Chapter 4-3

U.S. AND CHINESE TEACHERS' CULTURAL VALUES OF REPRESENTATIONS IN MATHEMATICS EDUCATION

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1. INTRODUCTION

A major goal of educational research is to improve learning opportunities for all students. Cross-national studies provide unique insights into issues in the teaching and learning of mathematics as well as diagnostic and decision-making information about how to improve students' learning. During the past several years, an attempt has been made to explore the impact of curriculum and instruction on U.S. and Chinese students' mathematical thinking. Previous studies have revealed remarkable differences between U.S. and Chinese students' mathematical thinking and reasoning (e.g., Cai, 2000; Cai & Hwang, 2002). By investigating factors that may contribute to these types of cross-national performance differences, we are just now beginning to understand these cross-national differences.

Although there is no universal agreement as to whether mathematics is a culturally bound subject, no one questions the idea that the teaching and learning of mathematics is a cultural activity (Bishop, 1988). Since teachers have an important effect on the ways students learn and think about mathematics, one may hypothesize that the differences between U.S. and Chinese students' thinking are related to the differences in their teachers' beliefs about mathematics and their conceptions about teaching mathematics. This paper is a progress report of a larger research project that examines the impact on students' thinking of U.S. and Chinese teachers' conceptions and their constructions of pedagogical representations. In particular, the purpose of this paper is to analyze U.S. and Chinese teachers' evaluations of a set of student responses in an attempt to understand U.S. and Chinese teachers' cultural values of representations and strategies in mathematics education.

2. THEORETICAL BASIS OF THE STUDY

In the field of educational research, there has been an increased interest in investigating how teaching and learning are connected (e.g., Fennema et al., 1996; Hiebert & Wearne, 1993; Stein & Lane, 1998). Rather than studying teaching and learning separately, educational researchers have started to study the mechanisms by which teaching and learning are related as well as the processes by which students construct meaning from classroom instruction. Since classroom instruction is a complex enterprise, researchers have attempted to identify important features of classroom instruction to investigate how teaching and learning are related. "Representation" is an important construct in the research about the teaching and learning of mathematics because it is both an inherent part of mathematics and an instructional aid for making sense of mathematics (Ball, 1993; NCTM, 2000; Perkins & Unger, 1994). In mathematics, a representation must necessarily be used to express any mathematical object, statement, concept, or theorem (Dreyfus & Eisenberg, 1996). "Virtually all of mathematics concerns the representation of ideas, structures, or information in ways that permit powerful problem solving and manipulation of information" (Putnam, Lampert, & Peterson, 1990, pp. 68).

2.1 Previous research on solution representations

Solution representations are both thinking and representational tools in problem solving. After a solver solves a mathematical problem, she/he communicates the thinking involved in the solution with certain representtations that express the solution processes. In other words, solution representations are the visible records generated by a solver to communicate his or her thinking about the solution processes. Clearly, solution processes can be recorded using different representations.

In previous studies related to this project, open-ended tasks were used to examine thinking and reasoning involved in Chinese and U.S. students' mathematical problem solving and problem posing (e.g., Cai, 2000; Cai & Hwang, 2002). These studies consistently revealed that Chinese students tended to use symbolic representations (e.g., arithmetic or algebraic symbols), U.S. students, on the other hand, tended to use visual representations (e.g., pictures). For example, when students were asked to find the number of blocks needed to build 20-step and 100-step staircases, over 20% of the U.S. 6th graders attempted to draw a 20-step staircase to arrive at an answer and nearly 10% of the U.S. 8th grade students tried to draw a 100-step staircase. By contrast, only a few of the Chinese 4th, 5th, and 6th graders answered the 20-step question this way and none of the Chinese students tried to draw 100-step staircase. When the U.S. and Chinese students were asked to generate mathematical problems based on a similar staircase situation, a considerable number of the U.S. students generated problems with pictures, but few Chinese students did.

