. There is no graduation without an exam in mathematics, and therefore it is expected that Hong Kong students spend more time out of school doing extra homework, extra lessons or studies in mathematics, than in other countries (Lam 2002).

Furthermore, teaching and learning must be focused on the public exam syllabus and on students' proficiency to work out problems of this syllabus. They perceive mathematics often as a terminology only and doing mathematics as applying a set of rules rather than thinking. (Lam et al. 1999). In international comparisons Hong Kong students perform well in mathematics tests, but do not necessarily show positive attitudes towards mathematics. (Mullis et al.1997)

In contrast, in Germany even the final exam of the most advanced level of schooling, at the university-bound high school, follows mostly the teacher-based assessment mode. The syllabus, similar to the Hong Kong syllabus, is rather focusing on abstract and technical aspects (e.g. of algebra

in our examples), on strict use of terminology and on following clearly stated formal rules, though the focus varies across the federal states. It is to be interpreted as a recommendation rather than a prescription. However, a common understanding of what mathematics teaching should be about among teachers is constructed and secured in the practical phase of teacher training, the pre-requisite to the entrance into the profession. Some see the most prominent and new challenges in Germany in the large groups of immigrants from different social, ethnic and cultural groups. This has increased the diversity of the students in many areas, but the university-bound Gymnasium is not yet very much affected.

The values assigned to teaching or learning mathematics in the intended curriculum can hardly be separated from the general educational values. If argumentation and critical thinking are educational values then mathematics is more likely to be seen as the outcome of deliberate intentions and valued as a means for analysing socially relevant problems. If functioning in the workplace is an important aim, then mathematics is more likely to be seen as a kind of generic tool-box (cf. Jablonka, 2003a).

In mathematics classrooms, mathematical practices are re-contextualised in a specific manner for the purposes of enculturation. The higher the value attributed to academic mathematics in an education system, the stronger we expect the orientation of the classroom practice to be towards the "quintessence" of this practice, such as mathematical argumentation, reasoning and proving. As to the intended curriculum, this applies to Germany in the university-bound strand of schooling, and particularly to Hong Kong, where mathematics is the core subject. In the mathematics classrooms under study, an observer would expect to find a high proportion of reasoning discourse, the stylistic characteristics varying according to the general educational values in operation: When a teacher explicates and elucidates the meaning of a new concept or demonstrates a mathematical proof, when students try to solve a problem in group work, or when in a "whole class" discussion mathematical meaning is negotiated. Consequently, reasoning discourse can be a sensible focus of analysis.

In classroom discourse "reasoning events" were identified on the level of utterances as parts of a discourse in which a person offers a reason for something because s/he interprets something as being not evident, doubtful or disputable, or is asked by another person to give a reason. The attempt to increase evidence or acceptance has to be visible for the other participants in the discourse. Some patterns in classroom discourse are already visible in small units containing just a few turns (e.g. the initiation-response-evaluation sequence), others can only be interpreted as parts of larger patterns. A focus of the analysis is to identify the distortions and systematic transformations of mathematical reasoning – as compared to reasoning in

situations in which knowledge and power are more equally distributed than in classrooms - with respect to commonalities and differences in the cultural environments of the classrooms under study.

2.1 Ideal and practice: distortions

The teacher – whose role is to mediate between the intended curriculum and the students – as well as the students, bring their own values to the classroom. Consequently, classrooms are not homogeneous in terms of values. But if there were not at least some shared values, interaction would not be possible. The students and the teacher find their ways of resolving the tension of competing obligations and constraints, which derive from their affiliations with various groups competing for their allegiance (cf. Clarke, 2001). In addition, the affiliations that an individual sees for herself may be quite different from that attributed to her by others.

In classrooms, subversive practices are likely to occur due to the role-related asymmetries. Students might choose to conform to a desirable behaviour without, at the same time, sharing the values (cf. Drew and Heritage, 1992).

The observation that social behaviour providing a warrant is more likely in a situation of breakdown (or when such a situation is anticipated) can also be made with respect to the mathematics, at least in the classrooms under study. The teachers are much more likely to provide a reason when something is considered as wrong. This seems natural because skilful participation – as reflected in obtaining a correct result when solving a task – is taken as a sign of mathematical understanding.

A study of classroom interaction can reveal the implicit values operating in a classroom. The values become more explicit if a conflict of values emerges. This is not necessarily only visible as a straightforward conflict referred to by the teacher or a student, but also in a more hidden way when the smooth flow of interaction is interrupted, when students or the teacher laugh or utter an ironic comment. Students more likely reveal their values in an interview with a researcher who is not directly collaborating with the teacher, while the teacher might be more cautious to make his or her values public especially if s/he conceives of these values as not desirable or in accord with the intended; these values are visible only in their consequences for classroom practice. Consequently, the interviews with the students do give easier access to students values not coinciding with those of the teacher, while analysis of classroom practice of both, teacher and students, reveal values of the teacher as well as those demonstrated by students as desirable behaviour.

This points to a general methodological problem of researching values. Such a research is predicated on the assumption that there is something stable behind the actions of people and that there are in fact some shared values. Because much of what constitutes a system of values that influences the behaviour of the participants of a distinct social practice is what is rarely questioned – that's just the way things are done – values are seldom made explicit in the form of evaluative, prescriptive or normative statements. As to mathematics teaching/learning this is usually the case only in curriculum documents. Most of the values remain implicit, and in addition, it is not even clear whether the participants of a social practice can easily articulate their values when being asked. There is still another level at which values are involved, that is that the interpretation of classroom episodes or interviews itself is value-driven. However, members of a research community may become more aware of their values in a setting in which these values are drawn from different cultural backgrounds.