Later, Cai and Hwang (2002) examined the nature of U.S. and Chinese students' generalized and generative thinking in mathematical problem solving and problem posing. Across the tasks, the Chinese students had higher rates of success than did the U.S. students. The disparities in the U.S. and Chinese students' problem-solving success rates were related to their use of different strategies and representations. Chinese students tended to choose abstract strategies and symbolic representations, while U.S. students tended to choose concrete strategies and drawing representations. If the analysis is limited to those U.S. and Chinese students who used concrete strategies, the success rates between the two samples become very similar. Therefore, the Chinese students' preference for abstract strategies seems to help them outperform the U.S. students on problems amenable to abstract strategies.

In these previous studies, the U.S. 6th grade students had not formally been taught algebraic concepts, but the Chinese 6th graders had received about 20 lessons on the topic. A recent study examined the extent to which U.S. and Chinese students' mathematical thinking is related to their opportunity to learn algebra (Cai, in press). The findings from the study showed that the Chinese 6th grade students' opportunity to learn algebra did not explain why they were less likely to use concrete visual representations than U.S. students. Even among the U.S. students who formally learned algebraic topics, a considerable number still used visual representations. In fact, U.S. 8th graders who had been taught formal algebra were more likely than Chinese 4th graders were to use concrete representations. These results suggest that we need to look beyond what was taught to understand the differences between U.S. and Chinese students' selection of strategies and representations.

2.2 Pedagogical representations

Adequate pedagogical representations play an important role in the way students learn and understand mathematics (Bransford et al., 2000; Hiebert & Carpenter, 1992). Pedagogical representations are the representtations teachers and students use in their classroom as carriers of knowledge and as thinking tools to explain a concept, a relationship, a connection, or a problem-solving process. As Dreyfus and Eisenberg (1996) indicated, "[a]ny representation will express some but not all of the information, stress some aspects and hide others" (pp. 267-268). In mathematics instruction, some representations might be more adequate than others as carriers of knowledge and thinking tools to explain a problem-solving process. Furthermore, pedagogical representations are effective in classroom instruction if they are known by students or are easily knowable. Indicators that students understand mathematics include their ability to use representations to express mathematical ideas and problems and their ability to move fluently within and between representations (Hiebert & Carpenter, 1992).

Although there is no universal agreement about what constitutes "good pedagogical representation" in mathematics teaching, no one questions the notion that teachers' beliefs, conceptions, and knowledge influence their selection of desirable pedagogical representations. There is no doubt that teachers' conceptions of what mathematics is affects their conceptions of how it should be presented (Thompson, 1992). A teacher's manner of presenting mathematics is both influenced by and indicative of what he/she believes to be most essential in it, thereby influencing the ways students understand and learn mathematics. However, we know very little about U.S. and Chinese teachers' conceptions and constructions of pedagogical representations in mathematics instruction. It is plausible that U.S. and Chinese students' use of different representations and strategies in problem solving reflects their teachers' differing views about various representations.

Although the Chinese 6th grade students were more likely than the U.S. 6th grade students to construct mathematical expressions and use symbols in their solutions, still a considerable number of them did not construct mathematical expressions or algebraic equations in their solutions. Nor did they choose to use a concrete, visual strategy. Why did these Chinese students not use concrete, visual approaches to solve the problems as the U.S. students did, since a concrete, visual strategy may provide entry-level, easily accessible tools for solving the problems? For example, about 80% of the Chinese 4th graders were unable to correctly conclude and justify that each boy gets more pizza than each girl if 2 pizzas were equally shared by 8 girls, and 1 pizza was equally shared by 3 boys. In fact, only 4% of the Chinese 4th graders used visual drawings even though a visual strategy might have benefited those Chinese students who did not use mathematical expressions (Cai, in press). Is it possible that teachers in China do not encourage visual strategies? If so, what are the Chinese teachers' cultural beliefs, if any, that made them discourage their students from using concrete, visual strategies?

A research project is currently underway to address these questions through extensive interviews and analyses of U.S. and Chinese teachers' lessons. The analysis of their lessons and interview transcripts contributes information about U.S. and Chinese teachers' cultural values of various representations from three aspects: (1) generating pedagogical representations for classroom instruction, (2) knowing students' representations and strategies in problem solving, and (3) evaluating students' representations and solution strategies. This paper reports some preliminary findings from a study analyzing how 11 U.S. and 9 Chinese teachers scored a set of 28 student responses.

3. METHOD

3.1 Selection of teachers

Eleven U.S. and 9 Chinese teachers participated in the study. The U.S. teachers were from Pennsylvania, North Carolina, and Wisconsin. The Chinese teachers were from Guiyang, Guizhou. U.S. and Chinese teachers were selected on the recommendations of a group of U.S. mathematics educators and a group of Chinese mathematics educators, respectively. All the selected U.S. and Chinese teachers were considered "distinguished" mathematics teachers in their respective regions according to local criteria. In particular, all selected U.S. teachers have taken leadership roles in their schools and/or school districts. They have led workshops or made presentations at regional or national mathematics education conferences. All of the U.S. teachers received at least one teaching award, such as "teacher leader," "district teacher of the year," or "the Presidential Award for Teaching Excellence in Mathematics and Science." All Chinese teachers have ranks of "first class teacher" or "special class teacher," the top two ranks in China for ranking teachers.

Teachers are recognized as distinguished in their respective regions because their teaching embodies the culturally accepted values of effective mathematics instruction. Therefore, the inclusion of distinguished mathematics teachers may help us understand U.S. and Chinese teachers' cultural values in mathematics education. During the time of this study, 8 of the U.S. teachers and all of the Chinese teachers were teaching 6^{th} grade mathematics; the remaining 3 U.S. teachers were teaching 7^{th} grade math. These 3 U.S. teachers had taught 6^{th} grade math the year before the study. Three of the U.S. teachers and 4 of the Chinese teachers were selected from schools that were involved in a previous study examining U.S. and Chinese students' mathematical thinking (e.g., Cai, 2000). The inclusion of these teachers allows the establishment of a link between the teachers' conceptions of mathematics and their students' mathematical thinking. All the other U.S. and Chinese teachers were from schools not involved in the previous studies.

3.2 Interview procedures

All the teachers were interviewed by asking them to score a set of 28 student responses using a general 5-point scoring rubric (0-4):

4 points - correct and complete understanding
3 points - correct and complete understanding, except for a minor error, omission, or ambiguity
2 points - partial understanding of the problem or related concept
1 point - a limited understanding of the problem or related concept
0 point - no understanding of the problem or related concept

These 28 responses consisted of various students' solutions to seven problems. Each student response had a correct answer (or a reasonable estimate for the answer) and an appropriate strategy that yielded the correct answer (or estimate), but representations and solution strategies in these responses were different. The teachers were asked to explain the reasons for their scoring. After they completed their scoring, they were asked to judge the sophistication of the representations and strategies used in the responses to each problem. It should be indicated that all the problems and student responses upon which the interview questions are based were from previous studies of U.S. and Chinese students' mathematical thinking. All interviews were videotaped.

3.3 Translation equivalence

In a cross-national study, it is absolutely essential to ensure the equivalence of the two language versions of the instruments. Although the 28 student responses were selected from students' actual work, both the Chinese and English versions of these responses were re-written by an educator to avoid possible biases and misinterpretations. To ensure the equivalency of the two versions, two people literate in both Chinese and English contributed to the translation of the student responses. One person first translated them from English into Chinese. The second person then compared the translated Chinese version with the originally-prepared Chinese version to ensure equivalence and consistency except for intentional changes involving culturally appropriate words like personal names, object names, contexts, and terminology. The presentations of the students' work and explanations were identical except that one was in Chinese and the other in English.