2.2 Patterns of whole class interaction

In the German as well as in the Hong Kong classrooms there are a couple of episodes of discourse that could – albeit only from the viewpoint of logic - be interpreted as a collective process of reasoning in order to obtain a previously unknown solution of a mathematical problem. The teacher and the students collectively provide a chain of (minor) premises ("reasons") and inferences, though the discourse does not necessarily contain any single utterance that can be interpreted as a request for, or a provision of, a reason. This is when the teacher exhaustively pre-structures the discourse by breaking down the chain of inferences into a series of closed questions, which – if answered correctly – in sum warrant the resulting conclusion, that is the solution of the problem. Typically, in such a discourse, the students' turns consist of providing short, mostly single word or single number sentences as the answers to the teacher's questions (cf. Voigt, 1995). In these episodes it is doubtful whether the students perceive the process as a collective process of argumentation as identified by Krummheuer & Brandt (2001) because in none of the single steps does anything seem to be not evident, doubtful or disputable.

Unlike normal questions, which aim at retrieving knowledge from the addressee that the speaker does not have, the questions of the teacher aim at conducting the students' thinking. The questions exactly specify the domain in which to answer. In normal questions this is the knowledge that the questioner thinks is shared by the addressee.

Example 1 (Hong Kong)

04:43	**T**	Class, please read the question. What does it remind you of?
04:59	**T**	What does it remind you of, Patrick?
05:06	**Patrick**	Addition and subtraction.
05:08	**T**	It reminds you of adding and subtracting numbers. Thank you. Why do you think about addition and subtraction?
05:17	**Patrick**	Simplify complicated things.
05:26	**T**	Who's more imaginative? What comes to your mind when you're reading this question, factorizing polynomials? What do you know and what do you want to know?
05:41	**Mark**	Learning factors.
05:42	**T**	Mark...huh?
05:43	**Mark**	Learning factors.
05:44	**T**	Learning factors. Factors...Thank you. We've learnt factors in primary school, like common factors, largest common factors etc. In this lesson, we'll talk about factorizing polynomials. This is a new topic. You haven't learnt it in form one or form two. First, we look at a simple example. [T writing m times brackets a plus b is equal to m times a plus m times b]

This episode can be interpreted as a strictly planned teacher presentation with allocated parts (cf. Ehlich and Rehbein, 1986). It can be reconstructed as a consistent chain of assertions uttered by one speaker. However, the teacher's questions aim at still maintaining a minimum of students' participation. In order to guarantee that it goes off smoothly, this pattern of interaction needs some skills and experience in formulating the directing questions. The decision about what exactly is stated as the known and as the unknown in the question is based on the expected mental operations performed by the students and aims at influencing these according to the direction in which the teacher wants the discourse to continue (cf. Jablonka, 2003b).

The reasoning involved in this pattern is not intended to be self-initiated by students. The teacher wants Patrick to *explain* his assertion; the answer does not fit the teacher's propositional plan. Consequently, a new directing question follows which is less ambivalent.

This pattern is common in all the classrooms under study. However, there are differences in the ways the teachers deal with unexpected comments and answers, that is with ruptures. Very tentatively it can be said that the Hong

Kong teachers have a tendency to repair the pattern by switching more to the presentation mode, while the German teachers tend either to open it by introducing normal questions instead of directing questions or switch to a presentation. In an arbitrarily chosen Hong Kong lesson, an episode of teacher talk in the course of a demonstration of a problem solution takes the form of a "dialogue" in which the teacher introduces the reasons he provides by rhetorical questions or rhetorical contradictions. In the classical European tradition of rhetoric this would be interpreted as a stylistic means used in speeches that aim at convincing the audience.

2.3 Barring empirical arguments

The following two episodes, one from a Hong Kong and one from a German classroom, are similar in that they contain self-initiated comments of students who introduce empirical arguments. In both episodes the students are asked to solve a word problem and both problems refer to the context of farming. The rejection of the empirical referents is not warranted. However, if the speaker decides not to utter a reason, the lack of understanding remains unproductive. It is likely to lead to frustration or disapproval on the side of the listener. This might be the case in the two episodes. In both cases the teacher decides not to give a reason for the fact that the points made by the students are not to be argued about or are not of interest (see example 2, 11:20; example 3, 33.15 – 33:30).

Example 2 (Hong Kong)

Some minutes before this, the teacher stated a problem to work on: A farmer has some rabbits and some chickens. He does not know the exact number of rabbits and chickens...but in total there are ten heads... remember...ten heads...and there are twenty-six legs. The teacher shows a strategy to find the answer by trial and error: for example start with the assumption that half of them are chickens, then there are ten chicken legs ...

[S wants to say something]
11:13 **Teacher** Okay...please say.
11:16 **S** Rabbits have only two legs as the other two are hands.
[whole class laughs]
11:18 **Keith** [laughs] He is impolite.
11:20 **Teacher** We usually say they are legs instead of hands. Don't argue about this point...okay?

It can be argued that the laughter of the other students shows that in this classroom the practice of solving a typical textbook problem is well

established. Most students know that the chicken is to be taken as a substitute for an animal with two legs and the rabbit for one with four, and that the farmer, who knows the sum of his animals' legs, is non-essential. In a previous conversation, in which Louis and Kelvin try to solve the problem, and in the interview, one of them talks about pigs instead of rabbits without noticing. Thus the student's self-initiated comment can only be taken as sarcastic by some. Interestingly Keith labels the student as being "impolite", which is exactly the attribute a person can expect when not conforming to the implicit rules of a distinct type of a routinised discourse. In another context, the introduction of an alternative assumption can be a legitimate move in a mathematical reasoning discourse. In the given example it would lead to a contradictory problem statement and could have been refused on these grounds. However, the teacher might have interpreted the comment as a subversive act that aims to distract participants from the proper theme of their conversation.