4. RESULTS

Table 1 below shows the means of the scores that U.S. and Chinese teachers assigned to the 28 student responses. The U.S. teachers assigned higher scores than did the Chinese teachers on a vast majority of the responses. In fact, the overall mean score for the 11 U.S. teachers is 3.47, while the overall mean score for the 9 Chinese teachers is 3.09. On 25 out of 28 student responses, the U.S. teachers gave higher scores than the group of Chinese teachers did. On 2 of the 28 student responses, the Chinese teachers gave higher scores than the U.S. teachers did, but the differences were very small. On the remaining response, the Chinese teachers (mean = 3.67) scored it much higher than the U.S. teachers did (mean = 2.73). This response involves a Number Theory Problem, which allows for multiple correct answers. The U.S. teachers scored it lower because four of them did not recognize the correctness of the answer in this response.

Table 4-3-1. Mean Scores Given by U.S. and Chinese Teachers

Response	China	US	Response	China	US
A	3.56	3.73	O	2.44	3.73
B	2.89	3.45	P	4.00	3.91
C	3.44	3.82	Q	2.44	3.82
D	3.78	3.55	R	3.44	3.73
E	2.33	2.55	S	2.89	3.27
F	3.89	3.91	T	3.78	3.91
G	3.44	3.73	U	3.00	3.82
H	1.33	1.82	V	3.78	3.82
I	2.56	3.64	W	3.89	3.91
J	3.67	3.91	X	3.56	3.64
K	1.56	2.00	Y	1.33	2.18
L	3.56	3.73	Z	3.44	3.91
M	2.67	3.36	AA	3.67	2.73
N	2.67	3.82	BB	3.56	3.73

Across the 28 responses, both U.S. and Chinese teachers showed very high internal consistency in their scoring. In fact, Cronbach's alpha is .7217 for the U.S. teachers and .7031 for the Chinese teachers. Although there is high internal consistency for both groups of teachers, the analysis of the interview transcripts reveals differences between the two groups' scoring of particular responses. In addition, the interview transcripts show there are different underlying reasons for their scoring. Generally speaking, the analysis of interview transcripts showed that the Chinese teachers focused their scoring on "what is missing", while the U.S. teachers focused their scoring on "what is there". Specific differences, categorized according to four themes, are described below.

4.1 Algebraic approach: It is valued highly, but should it be expected?

Algebraic approaches were used in 3 of the responses. For example, Response P to the Odd Number Pattern Problem, shown below, involves an algebraic approach. In Response P, the student found and used the general expression (2n – 1) to find the number of guests that entered on the n^{th} ring. To answer part C of the problem, the student set $2n - 1 = 99$ and solved for n, which is 50. All the U.S. and Chinese teachers, except 1 U.S. teacher, gave it 4 points.

Odd Number Pattern Problem: Sally is having a party. The first time the doorbell rings, 1 guest enters. The second time the doorbell rings, 3 guests enter. The third time the doorbell rings, 5 guests enter. The fourth time the doorbell rings, 7 guests enter. Keep going in the same way. On the next ring a group enters that has 2 more persons than the group that entered on the previous ring.

A. How many guests will enter on the 10^{th} ring? Explain or show how you found your answer.

B. In the space below, write a rule or describe in words how to find the number of guests that entered on each ring.

C. 99 guests entered on one of the rings. What ring was it? Explain or show how you found your answer.

Almost the all U.S. and Chinese teachers scored the responses with algebraic approaches the highest when compared to other responses to the same problem. For example, there are 5 responses (see below) for the following Map Ratio Problem:

Map Ratio Problem: The actual distance between Grantsville and Martinsburg is 54 miles. On the map, Grantsville and Martinsburg are 3 centimeters apart. On the map, Martinsburg and Rivertown are 12 centimeters apart. What is the actual distance between Martinsburg and Rivertown?

Response G: The student first found the many of miles that a centimeter on the map represents (54/3=18), then multiplied the result by 12 to get the actual distance that the 12 centimeters on the map represents (18 x 12=216).

Response H: The student used a finger to measure the distance between Martinsburg and Grantsville on the map, then used the measurement unit to measure the length between Martinsburg and Rivertown on the map and to find the number of the unit of the length. By multiplying the number of finger lengths by 54, the student found the distance between Martinsburg and Rivertown.