Example 3 (Germany)

31:01	**Teacher**	Well, now we want to address ourselves to the following problem.
31:17	**Teacher**	[draws a square on the board] For that purpose we look at this square, please. This is supposed to represent a square plot of land owned by a farmer. Now you have to tell the rest, would you please tell the story?
31:39	**Researcher**	The neighbour approaches the farmer and says 'It would be very advantageous for my planning if you could give me a strip of one meter of your land from the cross side [Teacher points to the upper horizontal edge of the square], I then would give you in return a strip of one meter from my property on the other edge' [Teacher points to the right vertical edge of the square] Would you agree?
32.20	**Teacher**	Okay.
32:26	**Anton**	No...it isn't the same...hm...hm.
32:28	**S**	Unfair.
32:41	**S**	No...well...because he then has less (...) You can...you can already see that from the drawing that you would then get less from that plot.
32:47	**S**	Well...somehow...well I would refuse because this is (...)//
32:48	**Teacher**	//Wait...I didn't get that...this is such a mumbling//

32:49	**S**	//I would refuse because somehow...ehm...you have a piece...you havn't such a corner (...) plot there around.
32:58	**Teacher**	No you have...ehm...what's the shape of the new plot...it's not a square anymore...but it's still at least a rectangle. So you would refuse because you'd prefer a square over a rectangle.
[Laughter]		
33:15	**S**	I mean...ehm...want to take away on one side and put on the other side...where...oh where is then the neighbour's garden...on the left side or there below?
33:22	**Martin**	Thus it is around.
33:23	**Teacher**	It is around.
33 24	**S**	But that's a bit illogical...isn't it?
33:27	**S**	Why?
33:29	**Teacher**	That's...that is...ehm//
33:30	**Martin**	//Man//
33:30	**Teacher**	//uninteresting. Our question is...ehm...whether the one concerned...well...whether he should exchange... whether this is favourable...or whether...ehm...it doesn't matter or...it is just this what it is about... whether for this very farmer who now is up to mischief in there...it is...ehm...yes a problem.
33:56	**Kerstin**	Well I would say...I would assume...ehm...if I...in this piece he wants to...I don't know...steal from me...if I hadn't anything there...I mean a shed and stuff...then I could as well take the meter down there because the area is the same.
34:15	**Martin**	Ey?
34:16	**Teacher**	Ah...that was...is the area the same?//
34:16	**Felix**	[putting up his hand] Yeah//
34:17	**Teacher**	//That is the good question.
34:20	**S**	No...isn't the same...because...ehm...before...well now...he is missing exactly one piece...he has the full side and now he has from his shorter plot also...ehm...only the shorter half from the bottom... and so he misses exactly one square meter.
34:33	**Teacher**	Hm.

In this episode some students immediately give the answer, one of them refers to the drawing. Perhaps because of lack of evaluation of these comments, the students then start to introduce different reasons and the

discourse moves from visual evidence to empirical arguments and eventually (not reproduced here) to an algebraic description of the drawing, which was intended by the teacher. The teacher explicitly keeps away the empirical argument of the student who is wondering about the shape of the neighbour's property, similar to the teacher in the Hong Kong classroom. He gets some help from Martin in doing this. But the comment is not interpreted as a diversionary tactic. However the student expresses her dissatisfaction with the answer, again self-initiated. This is one of the rare instances in this classroom in which a student expresses disagreement with a statement of the teacher.

2.4 Ambivalent reactions and tactical behaviour

Students and the teacher bring to the classroom their knowledge of the interactional pattern of reasoning in everyday contexts. This knowledge becomes explicit only in situations in which the co-operation of the interlocutors is at stake or breaks down. If a person utters a reason, this aims at transforming the knowledge or values of the addressee in a way that he or she accepts or understands an action or an assertion of the speaker. In an everyday context this is only necessary if the speaker has a reason to think that otherwise the listener would not understand her action or assertion. If the speaker wants to prevent a breakdown of co-operation, which would lead to a breakdown of the system of actions in which the participants are engaged, she decides to utter a reason. Such an attempt can be successful or unsuccessful. Success is assumed when the listener utters or shows a sign of appreciation and understanding. If unsuccessful, several cycles may follow. Success is only possible if the participants share a system of knowledge and values. If this were not the case, the reasoning would theoretically lead to an infinite recursion. In everyday contexts the breakdown usually happens after a few unsuccessful cycles (Ehlich & Rehbein, 1986).

In the classrooms under study, the pattern of reasoning is commonly missing the step in which the listeners utter or show a sign of appreciation and understanding when the teacher is addressing the whole class. It is usually not the case that all students get a turn to express their understanding. In the Hong Kong classroom from the third school the teacher frequently addresses the whole class by asking something like "Did you understand, class?" and is then confronted with no reaction. A non-reaction is ambivalent; it can be taken as both a sign of understanding or as a lack of understanding. It is likely that the students even use a non-reaction to pretend understanding in case of a lack of understanding. This helps them to circumvent or to undermine the formal and compulsive character of classroom interaction. This means the students are using their knowledge of the functioning of the

reasoning pattern in order to pretend approval and co-operation. The teacher might use the same reaction, that is neither showing a sign of understanding nor of not doing so after a student has uttered a reason, to provoke further reasoning on the side of the student. This might be the case in example 4. Such a use of the knowledge about patterns of interaction can be called *tactical behaviour*. It even takes the form of showing a reaction that expresses the opposite of the mental state (e.g. approval in the case of disapproval) instead of only performing the ambivalent action of not reacting.

In the following example, the ambivalent reaction is explicitly expressed by the student.