Response I: The student first multiplied 3 by 4 and got 12. Since 3 centimeters represent 54 miles, the actual distance represented by the 12 centimeters was 216 (4 x 54=216).

Response J: The student set up a formal proportional relationship to find the actual distance (i.e., 3/12 = 54/x, x = 216 miles).

Response K: A student used a paper clip as a unit to measure the distance between Martinsburg and Grantsville on the map, then used the measurement unit to measure the length between Martinsburg and Rivertown on the map and to find the number of the unit of the length. By dividing 54 by the number of the measurement unit between Martinsburg and Grantsville, the student found the number of actual miles per measurement unit. By multiplying the number of the measurement unit between Martinsburg and Rivertown by the number of actual miles per paper-clip unit, the student found the number of actual miles between Martinsburg and Rivertown.

Both the U.S. and Chinese teachers scored Response J the highest. A few U.S. and Chinese teachers deducted 1 point because they thought the explanation in Response J was not complete. For example, U.S. Teacher 1 wanted to see the proportion labeled and more explanation given about how the equation was set up. Chinese Teachers 2 and 3 gave a response involving the algebraic approach only 3 points because, in the response, the students did not explain what "x" meant. If these responses had included a sentence like "Let x be …," the responses would have been scored 4 points by the 2 Chinese teachers.

Although all the U.S. and Chinese teachers highly valued responses with algebraic approaches, the U.S. teachers seemed to have different expectations than the Chinese teachers did. All of the U.S. teachers, except for Teacher 6, believed that in general 6th grade students in the United States should not be expected to solve problems using algebraic approaches. For example, U.S. Teacher 9 said, "I wish my 6th graders could do this. But in our school, only 7th or 8th grade students are taught algebraic concepts, and 6th graders are only learning pre-algebra and are not expected to solve problems using this kind of approach involving x's. At this point, I am happy if they can do it no matter what they use." On the other hand, all the Chinese teachers expected their 6^{th} graders to solve problems using algebraic approaches.

4.2 Visual or concrete approach: It works, but is it efficient?

Chinese teachers consistently took the nature of the solution strategies into account in their scoring. If a response involved a visual or concrete strategy, Chinese teachers usually gave a relatively lower score even though the strategy was appropriate for the correct answer. For example, in order to find the number of blocks needed for building 5-step and 20-step staircases, Response N contains correctly drawn pictures of 5-step and 20-step staircases. The Chinese teachers gave only 2 or 3 points for this response, but 9 of the 11 U.S. teachers gave it 4 points, and the remaining two awarded 3 points. Most U.S. teachers acknowledged that the drawing in Response N was not a sophisticated strategy, and it was very time-consuming to use this strategy. However, they recognized that the drawing in Response N was a viable approach that produced correct answers. Moreover, almost all U.S. teachers stated that these visual drawings clearly showed how students thought about the problems and how they solved them.

Response Q involves the Odd Number Pattern Problem mentioned before. In Response Q, tables were created to solve the problem. In particular, a long table from ring number 1 to ring number 50 was created to list the number of guests entering on each ring, and then to determine the ring number when 99 guests entered. Like Response N, Chinese teachers gave only 2 or 3 points for Response Q, but 9 of the 11 U.S. teachers gave 4 points and the other 2 gave 3 points. Chinese teachers gave lower scores to the responses with visual or concrete approaches because "It is difficult to solve for larger numbers" (Chinese Teacher 1), "The approach is not efficient" (Chinese Teacher 4), "For [Response] N, the construction of the staircases are accurate, but the reasoning process is not as good as that in other responses" (Chinese Teacher 2), or "It is really troublesome to draw and not to find regularities among numbers" (Chinese Teacher 9).

The Chinese teachers seem to have a clear goal: students should learn more efficient strategies. The following excerpt from Chinese Teacher 7 is just one of the examples showing that Chinese teachers have such a goal: "Being able to solve a problem is good, but just the first step. Through mathematics instruction, we want students to learn generalized problem-solving methods. They should be able to 'Ju Yi Fan San' and 'Chu Lei Pang Tong (i.e., make generalizations and transfer them to other problem situations)." However, there is no evidence from the interviews that U.S. teachers have the clear goal that students should learn efficient strategies.

Instead, the U.S. teachers' goal seems to be that students solve a problem no matter what strategies they use.

Perhaps because the Chinese teachers believed that students should learn more efficient strategies, they seemed to have less internal consistency than did the U.S. teachers on scoring the responses involving drawing or making a list. The U.S. and Chinese teachers' scoring of Response U is a good example. Response U involves a drawing strategy to solve the following Hats Average Problem:

> **Hats Average Problem**: Angela is selling hats for the Mathematics Club. She sold 9 hats in Week 1, 3 hats in Week 2, and 6 hats in Week 3. How many hats must Angela sell in Week 4 so that the average number of hats sold is 7?"

In Response U, the student used a drawing to show how to use the leveling-off processes to solve the problem. The student viewed the average (7) as a leveling basis to line up the numbers of hats sold in Weeks 1, 2, and 3. Since 9 hats were sold in week 1, the drawing for Week 1 shows two extra hats beyond the average level. Since 3 hats were sold in week 2, 4 additional hats in Week 2 are needed in order to line up with the average. Since 6 hats were sold in Week 3, it needs 1 additional hat to line up with the average. In order to have enough hats to rearrange so that each week lines up with the average number of hats sold over the 4 weeks, 10 hats should be sold in Week 4.

The majority of the U.S. teachers gave it 4 points, and no one gave it less than 2 points. However, equal numbers of Chinese teachers gave it 2, 3, or 4 points. Chinese teachers seem to hold two different views regarding a response like this. At least three of the nine Chinese teachers felt that the drawing approach to leveling would be difficult to use when solving similar problems involving larger numbers, so it should not be scored 3 or 4 points. However, some other Chinese teachers maintained this approach should be scored 4 points because it shows students' creativity as well as their understanding of the averaging process.

4.3 Estimate of an answer: It is reasonable, but is it enough?

Responses H, K, and Y provided only estimates of answers. Both U.S. and Chinese teachers not only scored these responses the lowest, but also they scored them with the biggest variations. For Response H, 5 Chinese teachers scored it 2 points, 2 scored it 1 point, and 2 scored it 0 point. The U.S. teachers' scores for Response H ranged from 0 to 3 points. All the Chinese and 6 of the U.S. teachers liked the thinking processes involved in Response H and realized that the thinking processes in Response H were similar to those in Response I. However, these Chinese and U.S. teachers commented that the estimate of an answer is not enough even if the estimate is very reasonable and good. As U.S. Teacher 7 pointed out, "They've got everything to figure out the problem, but they seem not to care and don't use them. Regardless how good the estimate is, it is just a wrong answer." Four U.S. teachers gave it 3 points, citing that the approach of measuring with the paper clip is acceptable. "They could do better because of the information given to them, but I guess without using all the information they've attacked the problem well. This student proved an understanding of ratios" (U.S. Teacher 11).

The variation for each group of teachers was even bigger for Response Y than for Response H or K. Response Y involves an estimate for the following Score Average Problem:

> **Score Average Problem**: The average of Ed's ten test scores is 87. The teacher throws out the top and bottom scores, which are 55 and 95. What is the average of the remaining set of scores? Show how you found your answer.

In Response Y, the student said that the average for the remaining set of scores is between 55 and 95. But 87 is closer to 95 than 55. So the average for the remaining set of scores must be about 90. Chinese teachers gave this response either 0 or 2 points, and U.S. teachers' scores ranged from 0 to 3 points for this response. The 3 Chinese teachers who gave it 0 point expressed a concern about guessing instead of using rigorous mathematical reasoning. Eight of the 9 Chinese teachers explicitly commented that "it is a bad habit to guess for solving a problem like this and deducting points may help students overcome such a habit." The two U.S. teachers who gave it 0 or 1 point commented that students should provide precise answers for a problem like this. On the other hand, the remaining nine U.S. teachers, who gave it 3 points, thought that the response showed students' understanding of some properties about arithmetic mean. Because of that, they felt the response deserved some points.