Example 4 (Hong Kong)

36:07	**S**	[to S] How did you get that? [in Chinese]
37:10	**S**	(...)
38:27	**T**	Ida, Understand what I'm talking about?
42:19	**IDA**	Maybe.
43:27	**T**	Maybe?
44:27	**IDA**	Yeah.
45:17	**T**	Where don't you understand?
46:13	**S**	That simple? [in Chinese]
47:13	**S**	This was what he said. (He) didn't do the calculation himself. [in Chinese]
47:19	**IDA**	I don't know.
49:05	**S**	[Class giggles]

The following short conversation is typical of the first German classroom from the LPS. The teacher offers an explanation because the student has got an incorrect result and the student acts like being convinced, though by analysing his written productions and from the interview it can be said that this is tactical. In the interviews some other ways of acting as a "professional student" were discussed.

Example 5 (Germany)

09:20	**Teacher**	No that's not right...this negative sign here in the brackets...you have to...that means we have to envisage that it's multiplied by minus one...which means you write three minus z plus one half minus two.
09:38	**Günther**	Oh right...so I have to.

More interestingly, tactical behaviour also appeared when students discussed a problem in group work. It was only the interview that gave information about Otto's tactical approval and Norbert's strategic behaviour – both of them could not make a lot of sense of the task:

Example 6 (Germany)

31:53	**Norbert**	Yeah we've got that. I don't, but I don't know how we can – that's the measurement of the area. Oh we probably have to uh explain how we've arrived at *a* minus *b* in brackets squared. Here I mean.
32:15	**Tom**	It says here we only have to make a picture.
32:17	**Albert**	Finished?
32:17	**Norbert**	No wait, give me another sheet of paper [to Albert]. I haven't got one. I'm poor. No, I need that, I don't want that spoiled.
32:20	**Otto**	Haven't got a sheet.
32:24	**Norbert**	Sheet, sheet...Right look now we've got the square.
32:33	**Tom**	Yeah.
32:37	**Norbert**	That's the size here.
32:39	**Otto**	Yeah do it, get it done.
32:41	**Norbert**	Right and this is *a* squared, this is *b* squared, this is minus *ab* and minus *ab* right?
32:47	**Otto**	Yeah
32:48	**Norbert**	So, because that's, and because that's minus, because that's now minus, um this has, this has to go.
32:58	**Otto**	Yeah, yeah that's clear.
32:59	**Norbert**	This here then stays.
33:02	**Otto**	*a* squared?
33:03	**Norbert**	*a* squared then, this thing here and *b* squared.
33:10	**Norbert**	Yeah, yeah like that and then well like that, if you were to do that as a line, this here would be *a* and this bit *b*, then you take it away, then this piece here is *a* minus *b*.
33:25	**Otto**	Yeah.
33:26	**Norbert**	Yeah? Now when you have this bit *a* minus *b* and this bit *a* minus *b* here, and then the square of *a* minus *b* in brackets, then that must be the area.
33:45	**Otto**	Ah.
33:46	**Norbert**	Now look, *a* times *a* must be the area or *b* times *b*.
33:52	**Norbert**	[dismissive gesture in the direction of Otto and Seppi] Don't understand any of it, do you.
33:59	**Norbert**	But it's logical, look.
34:06	**Tom**	The microphone's not right.
34:09	**Norbert**	a minus b is this distance here, this here [emphasized]
34:14	**Otto**	And this as well (points to it on the sheet)

Consequently, when interpreting classroom discourse – be it as a participant or as a researcher – assuming sincerity can be problematic. Especially in a situation of unequal distribution of power and control over interaction, tactical behaviour is likely to occur. In mathematics classrooms this is often the case when the disapproval of a reason provided by a student can be taken as a lack of understanding, which, in turn, is evaluated in terms of achievement. This may establish a dynamic: the more tactical approval a student shows, the fewer reasons he/she is offered as explanations, the less chance he/she gets to make sense.

3. VALUES AND LEARNING MATHEMATICS

3.1 High achievers do (not) know mathematics better?

The German students in this selection of interviews attend their second year in a (university-bound) secondary school for the better achieving students, an inner city German Gymnasium, and belong to the upper third of the school population within the highly selective school system. Concerning their personal achievement in mathematics, their mathematics teacher considers them as a bit better than the average. Typically, their parents belong in majority to the middle class, among them quite a few academics, in particular teachers and lawyers. They value education very much; and they strongly support their children's schooling. Only very few students in the class come from an immigrant background. The 8th grade lessons in algebra are about transformations of algebraic terms.

It has to be emphasised here that students who have reached the Gymnasium, at the age of 10 or 12, are privileged in many aspects: They normally have a well-educated teacher specialised in mathematics (this is not regularly the case in the other secondary school types), and are by their teachers – and socially – considered as the high-achievers and possible future elite. In the TIMSS-results, students from this school type achieved much better results compared to students that have been placed into the German school types for low-achievers. They generally show much self-esteem and do not attribute problems of understanding mathematics or disliking it to themselves as a personal problem, they know that they can do well in other school subjects and are generally successful in school. For them it is just the mathematics that can cause trouble (if not the teacher). Given, that among these privileged students many utter their lack of sense-making, an aversion towards, or a frustration in mathematics, it can be assumed that mathematics must be more threatening as a selection means for

students in other school types, who feel that they have already lost their chances for prestigious professional careers or jobs, because they were placed in a school for low achievers. In the following, statements from German students are contrasted with those from the Hong Kong school.

3.2 Searching for meaning, importance and significance

The biggest struggle in mathematics is about meaning and significance, which is not revealed by the teaching practice. Why is mathematics to be learnt and taught? Why is mathematics applicable? These questions go beyond ordinary daily lesson plans and are rarely touched on by the teacher.