4.4 Focus on details: It is important, but what is the consideration behind?

When Chinese teachers scored the 28 students' responses, they consistently focused on details. Were units attached to the answers? Did the students write their responses in an appropriate format? For example, in Response I (3 x 4 = 12. 4 x 54 = 216. Therefore the answer should be 216), five Chinese teachers only gave it 2 points and only one Chinese teacher gave it 4 points. In contrast, for Response G (54/3 = 18. 12 x 18 = 216.

Therefore, the answer should be 216), all the Chinese teachers scored the response 3 or 4 points. The reason some Chinese teachers scored Response I lower was that students in the response used the number "4" (3 x 4 = 12) which was not given in the problem. The following excerpt is from the interview with Chinese Teacher 4.

Teacher:	It [Response I] is 2.
Interviewer:	Okay!
Teacher:	It has no units and 3 x 4 = 12 should be 12/3 = 4.
Interviewer:	Why should 3 x 4 = 12 be 12/3 = 4?
Teacher:	Four is not there [in the problem]. Where is the 4 from? Students must have guessed mentally about the number 4. It would be very hard to guess if the number is larger. In examinations, you will always have big numbers, and I don't think students should just guess. Otherwise, their scores will be deducted in their examinations. It is better to ask them to pay attention to little things in daily practice, so they will have good habits to write their solutions.
Interviewer:	What if the student has had 12/3 = 4?
Teacher:	It is going to be 3, but still not 4, because there are no units there.
Interviewer:	What do you mean about units?
Teacher:	4 x 54 = 216. What does 216 mean? What is the unit for 216? They need to understand the units.

The Chinese teachers' concerns about the details of the written format and the inclusion of units for answers might be related to the examination culture in China. In fact, every Chinese teacher mentioned the grading criteria in city-wide or region-wide common examinations at least once. In particular, Chinese Teacher 3 referred 11 times to writing requirements in examinations in her scoring of the 28 responses. The Chinese teachers' concerns about the details of the written format and the inclusion of units in answers are also related to their beliefs about understanding mathematics. Chinese teachers seem to believe that the use of an appropriate written format and the inclusion of units in problem solving can help students develop their abilities to think logically.

In contrast, U.S. teachers were not as concerned with details about written formats. For example, in Response O, the following expressions were included to find the number of blocks needed to build 20-step staircase: 1 + 2 = 3 + 3 = 6 + 4 = 10 + 5 = 15 + 6 = 21 + 7 = 28 + 8 = 36 + 9 = 45 + 10 = 55 + 11 = 66 + 12 = 78 + 13 = 91 + 14 = 105 + 15 = 120 + 16 = 136 + 17 = 153 + 18 = 171 + 19 = 190 + 20 = 210. Eight out of the 11 U.S. teachers gave it 4 points. Such imprecise writing did not seem to bother them at all. As U.S. Teacher 10 commented, "No, it does not bother me. The little fellow just put down what he thinks in his head. Isn't it the way we think?" However, no Chinese teachers gave it 4 points. The explanation provided by Chinese Teacher 5 was typical: "The result is correct, but there are some mistakes in the writing. Two sides of an equal sign should be equal."

5. DISCUSSION

By analyzing their scoring of 28 student responses, this paper shows how cultural values of U.S. and Chinese teachers affect their appraisal of solution representations and strategies. Overall, U.S. teachers are much more lenient than Chinese teachers are in their scoring. However, U.S. teachers' leniency cannot be detected in their evaluation of students' responses involving conventional approaches, such as using algebraic equations and other mathematical expressions. In fact, almost all the U.S. and Chinese teachers valued the responses with algebraic approaches the highest when compared to other responses in the same problem. Although all the U.S. and Chinese teachers highly valued responses with algebraic approaches, it is clear that U.S. and Chinese teachers hold different curricular expectations. Chinese teachers expect 6th graders to be able to use equations to solve problems, but for U.S. teachers this expectation only applies to 7th or 8th grade students.