3.2.1 Why do we learn mathematics and what is important? What is math about?

Students learn to abandon the question 'Why' and 'For what' and have nearly no ideas about where to apply math beyond the shopping mall; their knowledge about application and applicability is very limited, and mostly wrong.

Interviewer:	What does this sort of math mean to you essentially? What's it good for, what can you possibly do with it?
Steffi:	It's not clear to me at all.
Sharon:	Well, it just belongs to basic knowledge. You just have to be able to do it, and then you're allowed to forget it.
Interviewer:	But what's the use of knowing it?
Sharon:	Well, I don't know if//
Steffi:	//Perhaps if you want to become a mathematician
Sharon:	Or working in a bank. (Germany)
Interviewer:	Uh, you like this lesson?
Peggy:	Uh, a little bit. I can, I mean, I can discuss with my classmates.
Interviewer:	Uh, what do you think is the most important thing to learn in this mathematics lesson?
Peggy:	The most important? Calculation.
Interviewer:	Calculation is the most important?
Peggy:	Yep.
Interviewer:	What have you learnt?
Peggy:	I've learnt...(learnt). I don't know. (Hong Kong)

In the German interviews, the answer to the question if they like or dislike mathematics or are interested in doing mathematics has often nothing

to do with mathematics as a subject – which they feel they do not know about – but with mathematics as a school subject and the way it is taught. This can be also seen in the Hong Kong interviews: The mathematics teachers might be nicer than other teachers, they can chat with their classmates during the lesson, the atmosphere is more acceptable. Sometimes quite surprising values are assigned to mathematics; on the other hand, aversions against mathematics depend on the teaching style, their own lack of understanding and the helplessness they experience in the lesson.

Interviewer: Um... Do you like this lesson?
Rachel: I like it. Uh, I don't know why, but I am interested in mathematics lessons. All teachers, no matter now or before, are quite good.
...
Interviewer: Do you like Mathematics lessons?
Rachel: Yes.
Interviewer: Why?
Rachel: Well, because, I do not know how to tell you. But I think that mathematics can make me think faster.
Interviewer: Yep.
Rachel: That is, I need not to use my brain to think in a lesson usually. That is, I only need to memorize all the information in books. Also (in mathematics lessons), it can- that is I can find the answer when I see the question, so there is no need to find the information in books. (Hong Kong)

3.3 Enjoyment and math

'Doing math' offers extra practice and some advantage in comparison to other students. However, enjoyment or fun is rarely connected to intrinsic features of doing mathematics, in contrast to other school subjects. The fun might be rather limited, and does not offer sense making either, but a kind of substitute for a frustration that is going alongside with mathematics. To feel already some kind of enjoyment in being quicker than other students when solving some tasks, and then being able to recapitulate possible mistakes, is alarmingly modest. However, solving tasks or problems correctly, i.e. avoid mistakes or errors, is considered a very necessary action for the assessment procedure. Errors are not allowed, there is nothing to learn from them, but one has to hide them in front of the teacher.

Interviewer: Um, do you like studying mathematics or having mathematics lessons?

Peggy: Having mathematics lessons? Mm, I like the atmosphere of the lesson.
Interviewer: What atmosphere?
Peggy: That is very relaxed. He would not bind you inside the classroom and not allow you to say anything.
Interviewer: What about learning Mathematics?
Peggy: Learning Mathematics? Mm, I do not do that. I// seem to be, on average.
Interviewer: // Do you like that?
Peggy: Mm, if- if it is not too difficult, I am interested in it.
Interviewer: You are - are interested in (...)
Peggy: Yep, I like to learn new things, but when I learn the new thing, I learn in a slower speed. If he teaches too fast and I cannot catch up, I will not like this lesson lastly. (Hong Kong)

Interviewer: Do you like this lesson?
Polly: Yes. //We calculate in every mathematics lesson so I am not bored.
Interviewer: //Why... Why you are not bored?
Polly: I have to think during calculation. I can use my brain to think.
Interviewer: Others are boring? It cannot be recorded by nodding your head.
Interviewer: What do you think is important to learn?
Polly: This lesson or the normal lessons?
Interviewer: Normal lessons.
Polly: Uh, what is important to learn?
Interviewer: Yep. What do you think is important to learn?
Polly: My- my brain can think faster.
Interviewer: Yep, then this lesson?
Polly: Also the same.
Interviewer: Have you learnt something that makes you think faster?
Polly: Yep.
Interviewer: What is it?
Polly: For example, that parenthesis, I forget to write it and I could think of it. (Hong Kong)

....
Interviewer: Do you like mathematics or mathematics lesson?
Polly: I like.
Interviewer: Why? (laughing) It is difficult to answer.
Polly: Yep.

Interviewer: You can say if you do not.
Polly: Because this lesson is the one which let us think a lot. (Hong Kong)

Osbert: I didn't figure that out
Interviewer: You didn't figure that out. But why do you think it's important?
Osbert: Because I didn't understand.
Interviewer: It's important because you didn't understand?
Osbert: Yes.
Interviewer: What did you understand? You've said that you understand those methods at the beginning. Do you think it's important after you learnt those?
Osbert: I don't have to learn about it or think about it after I learnt it. (Hong Kong)

Interviewer: You enjoy doing (math)?
Friedrich: Most of the time, yeah.
Interviewer: Do you think you have a talent for it?
Friedrich: I don't think I am excessively talented. Because I enjoy doing it, I have the feeling that I get more practice, I don't know. Take these daily exercises that we always do, I usually get through them quite quickly and have, well I work out pretty quickly and then I've always enough time to check them through again. That is I can go through each problem twice and can usually spot most of the mistakes.
Interviewer: Is speed then the criterion so to speak, the quickest is the best.
Friedrich: Not necessarily, with me it's always much better if I finish quickly, as I've then still got time to go through it all again, because I've usually got one or two mistakes and then I've got more time for them. (Germany)

The Hong Kong teacher supports extra homework to foster students' proficiencies, but only few students can do so:

Interviewer: How often would your students do mathematics exercise at home?
Teacher: ...Usually, they will do homework. For the extras...they will do extra mathematics in advance. I've emphasized that students could do the following exercise if they could. For example, if they still have a lot of time after finishing their class work and homework, I allow them to do other

mathematics exercise. Some students have already finished half of the exercise in trigonometry although I haven't taught it.