The U.S. teachers' leniency was reflected on their rating of responses involving both visual strategies and estimates of answers. Chinese teachers consistently took the nature of the solution strategies into account in their scoring. If a response involved a visual or concrete strategy, Chinese teachers usually gave it a relatively low score even though the strategy could be used appropriately to arrive at a correct answer. While U.S. teachers acknowledged that the drawing strategy may not be a sophisticated strategy and may be very time consuming, they appreciated the fact that drawing is often a viable approach that produces correct answers. Therefore, U.S. teachers felt that a response with an appropriate concrete drawing strategy should not be penalized. The Chinese teachers seem to have a clear goal that students should learn more generalized strategies and they expect 6th grade students to use algebraic approaches. The U.S. teachers, on the other hand, seemed to be satisfied as long as their students were able to use an appropriate strategy to solve a problem. Furthermore, the U.S. teachers believed that in general 6th grade students in the United States should not be expected to solve problems using algebraic approaches.

Chinese teachers also gave lower scores for responses involving estimation than did U.S. teachers. For Chinese teachers, if a problem includes all of the information to provide an accurate answer, it is not desirable to simply estimate the answer. For U.S. teachers, if the process of solving a problem is sound and the process shows an understanding of the concept involved, the student response should receive a high score even though only an estimate of the answer is provided. In addition, Chinese teachers seem to be much more concerned about the details of the written format and the inclusion of units for answers than U.S. teachers are. Chinese teachers believe that the use of an appropriate written format and units in problem solving can help students develop their abilities to think logically. Such details are also required on Chinese examinations.

The fact that U.S. and Chinese teachers hold differing curricular expectations is not surprising since the curricula of the two countries are very different. However, on a deeper level, the differences in expectations may reflect the differences in cultural beliefs about mathematics and the learning of mathematics. Although both the U.S. and Chinese teachers agreed that mathematics has wide applications in the real world, the true beauty of mathematics for Chinese teachers was its purity, generality, and logic. Thus, a solution strategy that lacks generality (e.g., a visual approach) should be discouraged. In contrast, U.S. teachers heavily emphasized the pragmatic nature of mathematics: as long as it works, students can choose whatever strategies they like.

The differences in U.S. and Chinese teachers' scoring of responses involving visual approaches and estimates of answers appear to suggest the different cultural values of representations in mathematics education. Cultural beliefs do not dictate what teachers do. Nonetheless, teachers do draw upon their cultural beliefs as a normative framework of values and goals to guide their teaching (Bruner, 1996). Evaluation and scoring of student responses is a routine activity for both U.S. and Chinese teachers. This study indicates that U.S. and Chinese teachers may use such a routine activity to foster students' learning in very different ways. The U.S. teachers seemed to believe that as long as students can solve a problem using whatever viable strategies are available, the students should be encouraged by giving them full credit (positive reinforcement). On the other hand, Chinese teachers seemed to use negative reinforcement to help students form good habits by deducting points for less desirable solutions or written formats.

This paper includes only some preliminary results from a larger research project. Additional analyses are in progress to better understand U.S. and Chinese teachers' conception and construction of pedagogical representations. Nevertheless, the preliminary results not only demonstrate the U.S. and Chinese teachers' differential cultural values of representations in mathematics education, but the preliminary results also suggest the feasibility of using teachers' scoring of student responses as an effective way to examine teachers' values. In addition, findings from this study show the impact of U.S. and Chinese teachers' conceptions of representations on their students' thinking. It suggests that U.S. and Chinese students' use of differential solution strategies and representations may be due, at least in part, to their teachers' different cultural values of various representations.

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