Interviewer: Does it happen frequently? Does it happen frequently? Would a lot of students do exercise by themselves?

Teacher: Huh?

Interviewer: Would a lot of students do exercise by themselves?

Teacher: I...we call it beyond syllabus.

Interviewer: Beyond syllabus means...

Teacher: It means you go beyond the syllabus. About four or five students will do that. (Hong Kong 1, TI2)

3.3.1 Math is fun? An ironic remark

The ironic turning of "fun" in mathematics into magicians' or conductors' work shows the sad state of school mathematics: you do things correctly in following the rules, but it is just magic and does not make sense to you. And at the same time again: 'math has nothing to do' with the students, it is not their math, but is someone else's math, a strange or foreign propriety you have to master, but have no access in understanding.

Martin: Math is one of my favourite subjects. It is fun.

Interviewer: What is fun about it?

Martin: No idea. Numbers that nobody understands. You try to understand what you are supposed to do with all the x's and that's fun, juggling with numbers.

Interviewer: Like a sorcerer?

Martin: Yeah, to do something magical, you get a result, and then someone says that it is right. Yeah, but sometimes you get nothing out, just something you can't understand like why x equals y or the like. But it's fun anyway. Well...if the maestro says that it's so...then so be it....I really have to believe what he says. (Germany)

3.4 It is the assessment that counts: The only important goal is performance

Mathematics is considered even by the majority of these privileged German students at the Gymnasium as a compulsory enterprise without significance for them; it is for selection, for checking the mind, does not

make sense, is not understandable, an empty set of rules, and can become terrifying when turned into assessment tasks.

3.4.1 Dislike of mathematics as selection means: math is just terrible

Sabine: Classroom tests written classwork in math always drive me crazy in advance. These are the only ones which make me so nervous, in other school subjects I do not mind at all, but in math it is just....uaaah!

Interviewer: What is it that makes math so terrifying for you?

Sabine: No idea, it is just terrible. I do accept all other subjects, but math ... it's fully stupid. And I do not know why! (Germany)

Interviewer: Yep. Uh, do you like learning mathematics or having mathematics lessons?

Olivia: I do not like it very much.

Interviewer: You do not like it very much.

Olivia: I like Mr. Ng's lesson as I can chat. However, because I... I did not like mathematics since I was a child, I did not like listening to the lessons and my results were not good, so I did not like having the lesson. Also, mathematics is not like other subjects. You do not listen in the lesson of other subjects, then you pick up the book. There are some words in the book. If you revise it, you will pass the tests. However, mathematics is different. You... even if I pick up the book, but without the explanation of Mr. Ng and others who teach me: It changes like this and this changes like this and it substitutes like that. I cannot see that. Therefore... therefore, what have you asked?

Interviewer: Do you like having mathematics lesson or learning mathematics? (laughing)

Olivia: //Yes, I do not like it, I do not like it. (laughing) (Hong Kong)

3.4.2 Good performance and active participation in the classroom: one has to care for being recognised

In German schools, assessment of oral participation is an important aspect of evaluating performance in mathematics. Consequently one has to show that one has done correctly, and especially because of the teacher

based assessment modes, one has to constantly care for being recognised as belonging to those who can do so. To avoid to be recognised as somebody who did not do correctly is frequently shaping the pattern of classroom participation. It can be argued that the teacher-based assessment mode forces students and teachers to exhibit an ambivalent behaviour due to a conflict of values: On the one side the teacher is the facilitator of learning, maybe a friend who cares, on the other side, s/he is the (sole) responsible for assessment results, who can easily become the enemy of the students. Even if they like the teacher, by the very system, they might keep a distance.

Interviewer: Martin, I noticed you went to ask Mr. Reimer if your solution of the given task was correct. But you were completely sure already before that you had a correct solution. Why did you ask Mr. Reimer then?

Martin: I have to make sure that he knows and notes that I did it correctly, that I have well done. You have to care for that. (Germany)

To be recognised as a good and an actively participating student is considered as important in Hong Kong schools as well, but there are different perceptions of what might be honoured by teachers and valued by classmates:

Michael: Well, here. … I think we were not energetic enough.

Interviewer: Energetic means...

Michael: You were not answering the question actively. That means not - actually, it is not necessary to - not necessary to reach the level of active...the situation was that there was no one put up his or her hand. We only stood up to answer the question when teacher called us. This was not so good.

Interviewer: You think the whole class...

Michael: Yep, to see the whole class as a whole. Also including me. (laughing)

Interviewer: (laughing) How about you, why don't you put up your hand?

Michael: Ha?

Interviewer: You were shy?

Michael: No, think...I think that others would put up their hand to answer. Also, anybody else liked to answer- answer the questions. I don't know why.

Interviewer: Some students like answering the questions?

Michael: Yep, there is one. He would answer, so I gave the chance to him to answer. (laughing) He would like to be recorded. So. (laughing) (Hong Kong)

Interviewer: After watching the whole tape, do you have any other thing you would like to say, as a supplement or about the school, no… what the classmates and the teacher did and said that you think it is important?
Nina: In- I only think that there- there is one person speaks a lot. That means he always put up his hand.
Interviewer: (laughing)
Nina: Mm, there are only one or two people like this.
Interviewer: //(…) What?
Nina: //Not much
Nina: I don't know. He is also like this normally. He always likes...how to explain...want to be focused. (laughing)
Interviewer: (laughing) I see, I see.
Nina: Yep.
Interviewer: He put up his hand, why? //Answering the question?
Nina: //Answering the questions… always. Yep, sometimes he answers it wrong and he loses face.
Interviewer: (laughing) You think that this is special?
Nina: Yep. (Hong Kong)

Interviewer: What were you doing at that time? Were you afraid that Mr. Ng would ask you?
Osbert: No. He seldom asks me. Usually, he asks…the ones who often answer his questions
Interviewer: Huh? Whom does he ask usually?
Osbert: The students who often answer his questions (Hong Kong)
Interviewer: You didn't understand these?
Interviewer: Did you raise your hand?
Osbert: No. Not many students raised their hands.
Interviewer: Not many students raised their hands
Osbert: Not many students understood. (Hong Kong)

3.5 Meaning and understanding in learning and doing mathematics

Some of the German students recognise that they might need alternative ways of approaching a mathematical topic, but in most cases are only

offered one, which might be appropriate only for some, but not for the majority. If they do not know why they do not understand, they learn by heart.

3.5.1 Understanding why or learning by heart

Most of the German students exhibit in the interview that they have given up in trying to constructively participate, they follow the teacher's guidance and hide if they fail to understand the mathematics taught: It does not seem worthwhile to try to understand why mathematics is like it is, because you never find out by yourself.

Interviewer: What do you both prefer? Like this (a given discovery problem)? To reflect about, to draw something and to try to find out why? Or to calculate those other tasks.

Cordula: My, yeah, ... to find out why it is so, is more fun.

Interviewer: This is more fun for you?

Cordula: Yes.

Sabine: For me, I prefer tasks, it is too stressful for me to find out why, why it is just like this and not otherwise – alone, I would never have found out at all! So for me, it is enough to do tasks of which one just knows how to do them. It is too stressful to think about why – you never find it by yourself anyway. It is ok, when the teacher says it is ok.

Cordula: Yeah, but I would appreciate more when we now solve problems and also learn to know why this is like it is or not, the case that I just do problems and think yeah, it will be ok, this is not my favourite case. It makes much more fun to think about...why something does work or not. More than just do some algebraic transformations correctly.

Sabine: But it does not help me at all if there is a drawing or the like, I might have understood, but I would not be able to imagine, I am more such a type who can learn by heart.

Cordula: But when I have seen why, for example by a drawing, and understood why it is like this, it is much easier, I can reconstruct what we have done.

Sabine: But I cannot cope with those drawings, maybe there is another way that works for me – I don't know. If there is another way to work on this problem on your own and to find out, maybe that it seems to be more logical to me or the like, then I could probably also cope better and prefer it, but like this I cannot get a meaning. ... I have always

questioned why to calculate with letters and who invented something as strange as math! (Germany)

3.5.2 Collaboration and discussion might be a very good means in the struggle for understanding and meaning

The German teacher usually did not encourage collaboration, as 'assessment has to be individually' done anyway. He therefore also does not usually organise collaborative work besides some very few exceptions. Students more or less have to hide if they prefer to collaborate, even if they have developed collaboration into a very successful working pattern and found out some very good reasons for collaboration:

Interviewer: Do you work together on these problems?
Steffi: Yes, we do always. We talk about them as well. We help each other.
Sharon: Each one says a bit and then we agree on a happy medium.
Steffi: We work through together to see where a mistake is when we have different results or we just share work.
Sharon: Because I can actually multiply quite quickly on paper or something, you know, and then she does, she can write it down quickly, so we really share the work.... – But you can't do in a test. Yeah, really very impractical.
Steffi: It really helps to talk about because you recognize the mistakes before, well before it is done on the board; if you can go through the problems well with someone and find the mistakes, or where the mistakes have been made, it is really better. You put together what you know, this makes you secure and you feel competent, even at the black board! (Germany)

The Hong Kong teacher is referred to by students as encouraging or allowing student discussion in groups as an explicit value.

Michael: I think that this part is quite important. That is when students did not understand, they should er...discuss.
Interviewer: What was happening?
Michael: Er...her- and I, she asked me a question.//...
Interviewer: //Who asked you a question?
Michael: Natalie asked me.
Interviewer: Ha, what she asked you? Do you remember?

Michael: She asked me how to calculate that question and I said that she is stupid.(laughing)

Interviewer: Which question? (laugh)

Michael: I forgot, but she asked the one at the back and she was doing, she ...had not done. I said that she disturbed my rest.//...

Interviewer: //Disturbing your rest.(laugh)

Michael: Yep. I was closing my eyes and having a rest, then I said that you were stupid. It was like this. I have made her a new nickname. Then- then we called her by this nickname. I think that this is better because there is a gap between teacher and us. With a gap, we cannot ask so happily- cannot ask so easily. Asking our classmates can be very natural and easy and learn- I think we can learn faster. (Hong Kong)

REFERENCES

Bernstein, B., 1990, *The Structuring of Pedagogic Discourse*, Routledge, London.

Bishop, A., 1999, Mathematics Teaching and Values Education – An Intersection in Need for Research, Part 2, *Zentralblatt für Didaktik der Mathematik*, **99**(1):1-4.

Bishop, A.J., 1988, *Matematical Enculturation*, Kluwer Academic Publishers, Dordrecht, The Netherlands.

Burton, L., 1999, The Practices of Mathematicians: What Do They Tell Us about Coming to Know Mathematics?, *Educational Studies in Mathematics*, **37**:121-143.

Clarke, D., 2002a, The Learner's Perspective Study: Exploiting the Potential for Complementary Analyses. Paper presented as part of the symposium *Primary Research, Secondary Research and Research Synthesis: Theory, Value and Linkage* at the Annual Conference of the American Educational Research Association, April 2-5, New Orleans.

Clarke, D. et al, 2000b, Mathematics Classroom Practice: The Learners' Perspective. in: *Proceedings of the 10th Annual Conference of SAARMSTE* (Southern African Association for Research in Mathematics, Science and Technology Education), C. Lubisi and C. Malcolm, eds., University of Natal, Durban.

Clarke, D.J., ed., 2001, *Perspectives on Meaning in Mathematics and Science Classrooms*. Kluwer, Dordrecht.

Drew, P., and Heritage, J., eds., 1992, *Talk at Work. Interaction in Institutional Settings*, Cambridge University Press, Cambridge.

Ehlich, K., and Rehbein, J., 1986, *Muster und Institution. Untersuchungen zur schulischen Kommunikation*, Gunter Narr Verlag, Tübingen.

Ernest, P., 1999, Forms of Knowledge in Mathematics and Mathematics Education: Philosophical and Rhetorical Perspectives, *Educational Studies in Mathematics*, **38**(1-3):67-83.

FitzSimons, G.E., 1999, Values, Vocational Education and Mathematics: Linking Research with Practice, in: Changing Practice through Research: Changing Research through Practice, *Proceedings of the 7th Annual International Conference on Post-Compulsory*

Education and Training (Vol. 3), Centre for Learning and Work Research, Griffith University, Brisbane, pp.11-21.

Gellert, U., Jablonka, E., and Keitel, C., 2001, Mathematical Literacy and Common Sense in Mathematics Education, in: *Sociocultural Aspects of Mathematics Education*, B. Atweh et al., eds., Lawrence Erlbaum, New York, pp.57-73.

Jablonka, E., 2003a, Mathematical Literacy, in: *Second International Handbook of Mathematics Education*, A. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick and F. Leung, eds., Kluwer Academic Publishers, Dordrecht , pp.75-102.

Jablonka, E., 2003b, Constraints and Affordances of Reasoning Discourses in Mathematics Classrooms: Examples from Germany, Hong Kong and the United States. Paper presented as part of a symposium on social interaction and learning in mathematics classrooms at the 10th European Conference for Research on Learning and Instruction, Padova, Italy, Auguste 26-30, 2003.

Keitel, C., 2000, Cultural Diversity, Internationalisation and Globalisation: Challenges and Perils for Mathematics Education, in: *Cultural Diversity in Mathematics Education*, A. Ahmed, J.M. Kraemer, and H. Williams, eds., Ellis& Horwood, Chichester, pp.41-60.

Keitel, C., and Kilpatrick, J., 1999, Rationality and Irrationality of International Comparative Studies, in: *International Comparative Studies in Mathematics Education*, Kaiser, G., Huntley, I. and Luna, E., eds., Falmer, London, pp.241-257.

Krummheuer, G., and Brandt, B., 2001, *Paraphrase und Traduktion*, Beltz, Weinheim.

Lam, C.C., Wong, N.Y., and Wong, K.M.P., 1999, Students' Conception of Mathematics Learning: A Hong Kong Study, *Curriculum and Teaching*, **14**(2):27-49.

Lam, L., 2003, *Mathematics Education Reform in Hong Kong*, Department of Mathematics, Hong Kong Institute of Education, Hong Kong.

Lerman, S., 2002, Cultural, Discursive Psychology: A Sociocultural Approach to Studying the Teaching and Learning of Mathematics, *Educational Studies of Mathematics*, **46**(1-3): 87-113.

Lerman, S., and Tstsaroni, A., 1998, Why Children fail and What Mathematics Education Studies can do about it: The Role of Sociology, in: P. Gates, ed., *Proceedings of the First International Conference on Mathematics, Education and Society*, Centre for the Study of Mathematics Education, University of Nottingham, pp.26-33.

Love, E., Pimm, D., 1996, This is So: A Text on Texts, in: *International Handbook of Mathematics Education*, A. Bishop et al., eds., Kluwer Academic Publishers, Dordrecht, pp.371-410.

LPS, 2002, The Learners' Perspectives Study – An International Research Collaboration on Mathematics Classroom Practice, http://www.edfac.unimelb.edu.au/DSME/lps/.

Mullis, I.V.S., Martin, M.O., Beaton, A.E., Gonzales, E.J., Kelly, D.L., and Smith, T.A., 1997, *Mathematics Achievement in the Primary School Years: IEA's Third International Mathematics and Science Study (TIMSS)*. Boston College, Chestnut Hill.

Richards, J., 1991, Mathematics Discussions, in: *Radical Con-structivism in Mathematics Education*, E. von Glasersfeld, ed., Kluwer Academic Publishers, Dordrecht, pp.13-51.

Richards, J., 1991, Mathematics Discussions, in: *Radical Constreuctivism in Mathematics Education*, E. von Glasersfeld, ed., Kluwer Academic Publishers, Dordrecht, pp.13-51.

Seah and Bishop, [ICMI book] Trip for the Body, Expedition for the Soul: An Exploratory Survey of Tweo Asian Teachers of Mathematics in Australia.

van Oers, B., 2002, Educational Forms of Initiation in Mathematical Culture, *Educational Studies of Mathematics*, **46**(1-3):59-85.

Voigt, J., 1995, Thematic Patterns of Interaction and Sociomathematical Norms, in: *The Emergence of Mathematical Meaning: Interaction in Classroom Cultures*, P. Cobb and H. Bauersfeld, eds., Lauwrence Erlbaum, Hillsdale, N.J, pp.163-201.

Transcript protocol:

S Unidentified student

// Overlapping talk

... Pause of three or less seconds, respectively where it would be in the English version of the original

(...) Indecipherable words